ESTIMATION OF MULTIPLE FREQUENCY & PHASE USING OBSERVER AND ESTIMATOR

Deepali Verma*

Amit Goriyan**

Akhilesh Singh***

***M.Tech Student, Deptt. of EEE, GRDIMT, Dehradun. (er.deepaliverma@gmail.com)**

**** Assistant Professor, Deptt. of EEE, GRDIMT,Dehradun.**

***** Assistant Professor, Department of EEE, Simant Engg. College,Pithoraghar.(Uttarakhand).**

ABSTRACT

In signal & system, designing and development of off-line and on-line estimator is the basic problem. The recent advancement in control system developed on-line frequency estimators in various operations. The complication in estimating unknown frequencies of a complex sinusoidal signal simultaneously is our basic research purpose. In our work, different estimators are analyzed on the basis of on-line estimation. The presented estimators ensure a continuous time on-line simultaneous frequency estimation, which guarantees global boundedness and convergence of the state and frequencies estimation for all initial conditions and frequencies.

Keywords: *Adaptive signal processing, Estimation, Observer, sinusoidal signal*

I.INTRODUCTION

In this, we address the problem of simultaneous online estimation of the state and the frequency of a measurable multiple sinusoidal signal composed of the sum of n sinusoidal terms given by

$$y(t) = \sum_{i=1}^{n} A_i \ sin(a_{\ i}t + \emptyset_i)$$

Here Y(t) is a signal, $A_0 \neq 0$*,the frequency* $\omega_i \neq 0$ *and and and the initial phases are unknown for i=1,2……,n. The frequencies are different from each other. The frequency estimation is a most critical problem in control theory, due to the number of practical applications in rotational mechanical processes like, disk drivers, induction motors helicopters or controlling of vibration among others ([2], [4], [6], [7]).*

The issue of estimation of the frequency of any signal has been studied extensively by means of different techniques both in the offline case [11] and the online estimation [8], but just a while ago, a globally convergent estimator was proposed in [1] on the basis of an adaptive notch (AN) filter first proposed in the discrete-time version in [8] and adapted in for the continuous-time case [2]. A key feature of this Adaptive Notch filter was the scaling of the forcing term to normalize the parameters, which does not affect stability and ensures the positivity for all time of the estimate.

The problem of simultaneous online convergent estimation of the frequency and the state is a notable problem in system & control theory. In our work, we propose a solution to this well known problem and then extend the results naturally to the case of n unknown frequencies. More precisely, we present a new estimator which ensures a continuous-time online simultaneous frequency and state estimation, securing that all signals are globally bounded and the estimation of the frequencies and the states are asymptotically correct for all initial conditions and frequency values. In short we discuss this results and the difference with the approach presented in this paper .

II. ESTIMATION USING ADAPTIVE OBSERVER

Consider a sinusoidal signal

$$y(t) = A_1 \ \sin(a_1 t + \emptyset_1)$$

For this we can generate a state model

$\dot{x}_1$(t) = x_2 (t)

$\dot{x_2}(t) = -a^2 x_1(t)$

$y(t)=\frac{k_1}{\lambda}\, x_1 \;+\; \frac{k_2}{\lambda}\, x_2$

in which the parameter a^2 is unknown, k1, k2,$\lambda \neq 0$ and the initial conditions are also unknown. The parameter is introduced here to scale the magnitude of the signals.

We are interested in the estimation of the state(x_1,x_2) and the frequency a^2.

Let state of the estimator is denoted by $(s_1, s_2, s_3)^T$.

Error $e = (e_1, e_2, e_3)^T$.

With $e_1= x_1- s_1$

$e_2= x_2 - \lambda s_2$

and $e_3= s_3 - a^2$

Proposition 1: The estimator structure can be proposed as:-

$$s'_1 = \lambda s_2 + \frac{\lambda}{k_2}\,(y - \hat{y})$$

$$s'_2 = -\frac{s_1 s_3}{\lambda} + \zeta\,(y - \hat{y}\,)$$

$$s'_3 = -\gamma s_1 (y - \hat{y})$$

$$\hat{y} = \frac{k_1}{\lambda}\, s_1 \;+\; k_2 s_2$$

Where $\lambda, \zeta, \gamma > 0 \text{ and } k_1, k_2 \neq 0$ ensure that $\lim_{t\to\infty} e = 0$.

