Hamiltonian Laceability in Middle Graph of Cubic Graph $(Gc)_{2n}$ and $(W_{1,n}, k)$ Graphs

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ABSTRACT

A simple connected graph G is Hamiltonian laceable if there exists a Hamiltonian path between every pair of distinct vertices at an odd distance in it. G is Hamiltonian-t-laceable(t-laceable) if there exists a Hamiltonian path in G between every pair (at least one pair) of vertices u and v in G with the property $d(u,v)=t, 1 \le t \le diamG$. In this paper we explore the Hamiltonian laceability properties of the Middle graph of the Cubic Graph $(Gc)_{2n}$ and the $(W_{1,n}, k)$ graph for $k=1$.*

Keyword: Connected graph, Middle graph, Hamiltonian-t-laceable graph, Hamiltonian-t^*-laceable graph, t-laceability number.

1. Introduction

All graphs considered in this paper are finite, simple, connected and undirected. Let u and v be two vertices in a graph G. The distance between u and v denoted by $d(u,v)$ is the length of a shortest u-v path in G. G is a Hamiltonian-t-laceable if there exists in it a Hamiltonian path between every pair of vertices u and v with the property $d(u,v)=t, 1 \le t \le diamG$, where t is a positive integer. If this property is achieved for at least one pair of vertices u and v, the graph G is termed Hamiltonian-t^*-laceable. If G is Hamiltonian-t^*-laceable for all t such that $d(u,v)=t, 1 \le t \le diamG$ then G is called a t^*-connected graph. Various results on Hamiltonian laceability properties in graphs are available in [4], [7], [8], [9], [10], [11] and [13]. In [12] the authors have discussed laceability properties of the Middle graph of the Gear graph G_n, the Fan graph $F_{1,n}$, the Wheel graph $W_{1,n}$, the Path graph P_n and the Cycle C_n. In this paper we explore the Laceability properties of the Middle graph of the Cubic graph $(Gc)_{2n}$ and $(W_{1,n}, k)$ graphs for $k=1$.

Definition 1

The Middle graph of G denoted by $M(G)$ is defined as follows.

The vertex set of $M(G)$ is $V(G) \cup E(G)$. Two vertices x, y in the vertex set of $M(G)$ are adjacent in $M(G)$ in case one of the following holds:

i) x, y are in $E(G)$ and x, y are in adjacent in G.

ii) x is in $V(G)$, y is in $E(G)$ and x, y are incident in G.

Figure 1 below illustrates the Middle graph of a cubic graph G.

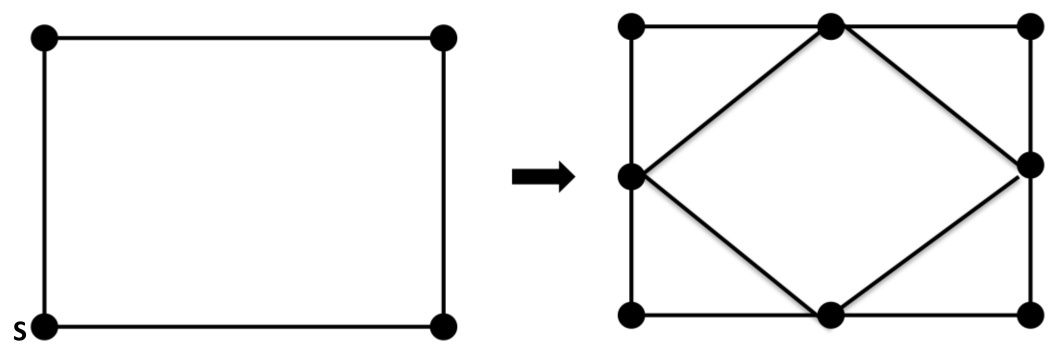

Fig1: A Graph and its Middle Graph

Definition 2

The graph $(Gc)_{2n}$ (which is a Cubic graph) is obtained by replacing the central vertex of Wheel $W_{1,n}$ by a cycle of length n.

Figure 2, below illustrates the Wheel graph $W_{1,8}$ and Cubic Graph G_{16}.

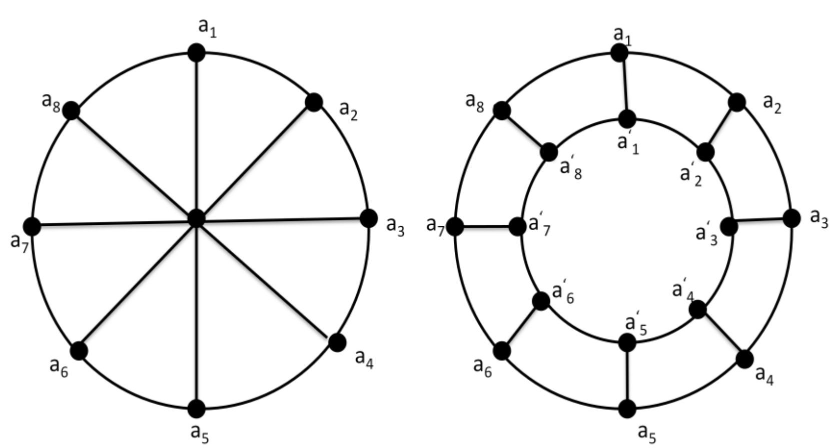

Fig2: The Wheel graph $W_{1,8}$ and the corresponding Cubic Graph $(Gc)_{16}$

Definition 3

Consider the Wheel graph $W_{1,n}$ .The graph $(W_{1,n}, k)$ is obtained from $W_{1,n}$ as follows.

Denote the vertices of $W_{1,n}$ by a_i, $1 \le i \le n$ and the central vertex as a_0. $(W_{1,n}, k)$ is obtained by taking the disjoint union of k copies of cycle C_k with the vertices $a_{k_1}, a_{k_2}, ..., a_{k_{(n-1)}}, a_{k_n}$. The edges of $(W_{1,n}, k)$ are obtained as follows:

(i) If $k = 1$, for $1 \le i \le n$, draw an edge connecting vertices a_i of $W_{1,n}$ to a_1

(ii) If $k \ge 2$, starting from $k = 2$ proceed recursively by joining the vertices $a_{(k-1)}$ to a_{k_1} by an edge for $1 \le i \le n$.

Figure 3 below illustrates the $(W_{1,6}, 2)$ graph.

