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INTEGRATED IMPERFECT PRODUCTION PROCESS WITH EXPONENTIAL DEMAND RATE

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ABSTRACT:

This paper is developed a three layer supply chain production-inventory model for supplier, producer and retailer. The model speculates exponential demand and production rate for all member of three-layer supply chain. The model deliberates the effect of commercial policies such as exponential demand rate; production rate is demand dependent, perfect order size of raw material, unit manufacturing price and idle times in assorted sectors on integrating marketing framework. The basic assumption of the model is that manufacturing items are of perfect and imperfect quality. This technique used in different types of production industries like electrical and electronics, paper, pharmaceutical, automobiles etc. Mathematical modeling is used to derive the production rate and order size of raw material for maximum total profit of supply chain. A numerical example including the sensitivity analysis is validated to the outcomes of the production-inventory model.

Keywords: ***Exponential Demand Rate, Imperfect Production Process, Integrated, Supply Chain***

1. INTRODUCTION:

In a supply chain system, experts have given attention to elaborate inventory control problems in production industries. The challenging aspects are price, superiority, supply and elasticity. These aspects connect to manufacturing method and governance, creativity, production, services, manpower, schedule etc. This paper consider the influence of exponential production rate, manufacturing period, idle time of the framework, inventory level and sequence stuff money. Sequence stuff money is elucidated as sequence stuffed entire as a bit of the absolute quantity of sequences. In general, all manufactured objects are perfect in economic production quantity model but a specific percentage of manufactured objects are of defective quality. Salameh and Jaber [2000] developed an economic production quantity model for defective quality objects. A modification prepared by Cardens-Barron [2000] in Salameh and Jaber's [2000] model and proved that the inaccuracy only affects the perfect price of the sequence extent. Goyal and Carden-Barron [2002] accumulated Salameh and Jaber's [2000] model and implied a realistic approach to establish economic production quantity for defective quality objects. The models of Salameh and Jaber [2000] simplified by Yu et al. [2005] allowing the impact of defective quality, degeneration and superficial backlogging in the provider selection. Liu and Yang [1996] considered a solitary manufacturing method with defective route supplying two classes of imperfections: reworkable and non-reworkable objects. The reworkable objects are sent for modification, while non-reworkable objects are instantly eliminated from the framework. The model of Salameh and Jaber [2000] assimilated by Huang [2004] in a united manufacturing and despatching position. Wee et al. [2007] and Eroglu and Ozemir [2007] accumulated the model of Salameh and Jaber [2000], letting shortages. In a realistic manufacturing surroundings, the defective quality objects could be revised and reconstructed (Sana and Choudhary [2010]; Sana [2010, 2010a]; Sarkar et al. [2010]); therefore, generally manufacturing inventory charges can be condensed prominently (Hayek and Salameh [2001]; Chiu [2003]; Chiu et al. [2004, 2006]). Whole phases from delivery of raw materials to finished goods can be incorporated in a supply chain, consolidating supplier, manufacturer, retailer and customers. A three layer supply chain depends on better performance of business strategies like price, better delivery of goods, quantity deduction, quantity up and down, guarantee of purchase quantity etc. Goyal and Gunasekaran [1995] extended an integrated model for defining the economic production quantity and economic order quantity for raw materials in multi-stage manufacturing framework. Aderohunmu et al. [1995] accomplished the savings of both the retailer and customer when they monitored a supportive approach and mutual cost message along with other information in time. Thomas and Grifin [1996] studied that effective supply chain system needs arrangement and management between the system followers including producers, vendors and mediators if any. Narsimhan and Carter [1998] expressed, a good supply chain system totally depends on the combination of supply of goods and management between providers, producers and consumers. Munson and Rosenblatt [2001] developed two level supply chains to a three level supply chain, including a provider, a producer and a vendor where producer was leader of the supply chain system. Yang and Wee [2001] developed a three-layer supply chain model including supplier, manufacturer and retailer. They proved that cost reduction is more important instead of individual decision of each member of supply chain. Khouja [2003] presumed three-layer system between the followers of the supply chain that lead to significant contraction in total cost. Cardenas-Barron [2007] accumulated the model of Khouja [2003] by numerical technique, including n-stage multi-customer supply chain inventory system. Yao et al. [2007] developed an analytical model that allocate the important supply chain limits, specifically ordering costs and carrying charges, affects the inventory cost savings.

The projected model deliberates a three-layer supply chain including supplier, producer and retailer, who are reliable for execution of the raw materials into finished goods and mark them obtainable to gratify consumer's demands in time. Inventory and manufacturing policies are prepared at the supplier, producer and retailer stages. The challenge is to accomplish inventory and manufacturing policies through the supply chain so that the total anticipated profit of the supply chain is maximized.

The remnant of this paper is arranged as follows: Section 2 describes essential assumptions and notations. Section 3 describes mathematical model and solutions. Section 4 describes numerical example. Section 5 describes sensitivity analysis. Section 6 concludes the paper. A list of references is also provided.

2. ESSENTIAL ASSUMPTIONS AND NOTATIONS

The mathematical model is developed on the bases of following assumptions:

1. The demand rate is exponential increasing function of time.
2. Production rate is demand dependent
   i.e. $P(t) = \lambda D(t)$, where $D(t) = be^{at}$ *and* $\lambda > 1$
3. Production cost per unit is a function of production rate.
4. Idle time costs are assumed at supplier's and producer's level.
5. Different probability distributions functions are considered for defective items at supplier's and producer's level.

6. Single item products are considered for joint effect of supplier, producer and retailer in a three layer supply chain.
7. Replenishment rate is instantly infinite but its size is finite at producer's level.
8. Insignificant lead time.

The mathematical model is developed on the bases of following notations:

Q Supplier's replenishment lot size.

λbe^{at} Producer's production rate that is equal to supplier's demand rate.

α Supplier's proportional probability of imperfect items with probability density function $f(\alpha)$.

S_A Supplier's set up cost.

s_r Supplier's screening rate per unit time.

S_s Supplier's screening cost per unit item.

s_h Supplier's holding cost per unit time.