Proof: The error dynamics takes the form:-

$$e'_1(t) = \frac{-k_1}{k_2}\, e_1$$

$$e'_2(t) = -e_1(a^2 + \zeta\, k_1) - \zeta k_2 e_2 + e_3 s_1$$

$$e'_3(t) = -\frac{\gamma}{\lambda} s_1 (k_1 e_1 + k_2 e_2)$$

Let us take Lyapunov candidate function

$$V(e) = e^T M e = e^T \begin{pmatrix} \frac{c_1}{2} & \frac{k_1}{2} & 0 \\ \frac{k_1}{2} & \frac{k_2}{2} & 0 \\ 0 & 0 & \frac{\lambda}{2\gamma} \end{pmatrix} e$$

M is Hermitian matrix, we know that **M** is definite-positive if $c_1 > 0$ and $c_1 k_2 > k_1^2$ from which k_2 must be positive.

The derivative of $V(e)$ is now calculated as

$$\dot{V}(\mathrm{e}) = \frac{c_1}{2}(\dot{e}_1)^2 + \frac{k_2}{2}(\dot{e}_2)^2 + \frac{\lambda}{2\gamma}(\dot{e}_3)^2 + k_1(\dot{e_1})(\dot{e_2})$$

$$= \frac{c_1}{2}(\dot{e}_1)(e_1) + \frac{k_2}{2}(\dot{e}_2)(e_2) + \frac{\lambda}{2\gamma}(\dot{e}_3)(e_3) + \quad k_1(\dot{e_1})(e_2) + k_1(\dot{e_2})(e_1)$$