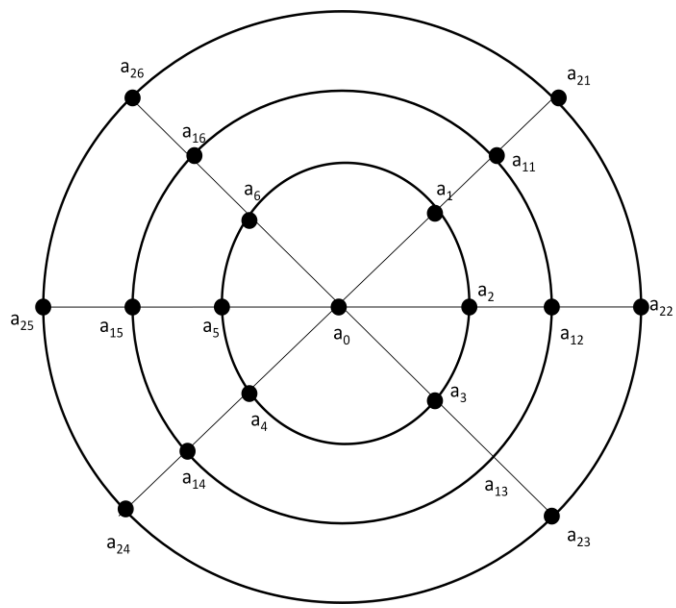

Fig3: The Graph $(W_{1,6}, 2)$

Definition 4

Let P be a path from the vertices a_i to a_j in a graph G and let P' be a path from a_i to a_k. Then the path $P \cup P'$ is the path obtained by extending the path P from a_i to a_j to a_i to a_k through the common vertex a_j (i.e., if $P: a_i ......a_j$ and $P': a_j ......a_k$, then $P \cup P': a_i ...a_j ....a_k$).

In [14], Girisha obtained the following results.

Theorem 1: The Graph $(Gc)_{2n}$ is Hamiltonian-t*-laceable for odd $n \geq 3$, where $1 \leq t \leq 4$.

Theorem 2: The Graph $(Gc)_{2n}$ is Hamiltonian-t*-laceable for even $n \geq 4$, where $2 \leq t \leq 4$.

Theorem 3: The Graph $G = (W_{1,n}, k)$, $n \geq 3$, $k \geq 1$ is Hamiltonian-$t*$-laceable for $1 \leq t \leq 3$.

We now prove the following results.

2. Results

Theorem 4: The Graph $G = M(Gc)_{2n}$ is Hamiltonian-$t*$-laceable for $t = 1, 2$ and 3 if n is odd, $n \geq 5$.

Proof: Let $V((Gc)_{2n}) = \{a_1, a_2, ..., a_n\} \cup \{a'_1, a'_2, ..., a'_n\}$ and

$E((Gc)_{2n}) = \{b_i : 1 \leq i \leq n\} \cup \{c_i : 1 \leq i \leq n-1\} \cup \{c_n\} \cup \{b'_i : 1 \leq i \leq n\}$ where b_i is the edge $a_i a_{i+1}$, c_i is the edge $a_i a'_i$, c_n is the edge $a_n a'_n$ and b'_i is the edge $a_n a'_{n-1}$. By the definition of Middle graph, $V(M((Gc)_{2n})) = V((Gc)_{2n})) \cup E((Gc)_{2n})) = \{a_i : 1 \leq i \leq n\} \cup \{a'_i : 1 \leq i \leq n\} \cup \{b_i : 1 \leq i \leq n\} \cup$

$\{c_i : 1 \leq i \leq n\} \cup \{b'_i : 1 \leq i \leq n\}$.

We have the following cases.

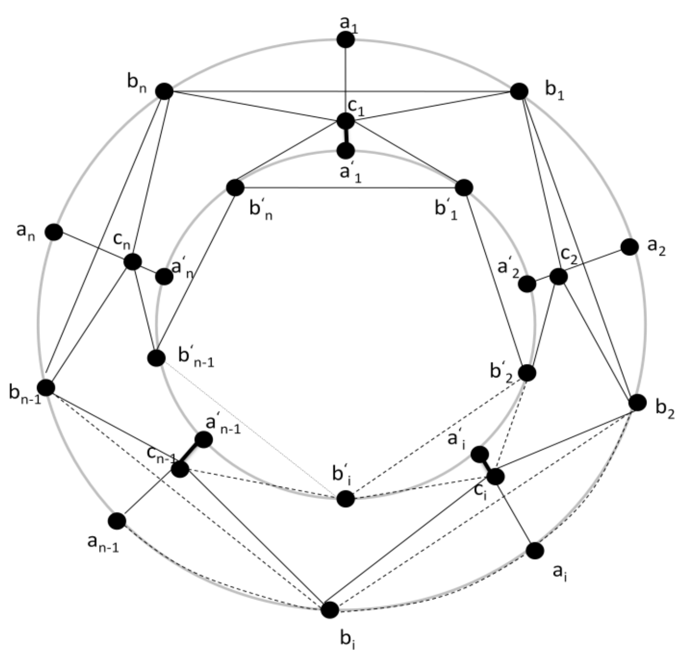

Fig 4: The Middle graph $M((Gc)_{10})$

Case (i): $t=1$

In $G, d(a_1,b_1)=1$ and the path

$P:(a_1,b_n)\cup(b_n,a_n)\cup(a_n,b_{n-1})\cup(b_{n-1},a_{n-1})\cup(a_{n-1},b_{n-2})\cup\ldots\ldots\ldots\ldots\cup(b_3,a_3)\cup(a_3,b_2)\cup(b_2,a_2)\cup(a_2,c_2)\cup(c_2,b_1')\cup(b_1',a_2')\cup\ldots\ldots\ldots\ldots\cup(c_3,b_3')\cup\ldots\ldots\ldots\ldots\cup(a'_{(n-3)},c_{(n-3)})\cup(c_{(n-3)},b'_{(n-3)})\cup(b'_{(n-3)},a'_{(n-2)})\cup\ldots\ldots\ldots\cup(a'_{(n-1)},c_{(n-1)})\cup(c_{(n-1)},b'_{(n-1)})\cup(b'_{(n-1)},a'_n)\cup(a'_n,c_n)\cup(c_n,b'_n)\cup(b'_n,a'_1)\cup(a'_1,c_1)\cup(c_1,b_1)$ is a Hamiltonian path between the vertices a_1 and b_1. Hence G is Hamiltonian-1*-laceable.