S_I Supplier's cost per unit idle time.

C_s Supplier's purchasing cost per unit item.

s_w Supplier's selling price per unit perfect items.

$\overline{s_w}$ Supplier's selling price per unit imperfect items.

$E(x)$ Expectation of variable x.

SAP Supplier's average profit.

$ESAP$ Supplier's expected average profit.

β Producer's proportional probability of imperfect items with probability density function $g(\beta)$.

A_p Producer's set up cost.

r_p Producer's screening rate per unit time.

S_p Producer's screening cost per unit item.

h_p Producer's holding cost per unit time.

p_I Producer's cost per unit idle time.

$P(C)$ Per unit item production cost.

w_p Producer's selling price per unit perfect item.

$\overline{w_p}$ Producer's selling price per unit imperfect item.

PAP Producer's average profit.

$EPAP$ Producer's expected average profit.

be_c^{at} Customer's demand rate.

be_r^{at} Retailer's demand rate.

r_A Retailer's set up cost.

r_h Retailer's holding cost per unit time.

r_w Retailer's selling price per unit item.

RAP Retailer's average profit.

$ERAP$ Retailer's expected average profit.

T Retailer's cycle length.

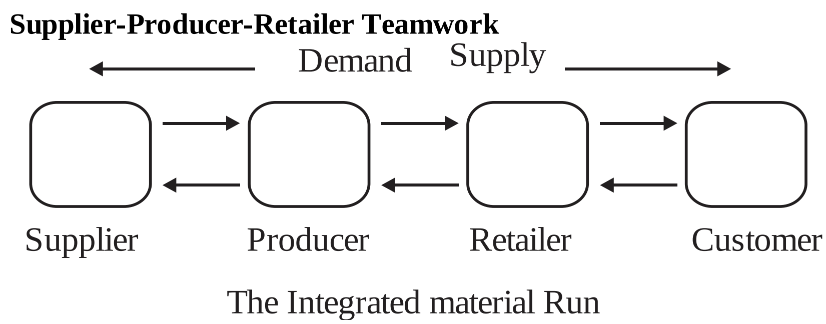

Fig. 1

3. MATHEMATICAL MODEL

In this projected model, supplier deliveries the raw materials at rate $\lambda b e^{at}$ to the producer up to manufacturing run time t_1. The imperfect objects at supplier are sent back after finished the assessment at one lot with sales rate $\overline{s_w}$ per unit item to the external supplier where the raw materials are bought. The producer meets the demand of retailer at a rate be_r^{at} upto time $kT\left(k<1\right)$. Throughout manufacturing run time $\left(0,t_1\right)$, inventory of best items stacks up with rate $\left[\left(1-\beta\right)\lambda b e^{at}-be_r^{at}\right]$.The collected inventory $\left[\left(1-\beta\right)\lambda b e^{at}-be_r^{at}\right]t_1$ gratifies the demand of retailer through $\left[t_1,kT\right]$. The imperfect items $\beta\lambda b e^{at}$ are collected at time t_1 those are sold at lesser rate in one lot. Thus, inventory cost for imperfect items up to time t_1 is measured. The demand of customers is met with rate be_c^{at} by retailer where the supply rate of produced objects is sustained up to time kT. The collected inventory at time kT is $\left(be_r^{at}-be_c^{at}\right)kT$ that gratifies the demand for the period $\left[kT,T\right]$ (See Fig. 1.). The leading differential equations at supplier, producer and retailer level are as follows:

3.1. Specific average profit of supplier

$I_S\left(t\right)$ is an inventory of perfect objects. In this case, the lot size Q is detached with rate r_s at cost S_s per unit item, after compilation of detaching, the total imperfect items are sent back to the retailers where supplier bought at a price $\overline{s_w}$ per unit item. The leading differential equation is

$$I_s^{'}\left(t\right)=-\lambda b e^{at},0\le t\le t_1 \qquad \ldots (1)$$

With $I_S\left(0\right)=\left(1-\alpha\right)Q$ and $I_S\left(t_1\right)=0$.

From equation $\left(1\right)$, we have $I_s\left(t\right)=\dfrac{\lambda b}{a}\left(1-e^{at}\right)+\left(1-\alpha\right)Q,0\le t\le t_1$ … (2)

Now, $I_S(t_1)=0$, we have $t_1=\frac{1}{a}\log\left(1+\frac{a(1-\alpha)Q}{\lambda b}\right)$ ...(3)

The inventory cost of perfect items is

$$s_h\int_0^{t_1} I_s(t)\,dt = s_h\int_0^{t_1}\left(\frac{\lambda b}{a}\left(1-e^{at}\right)+(1-\alpha)Q\right)dt = \frac{s_h}{a}\left[\begin{array}{l}\left(\frac{\lambda b}{a}+(1-\alpha)Q\right)\log\left(1+\frac{a(1-\alpha)Q}{\lambda b}\right)\\ -(1-\alpha)Q\end{array}\right]$$

Inventory cost of imperfect items is $\frac{s_h\alpha Q^2}{s_r}$

The price of screening is S_cQ .

The earning from selling the perfect and imperfect items is $\left(s_w(1-\alpha)Q+\overline{s_w}\alpha Q\right)$.

The buying price of Q objects is C_sQ .

The price of arrangement is S_A .

The idle time cost is $S_I(T-t_1)$.