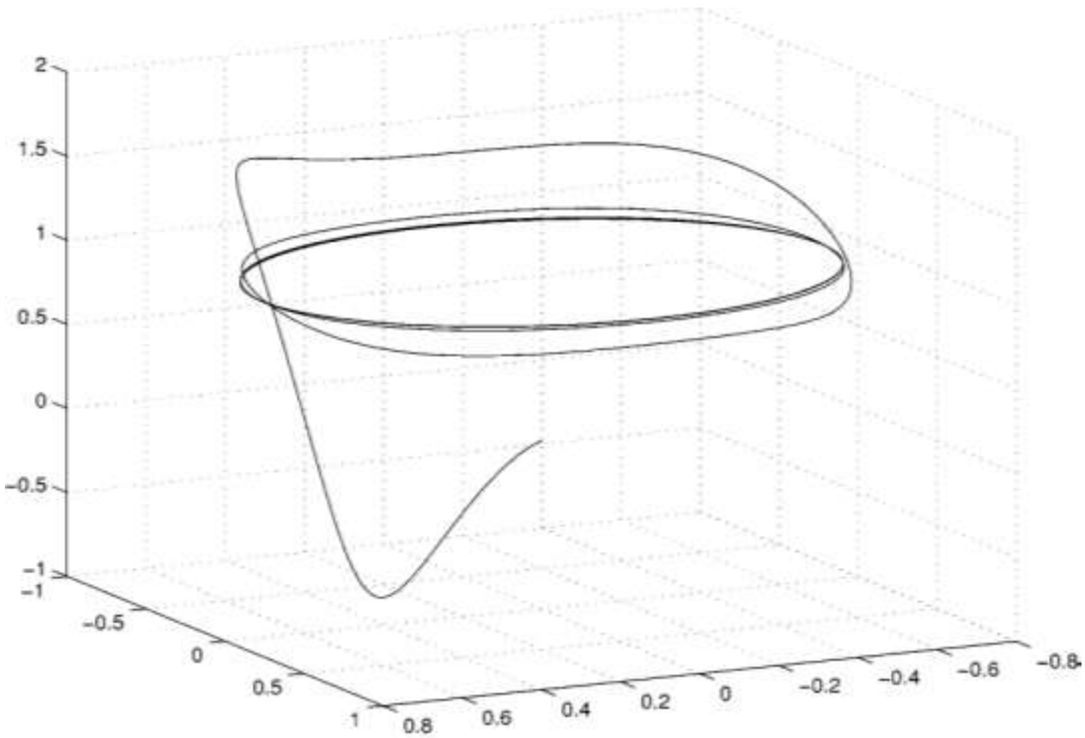

Fig. 1: Signals S_1, S_2, S_3 for a = 1

$\dot{V}(\mathrm{e}) = \left[\frac{c_1}{2}(e_1)\left\{\frac{-k_1}{k_2}e_1\right\}\right] + \left[\frac{k_2}{2}\{-e_1(a^2 + \zeta\, k_1) - \zeta k_2 e_2 + e_3 s_1\}(e_2)\right] + \left[\frac{\lambda}{2\gamma}\left\{-\frac{\gamma}{\lambda}s_1(k_1 e_1 + \right.\right.$
k2e2e3+[k1–k1 k2 e1e2]+[k1{–e1a2+ζ k1–ζk2e2+e3s1}(e1)]

By solving this equation:-

$$\dot{V}(\mathrm{e}) = -{e_1}^2\left[\frac{c_1 k_1}{k_2} + a^2 k_1 + \zeta\, {k_1}^2\right] - \zeta {k_2}^2 {e_2}^2 - e_1 e_2\left[k_2 a^2 + \frac{{k_1}^2}{k_2} + 2\zeta k_1 k_2\right]$$

To ensure that $\dot{V}$ is negative semi-definite, one need to ensure that

$\frac{c_1 k_1}{k_2} + a^2 k_1 + \zeta\, {k_1}^2 > \frac{k_2 a^2 + \frac{{k_1}^2}{k_2} + 2\zeta k_1 k_2}{4\zeta {k_2}^2}$

To satisfy both conditions $c_1 k_2 > k_1^2$ and (3.16), following constraint is imposed:

$$c_1 > max\left[\frac{{k_1}^2}{k_2}, \frac{k_2}{k_1}\left(\frac{k_2 a^2 + \frac{{k_1}^2}{k_2} + 2\zeta k_1 k_2}{4\zeta {k_2}^2}\right)^2 - k_1 a^2 - \zeta {k_1}^2\right]$$

Since $\dot{V}\ is\ not\ negative\ definite$

Now, $\dot{V}$ =0 is the set

Ω = {(e,s) I $e_1 = e_2 = 0$}

To show the stability of error dynamics, the solution of the error dynamics and estimator in set Ω is given by

$\dot{s_1}$=λ s

$\dot{s_2}$= -$\frac{s_1\ s_2}{\lambda}$

$\dot{s_3}$=0

$\dot{e_2} = 0 = s_1 e_3$

$\dot{e_3}$= 0

From this, we can observe that, e_3 must be zero, since $e_1 = 0$ so $e_3 = 0$ and the error dynamics tends asymptotically to zero. It leads to conclusion that s_3 tends asymptotically to the value of a^2 . So, in the invariant set, the only solution for the error dynamics is the trivial solution and for the observer dynamics, the only solution is a limit cycle and the output is precisely the exact output of the original system. So, since $V(e)$ is radically unbounded for all the values of $\boldsymbol{e}$, by using the Lasalle-Krasovskii theorem [4], we can conclude that the observer ensures global convergence of the estimator error.

III. For n-FREQUENCIES

In this, we depict the extension of the signal containing

n frequencies and given by

$$y(t) = \sum_{i=1}^{n} A_i \ \sin(\ _i t + \emptyset_i)$$

This signal may be shown as Output of the dynamic system is:-

$\dot{x}$(t) =W x(t)

Where W= $\begin{bmatrix}\begin{bmatrix} w_i & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & w_n \end{bmatrix}\end{bmatrix}$; $w_i = \begin{bmatrix} 0 & 1 \\ -a_i{}^2 & 0 \end{bmatrix}$

Y(t)=$\sum_{i=1}^{n} c_{1i}x_{1i}(t) + \ c_{2i}x_{2i}(t)$

for some constants $c_{1i} \neq 0, c_{2i} \neq 0$.