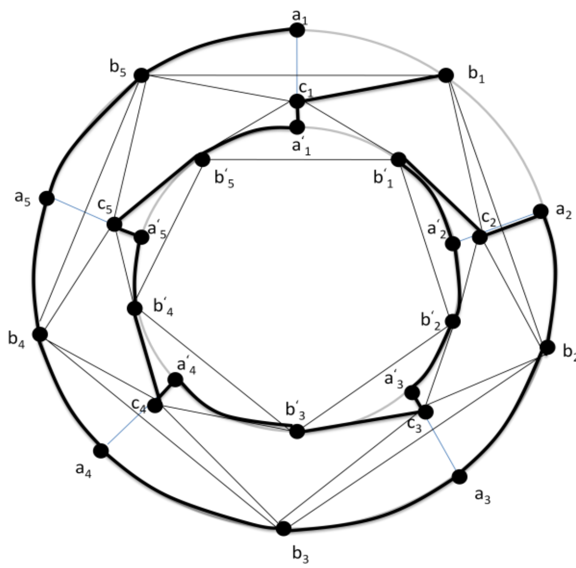

Fig 5: Hamiltonian path from the vertex a_1 to b_1 in the Middle graph $M((Gc)_{10})$

Case (ii): $t=2$

In G, $d(a_1,a_2)=2$ and the path,

$P:(a_1,b_n)\cup(b_n,a_n)\cup(a_n,b_{n-1})\cup(b_{n-1},a_{n-1})\cup(a_{n-1},b_{n-2})\cup\ldots\ldots\ldots\ldots\cup(b_3,a_3)\cup(a_3,b_2)\cup(b_2,c_2)\cup(c_2,b_1')\cup(b_1',a_2')\cup\ldots\ldots\ldots\cup(c_3,b_3')\cup\ldots\ldots\ldots\ldots\cup(a'_{(n-3)},c_{(n-3)})\cup c_{(n-3)},b'_{(n-3)})\cup(b'_{(n-3)},a'_{(n-2)})\cup\ldots\ldots\ldots\ldots\cup(a'_{(n-1)},c_{(n-1)})\cup(c_{(n-1)},b'_{(n-1)})\cup(b'_{(n-1)},a'_n)\cup(a'_n,c_n)\cup(c_n,b'_n)\cup(b'_n,a'_1)\cup(a'_1,c_1)\cup(c_1,b_1)\cup(b_1,a_2)$ is a Hamiltonian path between the vertices a_1 and a_2. Hence G is Hamiltonian-2*-laceable.

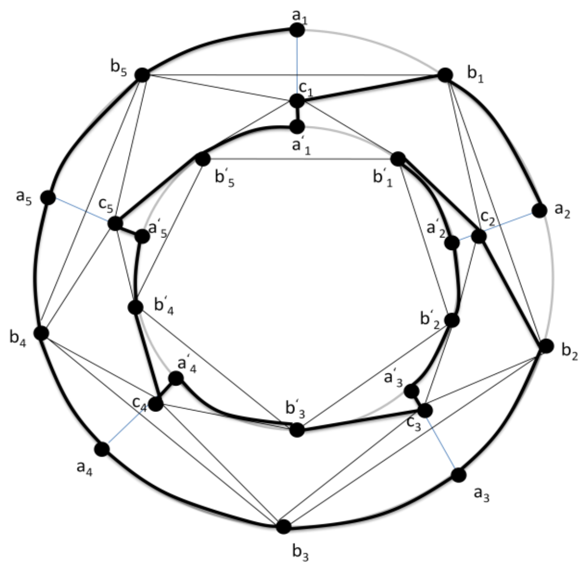

Fig 6: Hamiltonian path from the vertex a_1 to a_2 in the Middle graph $M((Gc)_{10})$

Case (iii): $t = 3$

In $G,\ d(a_1, a_3) = 3$ and the path

$P:(a_1,b_n)\cup(b_n,a_n)\cup(a_n,b_{n-1})\cup(b_{n-1},a_{n-1})\cup(a_{n-1},b_{n-2})\cup..........\,......\cup(b_5,a_5)\cup(a_5,b_4)\cup$ $(b_3,a_3)\cup(b_4,a_4)\cup(a_4,b_3)\cup(b_3,b_2)\cup(b_2,a_2)\cup(a_2,b_1)\cup(b_1,c_2)\cup(c_2,b_2')\cup(b_2',a_2')\cup$ $(a_2',b_1')\cup(b_1',a_1')\cup(a_1',c_1)\cup(c_1,b_n')\cup(b_n',c_n)\cup(c_n,a_n')\cup(a_n',b_{n-1}')\cup(b_{n-1}',c_{n-1})\cup(c_{n-1},a_{n-1}')\cup$ $(a_{n-1}',b_{n-2}')\cup(b_{n-2}',c_{n-2})\cup(c_{n-2},a_{n-2}')\cup(a_{n-2}',b_{n-3}')\cup(b_{n-3}',c_{n-3})\cup..........\,........\cup(b_5',c_5)\cup$ $(c_5,a_5')\cup(a_5',b_4')\cup(b_4',c_4)\cup(c_4,a_4')\cup(a_4',b_3')\cup(b_3',a_3')\cup(a_3',c_3)\cup(c_3,a_3)$ is a Hamiltonian path between the vertices a_1 and a_3. Hence G is Hamiltonian-3*-laceable.

Hence the proof.

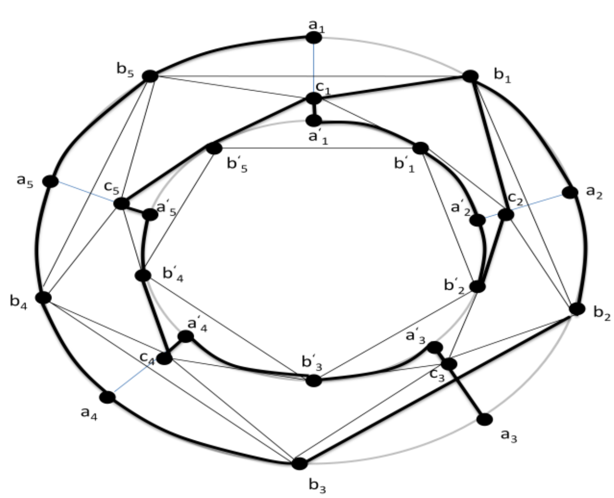

Fig 7: Hamiltonian path from the vertex a_1 to a_3 in the Middle graph $M((Gc)_{10})$

3. Remark

For n=3, the diameter of G is 2. In this case G is t*-connected. This is proved below.