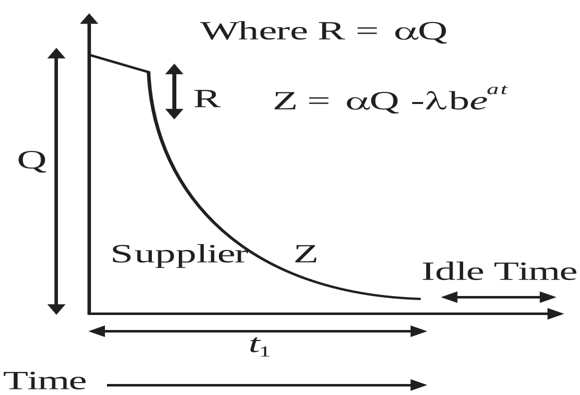

Fig. 2

The supplier's average profit is, using $t_1=\frac{1}{a}\log\left(1+\frac{a(1-\alpha)Q}{\lambda b}\right)$

$$SAP=\frac{s_w(1-\alpha)Q+\overline{s_w}\alpha Q}{T}-\frac{S_A+(S_s+C_s)Q}{T}-\frac{S_I}{T}\left(T-\frac{1}{a}\log\left(1+\frac{a(1-\alpha)Q}{\lambda b}\right)\right)$$
$$-\frac{s_h}{aT}\left\{\left(\frac{\lambda b}{a}+(1-\alpha)Q\right)\log\left(1+\frac{a(1-\alpha)Q}{\lambda b}\right)-(1-\alpha)Q+\frac{a\alpha Q^2}{s_r}\right\} \quad \ldots (4)$$

3.2. Specific average profit of producer

$I_p(t)$ is on-hand inventory of perfect items. In this stage, producer produces λbe^{at} objects per unit time where raw material is delivered with λbe^{at} rate up to the manufacturing run time t_1. Then, the differential equations are

$$I_p^{'}(t)=(1-\beta)\lambda be^{at}-be_r^{\,at},0\le t\le t_1 \quad \ldots (5)$$

with $I_p(0)=0$

and $I_p^{'}(t) = -be_r^{at}, t_1 \le t \le kT$ … (6)

with $I_p(kT) = 0$

From equations (5) and (6), we get

$$I_p(t) = \left\{\frac{b}{a}\left(1-e_r^{at}\right) - \frac{(1-\beta)\lambda b}{a}\left(1-e^{at}\right)\right\}, 0 \le t \le t_1 \quad \dots (7)$$

and

$$I_p(t) = \frac{b}{a}\left(1-e_r^{at}\right) - \frac{(1-\beta)\lambda b}{a}\left(1-e^{at_1}\right) \quad , t_1 \le t \le kT \quad \dots (8)$$

Now, $I_p(kT) = 0$, we have $t_1 = \frac{1}{a}\log\left\{1 - \frac{1-e_r^{akT}}{\lambda(1-\beta)}\right\}$

$$\therefore T = \frac{1}{ak}\log\left\{1 + \frac{a}{b}(1-\alpha)(1-\beta)Q\right\} \quad \dots (9)$$

The cost of arrangement is A_p

The earning from selling the perfect and imperfect items is

$$\left\{w_p(1-\beta)\lambda be^{at_1} + \overline{w_p}\beta\lambda be^{at_1}\right\} = \left\{w_p + \overline{w_p}\frac{\beta}{1-\beta}\right\} b\left\{\lambda(1-\beta) - 1 + e_r^{akT}\right\}$$

The cost of screening is $S_p\lambda be^{at_1} = \frac{1}{1-\beta}S_p b\left\{\lambda(1-\beta) - 1 + e_r^{akT}\right\}$

The inventory cost of perfect items is

$$HG_p = h_p\left[\int_0^{t_1} I_p(t)\,dt + \int_{t_1}^{kT} I_p(t)\,dt\right] = \frac{bh_p}{a^2}\left[\begin{array}{l}\left\{1 + akT - e_r^{akT}\right\} - \lambda(1-\beta) \\ \left\{1 + akT\right\} + \lambda(1-\beta)e^{at_1}\left\{1 + akT - at_1\right\}\end{array}\right]$$

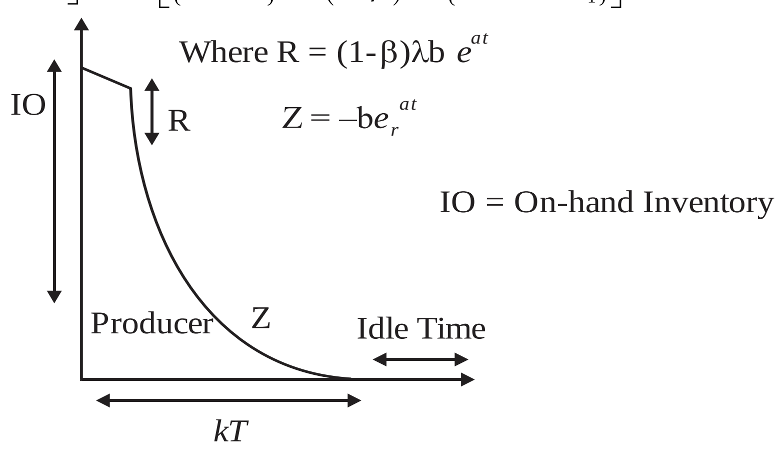

Fig. 3

The inventory cost of imperfect items is

$$HD_p = h_p\left[\int_0^{t_1} \beta\lambda b e^{at}\left(t_1 - t\right)dt + \frac{\beta\lambda^2 b^2 e^{2at_1}}{r_p}\right] \quad = \frac{h_p\beta\lambda b}{a^2}\left[\begin{array}{l} -\dfrac{1-e_r^{\ akT}}{\lambda(1-\beta)} - \log\left(1-\dfrac{1-e_r^{\ akT}}{\lambda(1-\beta)}\right) \\ +\dfrac{a^2\lambda b}{r_p}\left(1-\dfrac{1-e_r^{\ akT}}{\lambda(1-\beta)}\right)^2 \end{array}\right]$$

The total cost of manufacturing is $P(C)\lambda b e^{at_1} = P(C)\lambda b\left\{1-\dfrac{1-e_r^{\ akT}}{\lambda(1-\beta)}\right\}$

The unit manufacturing cost of finished product is

$$P(C) = s_w + \delta_p + \frac{L}{\lambda b e^{at}} + \gamma\lambda b e^{at}$$

Where w_s , the supplier's buying cost per unit raw material, which is static,

δ_p is the static cost of finished product per unit.

$\dfrac{L}{\lambda b e^{at}}$ is equally distributed labor and energy cost over production size $\left(\lambda b e^{at}\right)$.