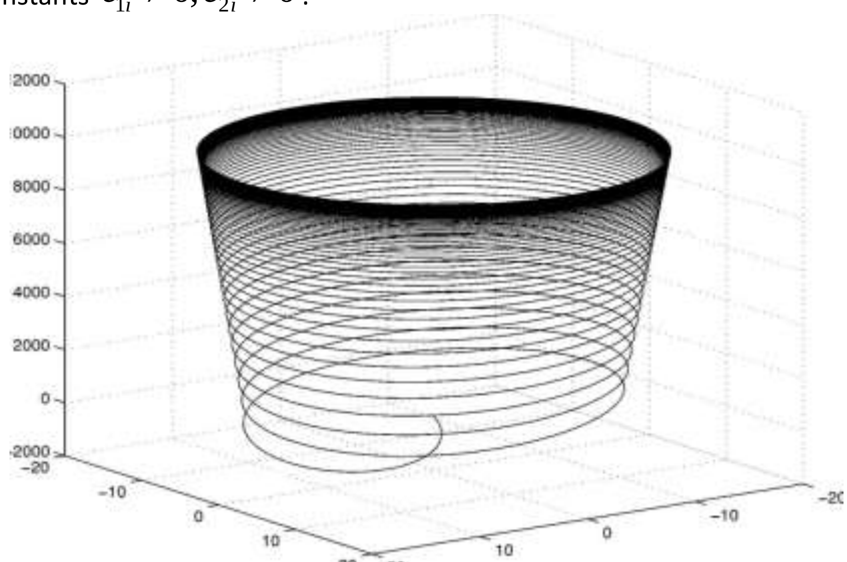

Fig. 2: signals s_1, s_2, and s_3 for a=100.

State model can be developed as:-

$\dot{x_1} = x_2$

$\dot{x_2} = x_3$

$\vdots$

$\dot{x}_{2n-1} = x_{2n}$

$\dot{x_{2n}} = -a_0x_1 - a_2x_3 - \cdots - a_{2n-2}x_{2n-1}$

Where $a_0 = \prod_{i=1}^{n} a_i{}^2$ and $a_2 = \sum_{i=1}^{n} a_i{}^2$

Output $y(t) = \frac{1}{\prod_{j=1}^{2n-1}\lambda_j}\left[\sum_{i=1}^{2n} k_i x_i\right]$

a_0 and a_2 are the coefficients of the characteristics polynomial

<u>Theorem-3:-</u> The estimator is shown as:-

$$\dot{s}(t) = \begin{pmatrix} 0 & \lambda_1 & 0 & \cdots & 0 & 0 \\ 0 & 0 & \lambda_2 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & 0 & \lambda_{2n-1} \\ \frac{-S_{2n+1}}{\prod_{j=1}^{2n-1}\lambda_j} & 0 & \frac{-S_{2n+2}}{\prod_{j=3}^{2n-1}\lambda_j} & \cdots & \frac{-S_{2n+n}}{\lambda_{2n-1}} & 0 \end{pmatrix} s(t) + \begin{pmatrix} 0 \\ 0 \\ \vdots \\ g_1 \\ g_2 \end{pmatrix} (y-\hat{y})$$

$\dot{S}_{2n+1} = -g_3 s_1(y-\hat{y})$

$\dot{S}_{2n+2} = -g_4 S_3(y-\hat{y})$

$\vdots$

$\dot{S}_{2n+n} = -g_{n+2}S_{2n-1}(y-\hat{y})$

With $g_i > 0$ for i=2,3,4….n+2 , $g_1 = \frac{\lambda_{2n-1}}{k_{2n}}$

$$\hat{y} = \sum_{i=1}^{2n-1}\frac{k_i}{\prod_{j=i}^{2n-1}\lambda_j} + k_{2n}S_{2n}$$

And k_i chosen such that polynomial:-

$P(w) = \omega^{2n-1} + \frac{K_{2n-1}}{k_{2n}}\omega^{2n-2} + \frac{k_{2n-2}}{k_{2n}}\omega^{2n-3} + \cdots + \frac{K_2}{K_{2n}}\omega + \frac{K_1}{K_{2n}}$

Is stable with all its roots distinct is such that $\hat{y} = y$,

$s_{2n+i} \rightarrow a_{2i-2}$, For i=1,2 ....n when t→ ∞

Proof:- Consider the error system first:-

$e_1 = x_1 - s_1$

$\dot{e_1} = e_2 = x_2 - \lambda_1 s_2$

$\dot{e_2} = e_3 = x_3 - \lambda_1 \lambda_2 s_3$

$$\vdots$$

$$e_{\dot{2n-1}} = x_{2n} - \lambda_1 \lambda_2 \ldots \lambda_{2n-1} s_{2n} - \lambda_1 \lambda_{2\ldots} \lambda_{2n-2} g_1 (y - \hat{y})$$