Theorem 5: The Graph $G = M((Gc)_{2n})$ is t*-connected.

Proof: We have the following two cases.

Case (i): $t = 1$

In G, $d(a_1, b_1) = 1$, and the path,

$P:(a_1,b_3)\cup(b_3,a_3)\cup(a_3,b_2)\cup(b_2,a_2)\cup(a_2,c_2)\cup(c_2,b_1')\cup(b_1',a_2')\cup(a_2',b_2')\cup(b_2',a_3')\cup(a_3',c_3)\cup(c_3,b_3')\cup(b_3',a_1')\cup(a_1',c_1)\cup(c_1,b_1)$ is a Hamiltonian path between the vertices a_1 and b_1. Hence G is Hamiltonian-1*-laceable.

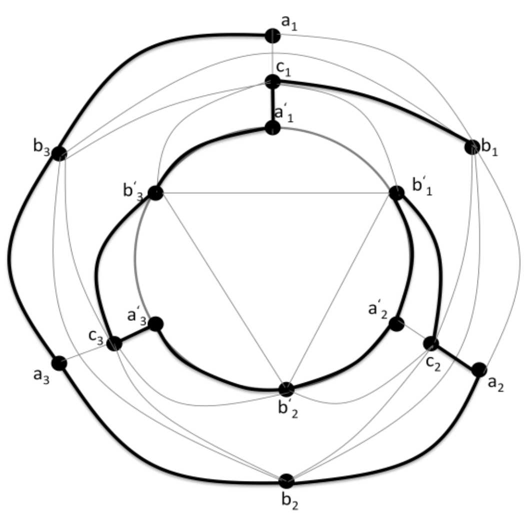

Fig 8: Hamiltonian path from the vertex a_1 to b_1 in the Middle graph $M((Gc_6))$

Case (ii): $t = 2$

In G, $d(a_1, a_2) = 2$, and the path,

$P:(a_1,b_3)\cup(b_3,a_3)\cup(a_3,b_2)\cup(b_2,c_2)\cup(c_2,b_1')\cup(b_1',a_2')\cup(a_2',b_2')\cup(b_2',a_3')\cup(a_3',c_3)\cup(c_3,b_3')\cup(b_3',a_1')\cup(a_1',c_1)\cup(c_1,b_1)\cup(b_1,a_2)$ is a Hamiltonian path between the vertices a_1 and a_2. Hence G is Hamiltonian-2*-laceable.

Hence the proof.

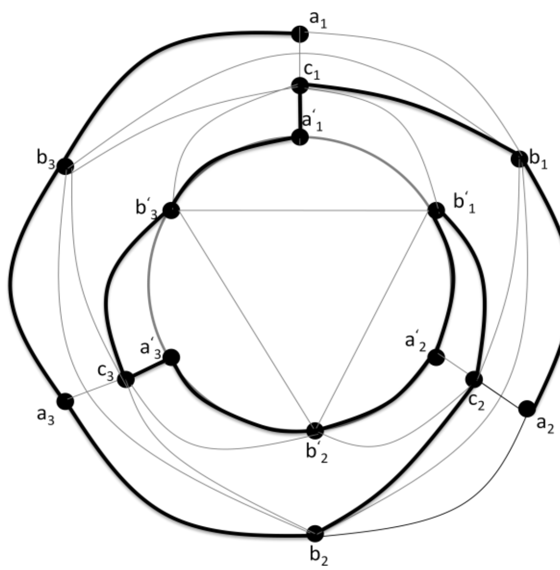

Fig 9: Hamiltonian path from the vertex a_1 to a_2 in the Middle graph $M((Gc)_6)$

Using the same vertex and edge set specified in Theorem 4, we now prove the following result.

Theorem 6: The Graph $G = M(Gc)_{2n}$ is Hamiltonian-t*-laceable for $t = 1, 2$ and 3 if n is even and $n \geq 4$.

Proof: We have the following cases.

Case (i): $t = 1$

In G, $d(a_1, b_1) = 1$ and the path

$P:(a_1,b_n)\cup(b_n,a_n)\cup(a_n,b_{n-1})\cup(b_{n-1},a_{n-1})\cup(a_{n-1},b_{n-2})\cup..........\cup(b_3,a_3)\cup(a_3,b_2)\cup(b_2,a_2)\cup(a_2,c_2)\cup(c_2,a_2')\cup(a_2',b_2')\cup(b_2',c_3)\cup(c_3,a_3')\cup(a_3',b_3')\cup..............\cup(c_{n-3},a_{n-3}')\cup(a_{(n-3)}',b_{(n-3)}')\cup(b_{(n-3)}',c_{(n-2)})\cup.......\cup(c_n,a_n')\cup(a_n',b_n')\cup(b_n',a_1')\cup(a_1',b_1')\cup(b_1',c_1)\cup(c_1,b_1)$ is a Hamiltonian path between the vertices a_1 and b_1. Hence G is Hamiltonian-1*-laceable.

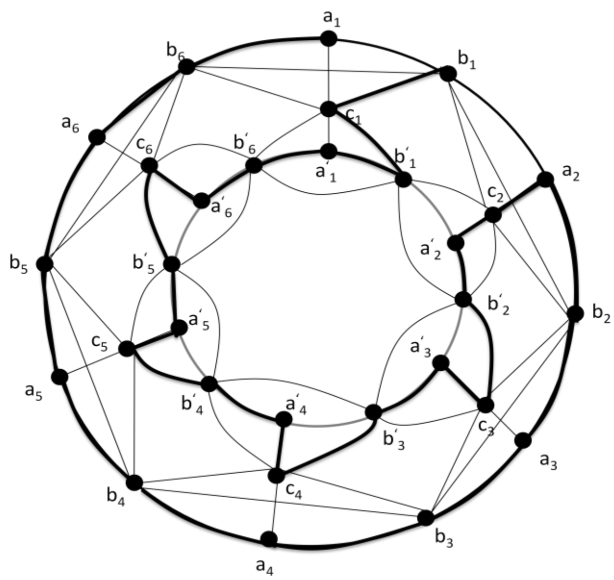

Fig 10: Hamiltonian path from the vertex a_1 to b_1 in the Middle graph $M(Gc)_{12})$