$\gamma\lambda b e^{at}$ is per unit instrument cost of finished product which is proportional to the magnitude of manufacturing rate $\left(\lambda b e^{at}\right)$.

The producer's average profit is

$$\begin{aligned} PAP = & -\frac{S_p b\left\{\lambda(1-\beta)-1+e_r^{\ akT}\right\}}{(1-\beta)T} - \frac{A_p}{T} + \frac{\left(w_p + \overline{w_p}\dfrac{\beta}{1-\beta}\right) b\left\{\lambda(1-\beta)-1+e_r^{\ akT}\right\}}{T} \\ & -\frac{bh_p}{a^2T}\left\{\begin{array}{l} \left(1+akT-e_r^{\ akT}\right)-\lambda(1-\beta)(1+akT)+\left(\lambda(1-\beta)+e_r^{\ akT}-1\right) \\ \left(1+akT-\log\left(1-\dfrac{1-e_r^{\ akT}}{\lambda(1-\beta)}\right)\right) \end{array}\right\} \\ & -\frac{h_p\beta\lambda b}{a^2T}\left\{-\frac{1-e_r^{\ akT}}{\lambda(1-\beta)} - \log\left(1-\frac{1-e_r^{\ akT}}{\lambda(1-\beta)}\right) + \frac{a^2\lambda b}{r_p}\left(1-\frac{1-e_r^{\ akT}}{\lambda(1-\beta)}\right)^2\right\} \\ & -\frac{P(C)\lambda b}{T}\left(1-\frac{1-e_r^{\ akT}}{\lambda(1-\beta)}\right) - \frac{p_I}{T}\left(T-\frac{1}{a}\log\left(1-\frac{1-e_r^{\ akT}}{\lambda(1-\beta)}\right)\right) \end{aligned} \quad \ldots (10)$$

3.3. Specific average profit of retailer

$I_r(t)$ is on-hand inventory where producer deliver at rate $be_r^{\ at}$ to the retailer up to time kT . After receiving customers demand at rate $be_c^{\ at}$, inventory stacks up with rate $be_r^{\ at} - be_c^{\ at} = b\left(e_r^{\ at} - e_c^{\ at}\right)$ up to time kT . The collected inventory at time kT reduces and reaches to zero level at time T . Then, the leading differential equations are

$$I_r^{'}(t) = b\left(e_r^{\ at} - e_c^{\ at}\right), 0 \le t \le kT \quad \ldots (11)$$

with $I_r(0) = 0$

and $I_r^{'}(t) = -be_c^{\ at}, kT \le t \le T \quad \ldots (12)$

with $I_r(T)=0$

From equations (11) and (12), we have $I_r(t)=\frac{b}{a}\left(e_r^{\ at}-e_c^{\ at}\right), 0\le t\le kT$ … (13)

and $I_r(t)=\frac{b}{a}\left(e_r^{\ akT}-e_c^{\ at}\right), kT\le t\le T$ …(14)

Now, $I_r(T)=0 \Rightarrow k=\frac{be_c^{\ at}}{be_r^{\ at}}<1$ as $e_r^{\ at}>e_c^{\ at}$

For possibility of the model, $t_1\le kT<T$ must be gratified. As stated, $kT<T$ grasps as $k<1$.

Now, $t_1\le kT$, we have $t_1\le\frac{(1-\beta)\lambda be^{at}t_1}{be_r^{\ at}}, \Rightarrow (1-\beta)\lambda e^{at}\ge e_r^{\ at},$

$\Rightarrow \lambda E(1-\beta)e^{at}\ge e_r^{\ at}$ … (15)

The cost of arrangement is r_A

The earning from selling items is $r_w be_c^{\ at}T$.

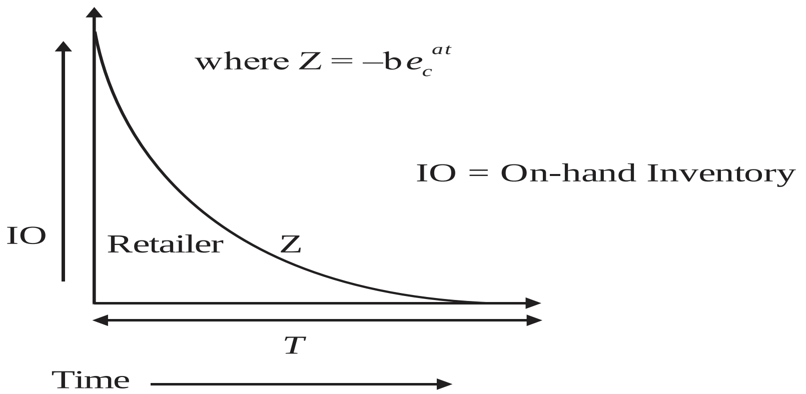

Fig. 4

The cost of buying objects is $w_p be_c^{\ at}T$.

The cost of inventory is

$$H_r=r_h\left[\int_0^{kT} I_r(t)\,dt+\int_{kT}^{T} I_r(t)\,dt\right] \quad =r_h\frac{b}{a^2}\left\{e_r^{\ akT}(1+aT-akT)-e_c^{\ aT}\right\}$$

The retailer’s average profit is

$$RAP=r_w be_c^{\ at}-\frac{r_A}{T}-w_p be_c^{\ at}-\frac{r_h b}{a^2T}\left\{e_r^{\ akT}(1+aT-akT)-e_c^{\ aT}\right\} \quad \text{… (16)}$$

3.4. Leader-Follower Association

In this case, producer is the leader; supplier and retailer are the followers. The producer bids followers to (i) buyback of the imperfect items, (ii) limited replacement charges with one time ordering cost (iii) assign buying costs, (iv) count the idle times of followers of the chain and (v) regular delivery agreement at every phase.