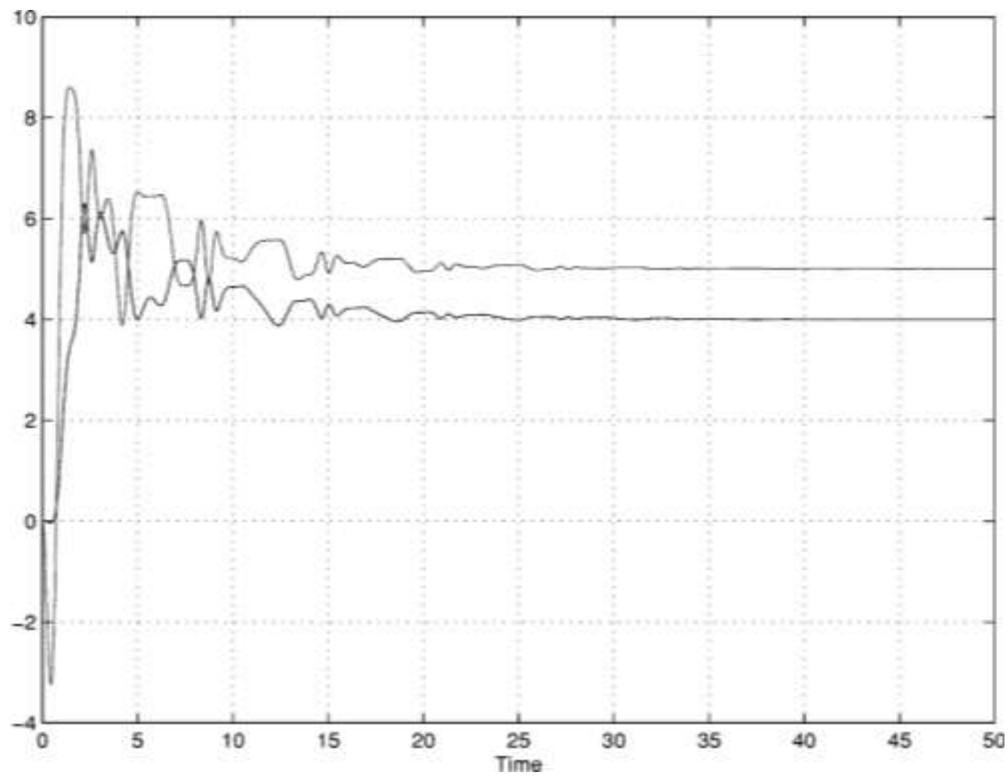

Fig.3: signals x_5, x_6, a_0, a_1

Characteristics polynomial:-

P(w)= $w^{2n-1} + \frac{k_{2n-1}}{k_{2n}} w^{2n-2} + \frac{k_{2n-2}}{k_{2n}} w^{2n-3} + \cdots + \frac{k_2}{k_{2n}} w + \frac{k_1}{k_{2n}}$ and is stable by hypothesis. Error dynamic may be rewritten as:- $\dot{\hat{e}}_1 = A\widehat{e_1}$

$$\dot{e_4} = -{\beta_1}^T \hat{e}_1 - \beta_2 e_{2n} + {\hat{s}_1}^T \hat{e}_3$$

$\dot{\hat{e}}_3 = -\Gamma \hat{s}_1 ({\hat{k}_1}^T \hat{e}_1 + k_{2n} e_{2n})$ Where A is a Hurwitz and given by

$$A = \begin{matrix} 0 & 1 & \cdots & 0 \\ 0 & 0 & \cdots & \vdots \\ 0 & 0 & \cdots & 1 \\ \frac{-K_1}{K_{2n}} & \frac{-K_2}{K_{2n}} & \cdots & \frac{-K_{2n-1}}{K_{2n}} \end{matrix}$$

Matrix A has distint eigen values then there exist a matrix T such that:-

$\tilde{A} = T^{-1} A T = Diag\ (\mu_1 \mu_2 \ldots \mu_{2n-1})$

$\hat{e}_1 = T \widetilde{e_1}$

And the error dynamics take the form:-

$\dot{e}_1 = \tilde{A} \tilde{e}_1$

$\dot{e}_{2n} = -{\hat{\beta}_1}^T \tilde{e}_1 - \beta_2 e_{2n} + {\hat{s}_1}^T \hat{e}_3$