Case (ii): $t = 2$

In G, $d(a_1, a_2) = 2$ and the path

$P: (a_1, b_n) \cup (b_n, a_n) \cup (a_n, b_{n-1}) \cup (b_{n-1}, a_{n-1}) \cup (a_{n-1}, b_{n-2}) \cup .... \cup (b_3, a_3) \cup (a_3, b_2) \cup (b_2, c_2) \cup (c_2, b'_1) \cup (b'_1, a'_2) \cup (a'_2, b'_2) \cup .... \cup (a'_{(n-3)}, c_{(n-3)}) \cup (c_{(n-3)}, b'_{(n-3)}) \cup .... \cup (b'_{(n-2)}, a'_{(n-1)}) \cup (a'_{(n-1)}, c_n) \cup (c_n, b'_{(n-1)}) \cup (b'_{(n-1)}, a'_n) \cup (a'_n, c_n) \cup (c_n, b'_n) \cup (b'_n, a'_1) \cup (a'_1, c_1) \cup (c_1, b_1) \cup (b_1, a_2)$ is a Hamiltonian path between a_1 and a_2. Hence G is Hamiltonian-2*-laceable.

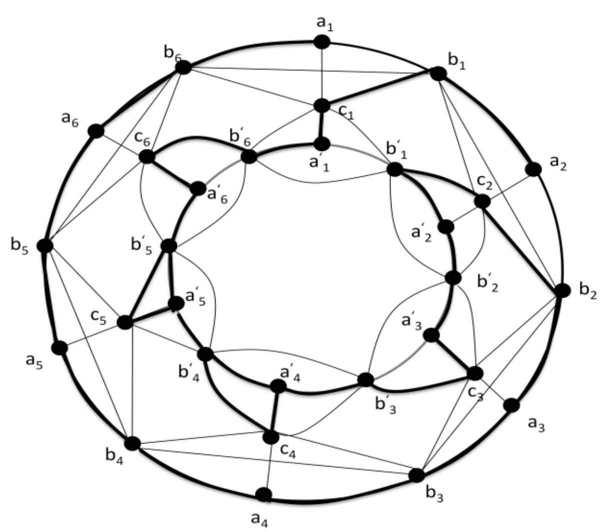

Fig 11: Hamiltonian path from the vertex a_1 to a_2 in the Middle graph $M((Gc)_{12})$

Case (iii): $t=3$

In $G,\ d(a_1,a_3)=3$ and the path

$P:(a_1,b_n)\cup(b_n,a_n)\cup(a_n,b_{n-1})\cup(b_{n-1},a_{n-1})\cup(a_{n-1},b_{n-2})\cup..........\ ......\cup(b_5,a_5)\cup(a_5,b_4)\cup(b_3,a_3)\cup(b_4,a_4)\cup(a_4,b_3)\cup..........\ ....\cup(a_2',b_2')\cup(b_2',b_1')\cup(b_1',a_1')\cup(a_1',c_1)\cup(c_1,b_n')\cup(b_n',a_n')\cup(a_n',c_n)\cup(c_n,b_{n-1}')\cup(b_{n-1}',a_{n-1}')\cup(a_{n-1}',c_{n-1})\cup..........\cup(b_5',a_5')\cup(a_5',c_5)\cup(c_5,b_4')\cup(b_4',a_4')\cup(a_4',c_4)\cup(c_4,b_3')\cup(b_3',a_3')\cup(a_3',c_3)\cup(c_3,a_3)$ is a Hamiltonian path between a_1 and a_3 .Hence G is Hamiltonian-3*-laceable.

Hence the proof.

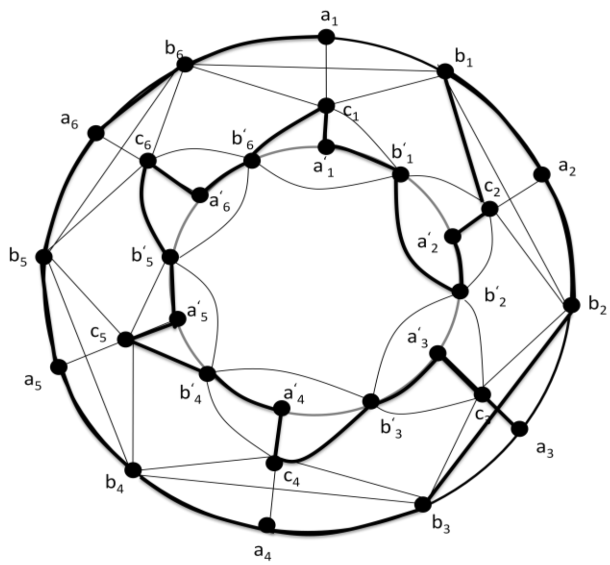

Fig 12: Hamiltonian path from the vertex a_1 to a_3 in the Middle graph $M((Gc)_{12})$

Theorem 7: The Graph $G=M(W_{1,n},k), k=1$ is Hamiltonian- $t*$ -laceable for $t=1$,2 and 3 if n is odd and $n\geq 3$.

Proof: Let $V((W_{1,n},k))=\{a_0\}\cup\{a_{11},a_{12},......,a_{1n}\}\cup\{a_1,a_2,......,a_n\}$ and

$E((W_{1,n},k))=\{b_i:1\leq i\leq n\}\cup\{a_i':1\leq i\leq n-1\}\cup\{a_n'\}\cup\{b_i':1\leq i\leq n-1\}\cup\{b_n'\}\cup\{a_{1i}:1\leq i\leq n\}$ where b_i is the edge $a_0a_i\ (1\leq i\leq n)$, a_i' is the edge $a_ia_{i+1}\ (1\leq i\leq n)$, a_n' is the edge $a_na_1\ (1\leq i\leq n)$, b_i' is the edge $a_ia_{i(i+1)}\ (1\leq i\leq n)$, b_n' is the edge $a_na_{1n}\ (1\leq i\leq n)$ and a_{1i}' is the edge $a_ia_{i(i+1)}'$.

By the definition of Middle graph $V(M((W_{1,n},k))=V((W_{1,n},k))\cup E((W_{1,n},k))=\{a_{1i}:1\leq i\leq n\}\cup\{a_i:1\leq i\leq n\}\cup\{b_i:1\leq i\leq n\}\cup\{a_i':1\leq i\leq n\}\cup\{b_i':1\leq i\leq n\}\cup\{a_{1i}:1\leq i\leq n\}$.