From equation (10), using $t_1=\frac{1}{a}\log\left(1+\frac{a(1-\alpha)Q}{\lambda b}\right), be_r^{\ akt}=be_c^{\ at}$ and $T=\frac{1}{ak}\log\left\{1+\frac{a}{b}(1-\alpha)(1-\beta)Q\right\}$

The producer’s expected average profit is

$$E[PAP] = \frac{ak\lambda b\left(w_p E(1-\beta)+\overline{w_p}E(\beta)-S_p-P(C)-\dfrac{h_p E(\beta)\lambda b}{r_p}\right)-akA_p}{\log\left\{1+\dfrac{a}{b}E(1-\alpha)E(1-\beta)Q\right\}}$$

$$+\frac{ak\left\{\left(w_p-\dfrac{h_p}{a^2}\right)E(1-\beta)-\left(S_p+P(C)\right)+\left(\overline{w_p}-\dfrac{h_p}{a^2}-\dfrac{2h_p\lambda b}{r_p}\right)E(\beta)\right\}aE(1-\alpha)Q}{\log\left\{1+\dfrac{a}{b}E(1-\alpha)E(1-\beta)Q\right\}}$$

$$-\frac{akh_p a^2 E(\beta)E\left((1-\alpha)^2\right)Q^2}{r_p\log\left\{1+\dfrac{a}{b}E(1-\alpha)E(1-\beta)Q\right\}}-ak\left(\frac{bh_p}{a^2}+\frac{h_p}{a}E(1-\alpha)E(1-\beta)Q+\frac{p_I}{ak}\right)$$

$$+\frac{ak\left(\dfrac{bh_p}{a^2}\lambda E(1-\beta)+\dfrac{h_p}{a}E(1-\alpha)E(1-\beta)Q+\dfrac{h_p E(\beta)\lambda b}{a^2}+\dfrac{p_I}{a}\right)\log\left(1+\dfrac{aE(1-\alpha)Q}{\lambda b}\right)}{\log\left\{1+\dfrac{a}{b}E(1-\alpha)E(1-\beta)Q\right\}}$$

$$E[PAP]=\frac{A}{\log(1+BQ)}[D+FGQ-HQ^2-(J+LQ+S)\log(1+BQ) \\ +(K+LQ+M+N)\log(1+PQ)] \quad \ldots(17)$$

where $A=ak$, $B=\dfrac{a}{b}E(1-\alpha)E(1-\beta)$, $M=\dfrac{h_p E(\beta)\lambda b}{a^2}$, $N=\dfrac{p_I}{a}$, $P=\dfrac{aE(1-\alpha)}{\lambda b}$, $G=aE(1-\alpha)$,

$$D=\left\{\lambda b\left(w_p E(1-\beta)+\overline{w_p}E(\beta)-S_p-P(C)-\frac{h_p E(\beta)\lambda b}{r_p}\right)-A_p\right\}$$

$$F=\left\{\left(w_p-\frac{h_p}{a^2}\right)E(1-\beta)+\left(\overline{w_p}-\frac{h_p}{a^2}-\frac{2h_p\lambda b}{r_p}\right)E(\beta)-\left(S_p+P(C)\right)\right\},\ S=\frac{p_I}{ak}\ \ H=\frac{h_p a^2 E(\beta)E\left((1-\alpha)^2\right)}{r_p},$$

$$L=\frac{h_p}{a}E(1-\alpha)E(1-\beta),K=\frac{bh_p}{a^2}\lambda E(1-\beta),J=\frac{bh_p}{a^2}$$

From equation (4), using $t_1=\dfrac{1}{a}\log\left(1+\dfrac{a(1-\alpha)Q}{\lambda b}\right)$, $be_r^{akt}=be_c^{at}$ and $T=\dfrac{1}{ak}\log\left\{1+\dfrac{a}{b}(1-\alpha)(1-\beta)Q\right\}$

The supplier's expected average profit is

$$E[SAP]=\frac{akQ\left(E(1-\alpha)\left(s_w-\dfrac{s_h}{a}\right)+E(\alpha)\overline{s_w}-S_s-C_s\right)-ak\left(S_A-\dfrac{aE(\alpha)Q^2}{s_r}\right)}{\log\left\{1+\dfrac{a}{b}E(1-\alpha)E(1-\beta)Q\right\}}-S_I$$

$$-\frac{k\left(\frac{\lambda b}{a}s_h+E\left(1-\alpha\right)Qs_h-S_I\right)\log\left(1+\frac{aE\left(1-\alpha\right)Q}{\lambda b}\right)}{\log\left\{1+\frac{a}{b}E\left(1-\alpha\right)E\left(1-\beta\right)Q\right\}} \qquad \dots (18)$$

From equation (16), using $t_1=\frac{1}{a}\log\left(1+\frac{a\left(1-\alpha\right)Q}{\lambda b}\right)$, $be_r^{akt}=be_c^{at}$ and $T=\frac{1}{ak}\log\left\{1+\frac{a}{b}\left(1-\alpha\right)\left(1-\beta\right)Q\right\}$

The retailer's average profit is

$$RAP=r_wbe_r^{akT}-\frac{r_A}{T}-w_pbe_r^{akT}-\frac{r_hb}{a^2T}\left\{e_r^{akT}\left(1+aT-akT\right)-e_r^{akT}\right\}$$

$$=b\left\{r_w-w_p-\frac{r_h}{a}\left(1-k\right)\right\}e_r^{akT}-\frac{r_A}{T}$$

$$=b\left\{r_w-w_p-\frac{r_h}{a}\left(1-k\right)\right\}\left\{1+\frac{a}{b}\left(1-\alpha\right)\left(1-\beta\right)Q\right\}-\frac{akr_A}{\log\left\{1+\frac{a}{b}\left(1-\alpha\right)\left(1-\beta\right)Q\right\}}$$

$$=b\left\{r_w-w_p-\frac{r_h}{a}\left(1-k\right)\right\}+a\left\{r_w-w_p-\frac{r_h}{a}\left(1-k\right)\right\}\left(1-\alpha\right)\left(1-\beta\right)Q$$

$$-\frac{akr_A}{\log\left\{1+\frac{a}{b}\left(1-\alpha\right)\left(1-\beta\right)Q\right\}}$$