$\dot{\hat{e}}_3 = -\Gamma\ \hat{s}_1 ({\tilde{k}_1}^T \widetilde{e_1} + k_{2n} e_{2n})$

Where ${\hat{\beta}_1}^T = {\beta^T}_1 T\ and\ {\hat{k}_1}^T = {\hat{k}^T}_1 T$

Since $\tilde{A}\ is\ diagonal$,there exist a Lyapunov function $v_0(\tilde{e}_1)$ satisfying :-

$v_0(\tilde{e}_1) = {\tilde{e}_1}^T P \tilde{e}_1$

$\dot{v}_0(\tilde{e}_1) = -{\tilde{e}_1}^T Q \tilde{e}_1$

For some P,Q diagonal and positive definite. Let us now consider the Lyapunov candidate function:-

V$(\tilde{e}_1, e_{2n}, \widehat{e_3})= v_0(\tilde{e}_1) + {C_{12}}^T \tilde{e}_1 e_{2n} + C_{2n} \frac{{e_{2n}}^2}{2} + \frac{1}{2} {\hat{e}_3}^T C_3 \hat{e}_3$

With $C_{12} = [C_{1,2n} \;\; C_{2,2n} \;\; C_{2n-2,2n}]^T$

$C_3 = Diag\,(C_{13} \;\; C_{23} \ldots C_{n3})$ Which is a positive definite for some P and $e_{2n} > 0$.

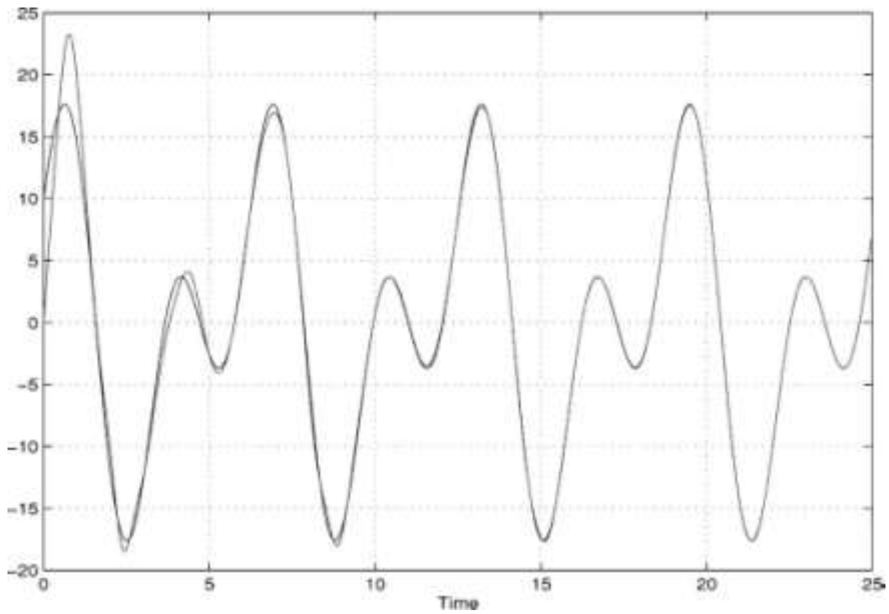

Fig.4: Signal y,$\tilde{y}$

From this, we have third equation:-

$\dot{V}(\tilde{e}_1, e_4, \hat{e}_3) =$

$-{\tilde{e}_1}^T (Q + C_{12}\, {\tilde{\beta}_1}^T)\, \tilde{e}_1 - C_{2n}\beta_2 {e_{2n}}^2 - ({C_{12}}^T \beta_2 - {C_{12}}^T \tilde{A} + {\tilde{\beta}_1}^T C_{2n}) \tilde{e}_1 e_{2n} + ({\tilde{e}_1}^T C_{12})({\hat{S}_1}^T \hat{e}_3) e_{2n}$ - $({\tilde{e}_1}^T \tilde{k}_1)({\hat{s}_1}^T \Gamma C_3 \hat{e}_3)$+ $C_{2n} ({\hat{s}_1}^T \hat{e}_3) e_{2n} - k_{2n} {\hat{s}_1}^T \Gamma C_3 \hat{e}_3 e_{2n}$