We have the following cases.

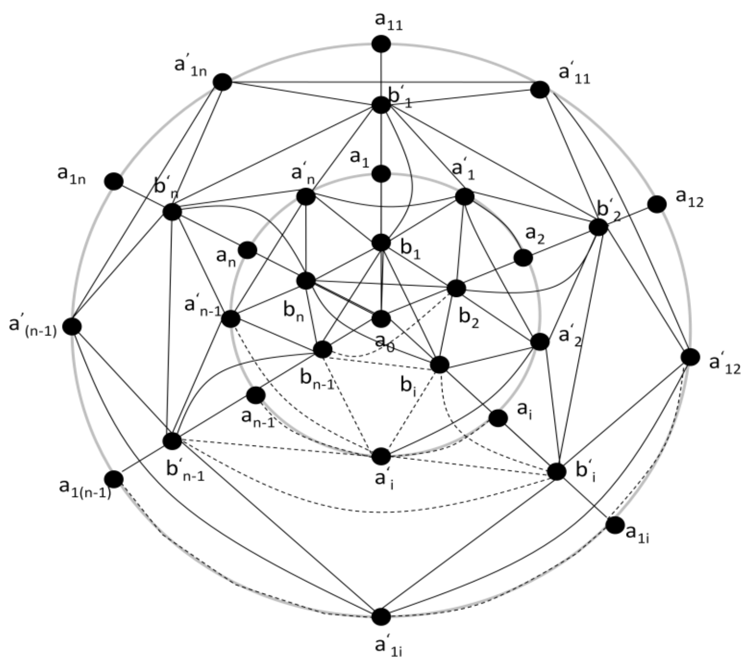

Fig 13: The graph $M(W_{1,n},k)$

Case (i): $t=1$

In G, $d(a_{k_1},a'_{k_1})=1$ and the path

$P:(a_{k_1},a'_{k_n})\cup(a'_{k_1},a_{k_n})\cup(a_{k_n},a'_{k_{(n-1)}})\cup(a'_{k_{(n-1)}},a'_{k_{(n-2)}})\cup\ldots\ldots\ldots\ldots\ldots\cup(a'_{k_3},a_{k_3})\cup(a_{k_3},a'_{k_2})\cup$ $(a'_{k_2},a_{k_2})\cup(b'_2,a'_1)\cup(a'_1,a_2)\cup(a_2,b_2)\cup(b_2,a'_2)\cup(a'_2,b'_3)\cup(b'_3,a_3)\cup(a_3,b_3)\cup(a'_2,b'_3)\cup$ $(b_3,a'_3)\cup(a'_3,b'_{n-1})\cup(b'_{n-1},a_{n-1})\cup\ldots\ldots\ldots..\cup(a'_n,a_n)\cup(a_n,b_n)\cup(b_n,a_0)\cup(a_0,b_1)\cup(b_1,a_1)\cup$ $(a_1,b'_1)\cup(b'_1,a'_{k_1})$ is a Hamiltonian path between the vertices a_{k_1} and a'_{k_1}. Hence G is a Hamiltonian-1*-laceable.

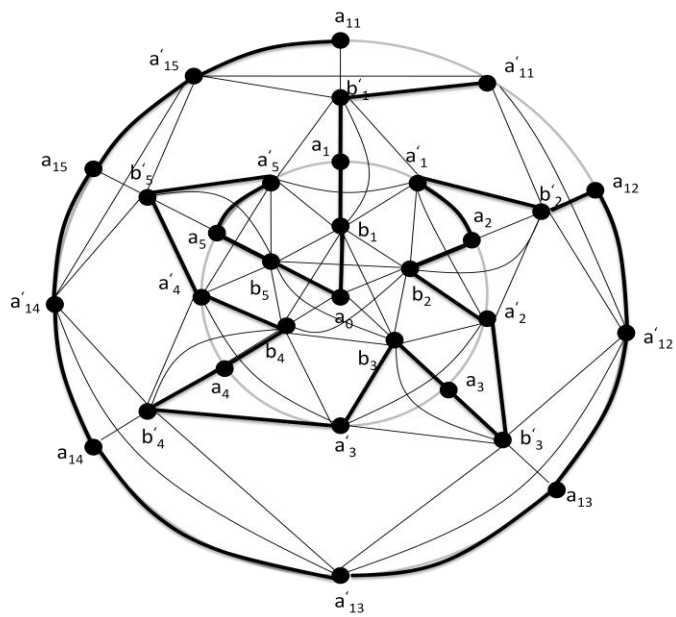

Fig 14: Hamiltonian path from the vertex a_{11} to a'_{11} in the graph $M(W_{1,5},1)$

Case (ii): $t = 2$

In G, $d(a_{k_1}, a_{k_2}) = 1$ and the path

$P:(a_{k_1}, a'_{k_n}) \cup (a'_{k_n}, a_{k_n}) \cup (a_{k_n}, a'_{k_{(n-1)}}) \cup (a'_{k_{(n-1)}}, a_{k_{(n-1)}}) \cup \ldots\ldots \cup (a'_{k_3}, a_{k_3}) \cup (a_{k_3}, a'_{k_2}) \cup (a'_{k_2}, a_{k_2}) \cup$ $(a_{k_2}, a'_{k_1}) \cup (a'_{k_1}, b'_1) \cup (a_1, b_1) \cup (a'_n, b'_n) \cup (b'_n, a_n) \cup (a_n, b_n) \cup (b_n, a'_{n-1}) \cup (a'_{n-1}, b'_{n-1}) \cup \ldots\ldots\ldots. \cup$ $(a'_3, b'_3) \cup (b'_3, a_3) \cup (a_3, b_3) \cup (b_3, a_0) \cup (a_0, b_2) \cup (b_2, a'_2) \cup (a'_2, a_2) \cup (a_2, a'_1) \cup (a'_1, b'_2) \cup$ (b'_2, a_{k_2}) is a Hamiltonian path between the vertices a_{k_1} and a_{k_2}. Hence G is a Hamiltonian-2*-laceable.