The retailer's expected average profit is

$$\text{E [RAP]}=b\left\{r_w-w_p-\frac{r_h}{a}\left(1-k\right)\right\}+a\left\{r_w-w_p-\frac{r_h}{a}\left(1-k\right)\right\}E\left(1-\alpha\right)E\left(1-\beta\right)Q$$

$$-\frac{akr_A}{\log\left\{1+\frac{a}{b}E\left(1-\alpha\right)E\left(1-\beta\right)Q\right\}} \qquad \dots (19)$$

Solution:

For optimum value of EAPP [Q], we have

$$\frac{\partial EPAP}{\partial Q}=\frac{A}{\log(1+BQ)}\begin{bmatrix}FG-2HQ-L\log(1+BQ)+(K+M+N+LQ)\log P\\+L\log(1+PQ)-(J+LQ+S)\log B\end{bmatrix}$$

$$-\frac{AB}{\left(\log(1+BQ)\right)^2}\begin{bmatrix}D+FGQ-HQ^2+\left(K+M+N+LQ\right)\log\left(1+PQ\right)\\-\left(J+LQ+S\right)\log\left(1+BQ\right)\end{bmatrix}$$

We consider $\frac{\partial EPAP}{\partial Q}=0$, which give us the value of Q

$$Q=\frac{\left(H+L\log B-L\log P\right)-\sqrt{\begin{matrix}(-H-L\log B+L\log P)^2-(-BH-L\left(\log B\right)^2+BL\log P)\\(-BD+FG-K\log B-M\log B-N\log B\\+K\log P+M\log P+N\log P)\end{matrix}}}{(-BH-L\left(\log B\right)^2+BL\log P)} \qquad \dots (20)$$

Again, we have the value of

$$\frac{\partial^2 EPAP}{\partial Q^2}=\frac{-2A}{\left(\log(1+BQ)\right)^3}\left[\begin{array}{l}\left(\log(1+BQ)\right)^2\left(H+L\log B-L\log P\right)\\+B\log(1+BQ)\\\left(\begin{array}{l}FG-2HQ-L\log(1+BQ)+L\log(1+PQ)\\+(K+M+N+LQ)\log P-(J+LQ+S)\log B\end{array}\right)\\+B^2\left(\begin{array}{l}-D-FGQ+HQ^2-(K+M+N+LQ)\log(1+PQ)\\+(J+LQ+S)\log(1+BQ)\end{array}\right)\end{array}\right]$$

Thus $\dfrac{\partial^2 EPAP}{\partial Q^2}<0$ grasps at Q satisfying inequality (15), then EPAP$\left[Q\right]$ is maximum.

3.5. Integrated expected average profit

The integrated expected average profit of the supply chain is

$$\text{EIAP}=\left\{br_w-bw_p-p_I-S_I-\frac{kbh_p}{a}-\frac{br_h}{a}(1-k)\right\}+\left\{ar_w-aw_p-kh_p-r_h(1-k)\right\}E(1-\alpha)E(1-\beta)Q_1$$
$$+\frac{ak\lambda b\left(w_pE(1-\beta)+\overline{w_p}E(\beta)-S_p-P(C)-\dfrac{h_pE(\beta)\lambda b}{r_p}\right)-akA_p-akr_A}{\log\left\{1+\dfrac{a}{b}E(1-\alpha)E(1-\beta)Q_1\right\}}$$
$$+\frac{\left\{\begin{array}{l}ak\left\{\left(w_p-\dfrac{h_p}{a^2}\right)E(1-\beta)-S_p-P(C)+\left(\overline{w_p}-\dfrac{h_p}{a^2}-\dfrac{2h_p\lambda b}{r_p}\right)E(\beta)\right\}aE(1-\alpha)\\-akS_A+ak\left(E(1-\alpha)\left(s_w-\dfrac{s_h}{a}\right)+E(\alpha)\overline{s_w}-S_s-C_s\right)\end{array}\right\}Q_1}{\log\left\{1+\dfrac{a}{b}E(1-\alpha)E(1-\beta)Q_1\right\}}$$
$$-\frac{\left\{akh_pa^2E(\beta)E\left((1-\alpha)^2\right)+\dfrac{a^2kE(\alpha)}{s_r}\right\}Q_1^2}{r_p\log\left\{1+\dfrac{a}{b}E(1-\alpha)E(1-\beta)Q_1\right\}}$$
$$+\frac{k\left(p_I+S_I-\dfrac{\lambda b}{a}s_h+\dfrac{bh_p}{a}\lambda E(1-\beta)+\dfrac{h_pE(\beta)\lambda b}{a}+\left(h_pE(1-\beta)-s_h\right)E(1-\alpha)Q_1\right)\log\left(1+\dfrac{aE(1-\alpha)Q_1}{\lambda b}\right)}{\log\left\{1+\dfrac{a}{b}E(1-\alpha)E(1-\beta)Q_1\right\}}$$

$$\text{EIAP}=A_1+B_1C_1Q_1+\frac{D_1+F_1Q_1-G_1Q_1^2}{\log\left(1+E_1C_1Q_1\right)}+\frac{\left(H_1+J_1Q_1\right)\log\left(1+K_1Q_1\right)}{\log\left(1+E_1C_1Q_1\right)} \quad \dots (21)$$

where $A_1 = \left\{ br_w - bw_p - p_I - S_I - \frac{kbh_p}{a} - \frac{br_h}{a}(1-k) \right\}$ $\quad B_1 = \left\{ ar_w - aw_p - kh_p - r_h(1-k) \right\}$,

$C_1 = E(1-\alpha)E(1-\beta)$, $\quad D_1 = ak\lambda b\left(w_p E(1-\beta) + \overline{w_p} E(\beta) - S_p - P(C) - \frac{h_p E(\beta)\lambda b}{r_p} \right) - akA_p - akr_A$,

$E_1 = \frac{a}{b}$,

$$F_1 = \left\{ \begin{array}{l} ak\left\{ \left(w_p - \frac{h_p}{a^2} \right) E(1-\beta) - S_p - P(C) + \left(\overline{w_p} - \frac{h_p}{a^2} - \frac{2h_p\lambda b}{r_p} \right) E(\beta) \right\} aE(1-\alpha) \\ -akS_A + ak\left(E(1-\alpha)\left(s_w - \frac{s_h}{a} \right) + E(\alpha)\overline{s_w} - S_s - C_s \right) \end{array} \right\}$$