Choosing $C_{2n}=k_{2n}$

$C_{12}=\hat{k}_1$

$C_3 = \Gamma^{-1}$

Finally, we get:-

$$\dot{V}(\tilde{e}_1, e_{2n,}\hat{e}_3) = -e_1{}^T\left(Q + \tilde{k}_1\tilde{\beta}_1{}^T\right)\tilde{e}_1 - k_{2n}\beta_2 e_{2n}{}^2 - (\tilde{k}_1{}^T\beta_2 - \tilde{k}_1{}^T A + \tilde{\beta}_1{}^T k_{2n})\,\tilde{e}_1 e_{2n}$$

This is negative semi definite for the any value of Q.Therefore we use the same analysis made in the single frequency case. In this case the invariant set in gain by $\tilde{e}_1 = 0$, e_{2n}=0 and $\hat{e}_3$ must be zero b'coz the assumption that frequencies are distint implies that the signals are linearly independent .

IV. SIMULATIONS RESULT

The performance of the estimator was tested by simulations, some of which are presented in the following figures. Firstly, we can show in Fig. 1 the behavior of the estimator for a single frequency

λ= 1, = 10, = 1, k1 = 1, k2 = 1, = 1 and y(t) = sin(a t) we can see the fact that if the frequency is increased, then the magnitude of x increased, therefore, we can increase to reduce the magnitude of s and reduce k2 and k1 for which the dynamics of s1 are faster. The response of the estimator for a= 100, where the parameters are k1 = 0:5, k2 = :25, λ= 100, γ= 1000, ζ= 1000 and y(t) = 10 sin(100t) is shown in the Fig. 2. We observe the good behavior of the estimator.

Taking now two frequencies, let us say a_1 = 1,a_2 = 2, we get a0 = 4 and a2 = 5 and the performance of the filter with parameters k1 = 6, k2 = 11, k3 = 6, k4 = 1,λ = 1 and y(t) = 10(sin(t) + sin(2t)), are shown in Figs. 3 and 4.As it may be seen, the estimator exhibits a good convergence property, so these results suggest the validity of the proposed given estimator.

V. CONCLUSION

In this paper the question of global state and frequency simultaneous estimation is discussed. We propose a new estimator which provides a solution to given important problem in control theory. This estimator is globally convergent for all initial conditions and frequency values and its dimension is 3n. The extensive performed simulation allows us to state the validity of the proposed solution.

REFERENCES

[1] P. Regalia, *IIR Filtering in Signal Processing and Control*. New York: Marcel Dekker, 1995.

[2] L. Hsu, R. Ortega, and G. Damm, "A globally convergent frequency es-timator," *IEEE Trans. Automat. Contr.*, vol. 44, pp. 698–713, Apr. 1999.

[3] M. Bodson and S. Douglas, "Adaptive algorithms for the rejection of sinusoidal disturbances with unknown frequency," in *Proc. 13th IFAC World Conf.*, San Francisco, CA, July 1–5, 1996.

[4] C. Fuller and A. von Flotow, "Active control of sound and vibration," *IEEE Contr. Syst. Mag.*, vol. 15, pp. 9–19, Dec. 1996.

[5] H. Khalil, *Nonlinear Systems*, 2nd ed. NJ: Prentice-Hall, 1996.

[6] S. Hal l and N. Wereley, "Performance of Higher harmonic control algorithms for helicopter vibration reduction," *J. Guid. Control Dyn.*, vol. 116, no. 4, pp. 793–797, 1995

[7] R. Herzog, P. Buhler, C. Gahler, and R. Larsonneur, "Unbalance com-pensation using generalized notch filters in the multivariable feedback of magnetic bearings," *IEEE Trans. Contr. Syst. Technol.*, vol. 4, pp. 580–586, Sept. 1996.

[8]P. Regalia, "An improved lattice-based adaptive IIR notch filter," *IEEE Trans. Signal Processing*, vol. 39, pp. 2124–2128, Sept. 1991.

[9] R. Marino and P. Tomei, "Global estimation of n unknown frequencies," in *39th Conf. Decision Control*, Sydney, Australia, Dec. 12–15, 2000.