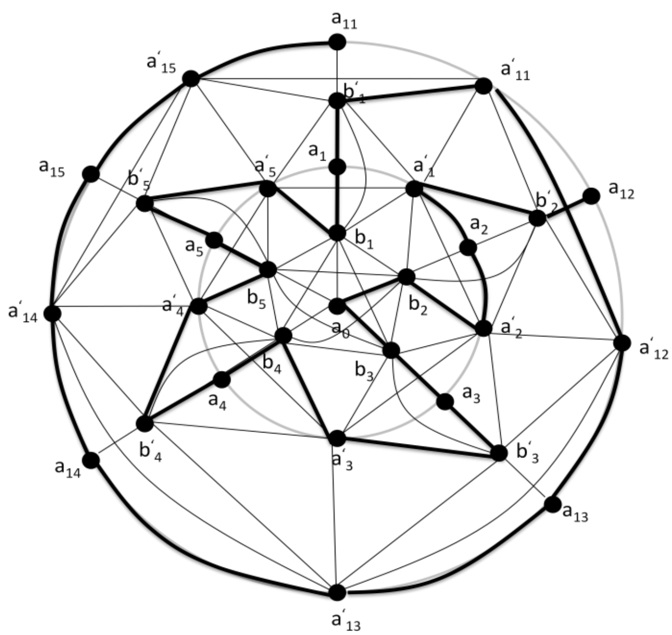

Fig 15: Hamiltonian path from the vertex a_{11} to a_{12} in the graph $M(W_{1,5},1)$

Case (iii): $t=3$

In G, $d(a_{k_1},a_0)=3$ and the path

$P:(a_{k_1},a'_{k_n})\cup(a'_{k_n},a_{k_n})\cup(a_{k_n},a'_{k_{(n-1)}})\cup(a'_{k_{(n-1)}},a_{k_{(n-1)}})\cup$ .... $\cup(a'_{k_3},a_{k_3})\cup(a_{k_3},a'_{k_2})\cup$ $(a'_{k_2},a_{k_2})\cup(a_{k_2},a'_{k_1})\cup(a'_{k_1},b'_1)\cup(b'_1,a_1)\cup(a_1,b_1)\cup(a'_n,b'_n)\cup(b'_n,a_n)\cup(a_n,b_n)\cup(b_n,a'_{n-1})\cup$ $(a'_{n-1},b'_{n-1})\cup$.......... ....$\cup(a'_3,b'_3)\cup(b'_3,a_3)\cup(a_3,b_3)\cup(b_3,a'_2)\cup(a'_2,b'_2)\cup(b'_2,a_2)\cup(a_2,a'_1)\cup$ $(a'_1,b_2)\cup(b_2,a_0)$ is a Hamiltonian path between the vertices a_{k_1} and a_0. Hence G is a Hamiltonian-3*-laceable. Hence the proof.

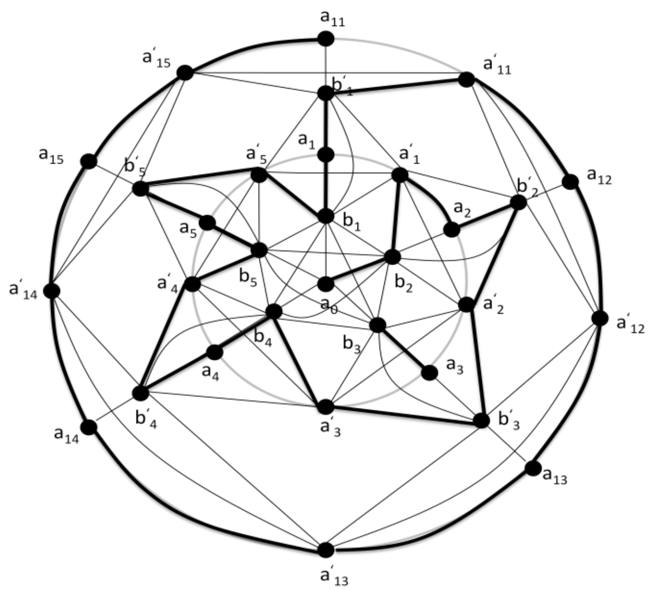

Fig 16: Hamiltonian path from the vertex a_{11} to a_0 in the graph $M(W_{1,5},1)$

Using the same vertex and edge set specified in Theorem 7, we now prove the following result.

Theorem 8: The Graph $G = M(W_{1,n},k), k = 1$, is Hamiltonian- t^* -laceable for t=1,2 and 3 if n is even and $n \geq 4$.

Proof: We have the following cases.

Case (i): $t = 1$

In G, $d(a_{k_1}, a'_{k_1}) = 1$ and the path

$P:(a_{k_1}, a'_{k_n}) \cup (a'_{k_n}, a_{k_n}) \cup (a_{k_n}, a'_{k_{(n-1)}}) \cup (a'_{k_{(n-1)}}, a'_{k_{(n-2)}}) \cup ...... \cup (a'_{k_4}, a_{k_4}) \cup (a_{k_4}, a'_{k_3}) \cup (a'_{k_3}, a_{k_3}) \cup$
$(a_{k_3}, a'_{k_2}) \cup (a'_{k_2}, a_{k_2}) \cup (a_{k_2}, b'_2) \cup (b'_2, a_2) \cup (a_2, b_2) \cup (b_2, a'_2) \cup (a'_2, b'_3) \cup (b'_3, a_3) \cup$
$(a_3, b_3) \cup .... \cup (b'_{n-2}, a_{n-2}) \cup (a_{n-2}, b_{n-2}) \cup (b_{n-2}, a'_{n-2}) \cup (a'_{n-2}, b'_{n-1}) \cup ...... \cup (a'_3, b'_{n-1}) \cup$
$(a_0, b_n) \cup (b_n, a'_{n-1}) \cup (a'_{n-1}, a_n) \cup (a_n, b'_n) \cup (b'_n, a'_n) \cup (a'_n, b_1) \cup (b_1, a_1) \cup (a_1, b'_1) \cup (b'_1, a'_{k_1})$ is a Hamiltonian path between the vertices a_{k_1} and a'_{k_1}. Hence G is a Hamiltonian-1*-laceable.