$G_1 = \frac{1}{r_p}\left\{ akh_p a^2 E(\beta) E\left((1-\alpha)^2\right) + \frac{a^2 kE(\alpha)}{s_r} \right\}$, $\quad H_1 = k\left(p_I + S_I - \frac{\lambda b}{a} s_h + \frac{bh_p}{a}\lambda E(1-\beta) + \frac{h_p E(\beta)\lambda b}{a} \right)$,

$J_1 = k\left(h_p E(1-\beta) - s_h \right) E(1-\alpha)$,

$K_1 = \frac{aE(1-\alpha)}{\lambda b}$

$$EIAP = \frac{1}{\log(1+E_1C_1Q_1)} \left[\begin{array}{l} D_1 + F_1Q_1 - G_1Q_1^2 + (H_1 + J_1Q_1)\log(1+K_1Q_1) + (A_1 + B_1C_1Q_1) \\ \log(1+E_1C_1Q_1) \end{array} \right]$$

Solution:

Differentiating $EIAP$ partially with respect to Q_1, we have

$$\frac{\partial EIAP}{\partial Q_1} = \frac{\begin{array}{l} F_1 - 2G_1Q_1 + C_1E_1 \log(A_1 + B_1C_1Q_1) + B_1C_1 \log(1+E_1C_1Q_1) + K_1 \log(H_1 + J_1Q_1) \\ +J_1 \log(1+K_1Q_1) \end{array}}{\log(1+E_1C_1Q_1)}$$
$$- \frac{E_1C_1\left(D_1 + F_1Q_1 - G_1Q_1^2 + \log(A_1 + B_1C_1Q_1)(1+E_1C_1Q_1) + \log(H_1 + J_1Q_1)(1+K_1Q_1) \right)}{\left(\log(1+E_1C_1Q_1)\right)^2}$$

For optimum value of $EIAP[Q_1]$, put $\frac{\partial EIAP}{\partial Q_1} = 0$,

$$\Rightarrow \left[\begin{array}{l} \frac{\begin{array}{l} F_1 - 2G_1Q_1 + C_1E_1 \log(A_1 + B_1C_1Q_1) + B_1C_1 \log(1+E_1C_1Q_1) + K_1 \log(H_1 + J_1Q_1) \\ +J_1 \log(1+K_1Q_1) \end{array}}{\log(1+E_1C_1Q_1)} \\ - \frac{E_1C_1\left(D_1 + F_1Q_1 - G_1Q_1^2 + \log(A_1 + B_1C_1Q_1)(1+E_1C_1Q_1) + \log(H_1 + J_1Q_1)(1+K_1Q_1) \right)}{\left(\log(1+E_1C_1Q_1)\right)^2} \end{array} \right] = 0$$

We have

$$Q_1 = \frac{2\left(G_1 - B_1C_1^2\log E_1 - J_1\log K_1\right) - \sqrt{\begin{matrix}\left(-2G_1 + 2B_1C_1^2\log E_1 + 2J_1\log K_1\right)^2 \\ -4\begin{pmatrix}-C_1D_1E_1 + F_1 + B_1\log C_1 - C_1H_1\log E_1 + \log J_1 \\ +H_1\log K_1\end{pmatrix} \\ \left(-C_1G_1E_1 + B_1C_1^3\left(\log E_1\right)^2 + C_1J_1E_1\log K_1\right)\end{matrix}}}{2\left(-C_1G_1E_1 + B_1C_1^3\left(\log E_1\right)^2 + C_1J_1E_1\log K_1\right)} \quad \ldots (22)$$

Again, we have

$$\frac{\partial^2 EIAP}{\partial Q_1^2} = -2\left[\begin{matrix}\dfrac{G_1 - B_1C_1^2\log E_1 - J_1\log K_1}{\log\left(1+E_1C_1Q_1\right)} \\ +\dfrac{E_1C_1\begin{pmatrix}F_1 - 2G_1Q_1 + E_1C_1\log\left(A_1 + B_1C_1Q_1\right) + B_1C_1\log\left(1+E_1C_1Q_1\right) \\ +K_1\log\left(H_1 + J_1Q_1\right) + J_1\log\left(1+K_1Q_1\right)\end{pmatrix}}{\left(\log\left(1+E_1C_1Q_1\right)\right)^2} \\ -\dfrac{E_1^2C_1^2\left(D_1 + F_1Q_1 - G_1Q_1^2 + \log\left(A_1 + B_1C_1Q_1\right)\left(1+E_1C_1Q_1\right) + \log\left(H_1 + J_1Q_1\right)\left(1+K_1Q_1\right)\right)}{\left(\log\left(1+E_1C_1Q_1\right)\right)^3}\end{matrix}\right]$$

Thus, $\dfrac{\partial^2 EIAP}{\partial Q_1^2} < 0$ hold at Q_1, and then $EIAP\left[Q_1\right]$ is maximum.

4. NUMERICAL EXAMPLE:

Example 1:

The following parameters in applicable units are considered as:
$S_A = \$400$, $s_r = 185{,}000$ units per unit time, $C_s = \$20$ per unit, $S_s = \$0.6$ per unit, $b = 2$ per unit, $k = 0.5$ per unit, $s_h = \$3.5$ per unit per unit time, $S_I = \$30$ per unit time, $a = 0.7$ per unit, $s_w = \$70$ per unit, $\overline{s_w} = \$40$ per unit, $A_p = \$5000$, $r_p = 180{,}000$ units per unit time, $S_p = \$0.8$ per unit, $h_p = \$4.5$ per unit per unit time, $p_I = \$20$ per unit time, $w_p = \$600$ per unit, $\overline{w_p} = \$400$ per unit, $be_c^{at} = 300$ units, $r_A = \$4000$, $r_h = \$6$ per unit per unit time, $r_w = \$620$ per unit, $\delta_p = \$2.5$ per unit, $\gamma = \$0.02$ per unit, $\lambda = \$1.5$ per unit, $L = \$4500$, $f\left(\alpha\right) = \dfrac{1}{0.3 - 0.04}$, $0.04 < \alpha < 0.3$, $g\left(\beta\right) = \dfrac{1}{0.3 - 0.05}$, $0.05 < \beta < 0.3$.

The ideal outcome for integrated framework is $Q = 21.90$ units, $ERAP = \$2960$, $ESAP = \$164.40$, $EPAP = \$1006.80$. Total profit of the supply chain is $\$4131.20$.

$0.04 < \alpha < 0.3$

$$E\left(\alpha\right) = \frac{1}{0.3 - 0.04}\int_{0.04}^{0.3}\alpha\, d\alpha = \frac{1}{0.26}\left[\frac{\alpha^2}{2}\right]_{0.04}^{0.3} = \frac{1}{2\left(0.26\right)}\left[\left(0.3\right)^2 - \left(0.04\right)^2\right]$$

$$= \frac{1}{0.52}\left[0.09 - 0.0016\right] = \frac{0.0884}{0.52} = 0.17$$

Example 2:

The following parameters in applicable units are considered as:

The values of the parameters in Example 1 are identical except $S_I = 0 \, and \; p_I = 0$.

The ideal outcome for integrated framework is $Q = 22.04$ units, $ERAP = \$2960$, $ESAP = \$179.80$, $EPAP = \$1017$. Total profit of the supply chain is $\$4156.80$.

From Example1 and Example2, expected average profit of the supply chain with exponential demand is maximum.

5. SENSITIVITY ANALYSIS:

To know, how the optimal solution is affected by the parameters, we derive the sensitivity analysis for all parameters. From the given numerical example, we derive the finest solution for a static subset. The finest values of all parameters in the subset increases or decreases by $5\%, -5\%$ and $10\%, -10\%$. The results of total profit are existing in the following table 1.

TABLE 1

Parameters	Values	Values $S_I = 0 \, and \, p_I = 0$	Values $+5\%$ changed	Values $+10\%$ changed	Values -5% changed	Values -10% changed
S_A	400	400	420	440	380	360
s_r	185,000	185,000	194,250	203,500	175,750	166,500
C_s	20	20	21	22	19	18
S_s	0.6	0.6	0.63	0.66	0.57	0.54
b	2	2	2.1	2.2	1.9	1.8
k	0.5	0.5	0.53	0.55	0.47	0.45
s_h	3.5	3.5	3.68	3.85	3.32	3.15
S_I	30	0	31.5	33	28.5	27
a	0.7	0.7	0.74	0.77	0.66	0.63
s_w	70	70	73.5	77	66.5	63
$\overline{s_w}$	40	40	42	44	38	36
A_p	5000	5000	5250	5500	4750	4500
r_p	180,000	180,000	189,000	198,000	171,000	162,000
S_p	0.8	0.8	0.84	0.88	0.76	0.72
h_p	4.5	4.5	4.73	4.95	4.27	4.05
p_I	20	0	21	22	19	18
w_p	600	600	630	660	570	540
$\overline{w_p}$	400	400	420	440	380	360
be_c^{at}	300	300	315	330	285	270
r_A	4000	4000	4200	4400	3800	3600
r_h	6	6	6.3	6.6	5.7	5.4
r_w	620	620	651	682	589	558
δ_p	2.5	2.5	2.63	2.75	2.37	2.25
γ	0.02	0.02	0.021	0.022	0.019	0.018
λ	1.5	1.5	1.58	1.65	1.42	1.35
L	4500	4500	4725	4950	4275	4050
$E(\alpha)$	0.17	0.17	0.18	0.19	0.16	0.15
$E(\beta)$	0.175	0.175	0.18	0.19	0.17	0.16
$E(1-\alpha)$	0.83	0.83	0.87	0.91	0.79	0.75
$E(1-\beta)$	0.825	0.825	0.87	0.91	0.78	0.74
$E((1-\alpha)^2)$	0.7	0.7	0.74	0.77	0.66	0.63
Q	21.90	22.04	21.50	23.75	19.30	19.80
$ESAP$	164.40	179.80	228.02	320.05	77.90	60.86
$EPAP$	1006.80	1017	1278.85	3054.91	204.96	−2.8
$ERAP$	2960	2960	3370.37	3902.56	2420.1	2133
Total Profit	4131.20	4156.80	4877.24	7277.52	2702.96	2191.06

6. CONCLUSION:

The main conclusions draws from the sensitivity analysis are as follows: the demand rate affects the three stages of the supply chain; the production rate affects the profit of producer. The production rate is demand depended, i.e. more sensitive to increases or decreases the total profit of the supply chain. The values of other parameters are least sensitive to total profit of the supply chain. A supply chain system is based on combined financial lot sizing problem. It is focused on integrated retailer-customer inventory models. Inventory strategies are used to maximize total profit of the supply chain. In current period supply chain management has terrific effect on inventory control problem. The objective of this paper is to extend an integrated supply chain including supplier, producer and retailer. The cycle time of every phase is equal. Idle time cost is considered at supplier's and producer's level. Deliveries are sent as soon as they are produced and there is no need to wait until a complete lot is ready. The imperfect items are considered at supplier's and producer's level. After screening, the imperfect items are buyback at low price. A uniform distribution function is followed by the proportional influence of imperfect items. The cost of a unit of manufacturing is a function of manufacturing budget. We consider that producer as leader of the chain and the supplier and retailer are the followers of that chain. The total profit of producer is maximized. The integrated profit is also maximized by adding the profit of supplier, producer and retailer. Chief contributions of this research are exponential demand rate, production rate is demand dependent, idle times, finite replenishment rate and effect of imperfect items on the three-layer supply chain. The unit production cost is increases because the servicing costs of remedial and preclude maintenance is considered. Mathematica is used to derive the optimal solution. A numerical example including the sensitivity analysis is validated to the outcomes of the production-inventory model. Future research can be done for Weibull distribution deterioration under inflation, reverse logistics and shortages are allowed.

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