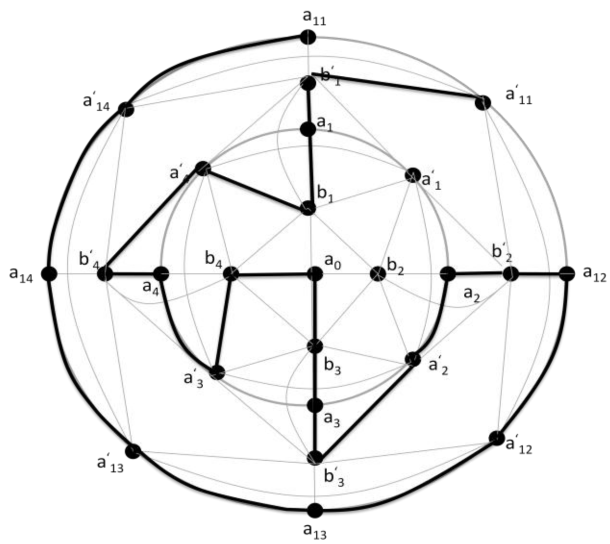

Fig 17: Hamiltonian path from the vertex a_{11} to a'_{11} in the graph $M(W_{1,4},1)$

Case (ii): $t = 2$

In G, $d(a_{k_1}, a_{k_2}) = 2$ and the path

$P:(a_{k_1},a'_{k_n})\cup(a'_{k_n},a_{k_n})\cup(a_{k_n},a'_{k_{(n-1)}})\cup(a'_{k_{(n-1)}},a_{k_{(n-1)}})\cup\ldots\ldots\cup(a'_{k_4},a_{k_4})\cup(a_{k_4},a'_{k_3})\cup(a'_{k_3},a_{k_3})\cup$ $(a_{k_3},a'_{k_2})\cup\ldots\ldots.\cup(a_1,b_1)\cup(b_1,a'_n)\cup(a'_n,b'_n)\cup(b'_n,a_n)\cup(a_n,b_n)\cup(b_n,a'_{n-1})\cup$ $(a'_{n-1},b'_{n-1})\cup\ldots..\cup(a'_3,a'_2)\cup(a'_2,b'_3)\cup(b'_3,a_3)\cup(a_3,b_3)\cup(b_3,a_0)\cup(a'_{n-1},a_n)\cup(a_0,b_2)\cup$ $(b_2,a_2)\cup(a_2,b'_2)\cup(b'_2,a_{k_2})$ is a Hamiltonian path between the vertices a_{k_1} and a_{k_2}. Hence G is a Hamiltonian-2*-laceable.

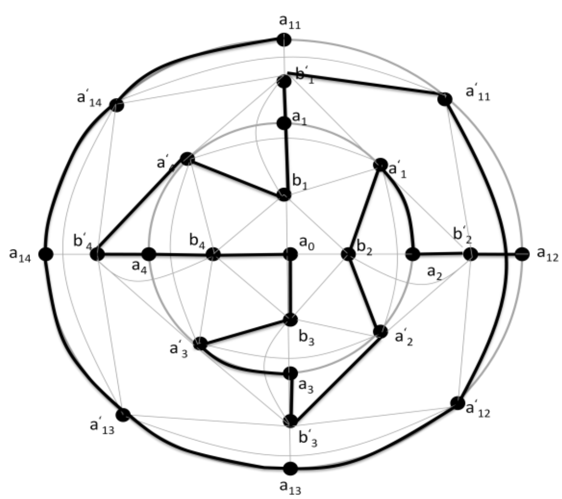

Fig 18: Hamiltonian path from the vertex a_{11} to a_{12} in the graph $M(W_{1,4},1)$

Case (iii): $t=3$

In G, $d(a_{k_1}, a_0)=3$ and the path

$P:(a_{k_1}, a'_{k_n}) \cup (a'_{k_n}, a_{k_n}) \cup (a_{k_n}, a'_{k_{(n-1)}}) \cup (a'_{k_{(n-1)}}, a_{k_{(n-1)}}) \cup .... \cup (a'_{k_4}, a_{k_4}) \cup (a_{k_4}, a'_{k_3}) \cup (a'_{k_3}, a_{k_3}) \cup (a_{k_3}, a'_{k_2}) \cup .... \cup (a_1, b_1) \cup (b_1, a'_n) \cup (a'_n, b'_n) \cup (b'_n, a_n) \cup (a_n, b_n) \cup (b_n, a'_{n-1}) \cup (a'_{n-1}, b'_{n-1}) \cup .... \cup (a'_4, b'_4) \cup (b'_4, a_4) \cup (a_4, b_4) \cup (b_4, a'_3) \cup (a'_3, b'_3) \cup (b'_3, a_3) \cup (a_3, b_3) \cup (b_3, a'_2) \cup (a'_2, b'_2) \cup (b'_2, a'_1) \cup (a'_1, a_2) \cup (a_2, b_2) \cup (b_2, a_0)$ is a Hamiltonian path between the vertices a_{k_1} and a_0. Hence G is a Hamiltonian-3*-laceable.

Hence the proof.

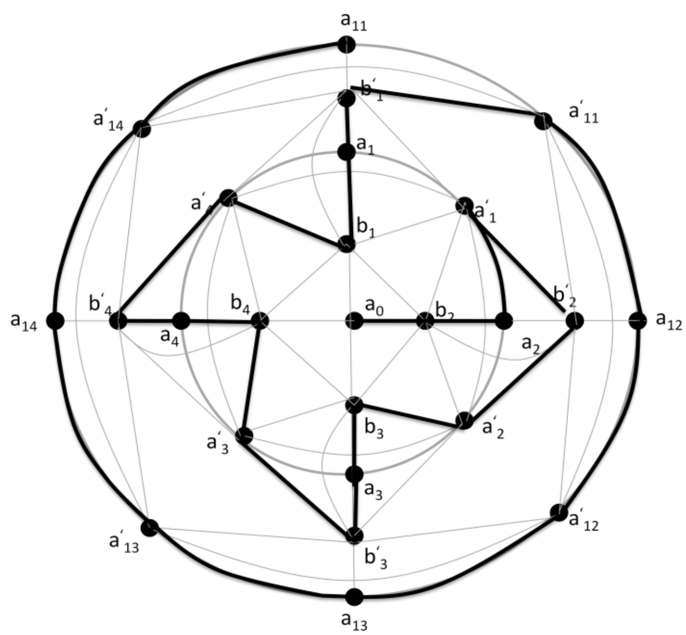

Fig 19: Hamiltonian path from the vertex a_{11} to a_0 in the graph $M(W_{1,4}, 1)$

4. Conclusion

In this paper, the Hamiltonian Laceability properties of the cubic graph $(Gc)_{2n}$ and the graph $(W_{1,n}, k)$ have been obtained. Whether these graphs are Hamiltonian-t-laceable for all t is an open problem.

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