SAFETY OF LAP LENGTHS PREDICTION IN REINFORCED CONCRETE STRUCTURES

***Sarki, Y. A.**
****Abejide, O. S**

ABSTRACT

The recommendations for lap lengths of steel bars in reinforced concrete as specified by CP110 (1972), BS8110 (1985) and EC2 (2008) have been reviewed and evaluated for their safety implications in the bar curtailment and detailing. Results indicate that the implied safety is not uniform especially when the bar size factor in the prevailing specified equations for lap length design is about half its value for all the codes. Also, the earlier provision of CP110 (1972) gave the best safety indices in all the cases evaluated. However, it is suggested herein that an adequate lap length of bars in reinforced concrete design could be about 50times the smallest bar size in the reinforced concrete member as opposed to the EC2 (2008) provisions of 40times the smallest bar size.

Key Words: reinforcement bars, lap lengths, concrete

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* Department of Civil Engineering, Ahmadu Bello University, Zaria, **NIGERIA**.
** Department of Civil Engineering, Ahmadu Bello University, Zaria, **NIGERIA**.

1. Introduction

For about a century, construction practices in the building of concrete structures have focused on the use of steel reinforcement to transfer tension and shear forces. Lap splicing has become the traditional method of connecting steel reinforcing bars, largely due to a misconception that lap splicing is cheap and sometimes may have no cost attached to it.

Steel reinforcement usually comes in 6m (200 ft) and 12m (40ft) lengths. In such cases where the steel reinforcement is required to exceed these lengths, or other cut lengths then a splice is required. The main purpose of the splice is to transform the stresses whether tensile or compression from one steel reinforcing bars or group of bundled bars to another in a manner to satisfy the governing local building/engineering codes and/or requirements of engineering plans and specification. The overlap load transfer mechanism takes advantage therefore of the load. The bond in one bar is transferred to the concrete, and then from the concrete to ongoing bar. The bond is largely influenced by deformations on the surface of the reinforcing bar.

In the construction of reinforced concrete, due to the limitations in available length of bars and due to constraints in construction, there are numerous occasions when bars have to be joined, some of which are detailed and hence the essence of overlapping two bars over at least a minimum specified length called lap length. This lap length as we would discuss varies depending on the bars sizes as there are various bar sizes and where the bars are lapped and/or which structural member or element the lapping occurs. This is the scope of this presentation; therefore this paper discusses the effect of lapping reinforced bars on few of these structural members e.g. beams; slabs; columns, etc. Technically, the lap length as would be detailed varies depending on concrete strength, the steel strength, size and spacing.

As a result of load transfer, the steel bars maybe either in axial tension or axial compression. Flexure, shear and torsion may occur as effect, but due to limitations of results and other factors, this paper would focus solely on tension and compression and how they vary with different lap lengths and bar sizes. Hence, the distribution of tensile stresses in the concrete normal to the axis of the bars is relevant. The overlap on the other hand transfers or generates additional forces in the concrete which tend to push the bars apart, so concrete cover must be strong enough to overcome this “bursting force”. Bursting force can cause spilling of the concrete cover and splice failure. Therefore because of the bursting force for longer size of reinforcing bars and additional transverse reinforcement is required by most design codes, at the laps.

Lapping in reinforced concrete can be approached in various ways but this paper attempts to deal basically with the variables of the lap splice which include lap length, the head-size and shape, and the bar spacing. Furthermore, we would go into detailing the effect of lapping under compression and tension for some structural members. How bars are arranged under lapping and finally the bond strength of lapped reinforced bars in all the structural members concerned.

2. 0 Lap Splice Design Equation.

A revised equation developed resulting from advancement in technology, submitted to building codes and reference documents and subsequently adopted is given in equation (2.1) below.

The minimum length of lap splices for reinforcing bars in tension or compression, l calculated by equation (2.1), but shall not be less than 300mm (12in) is given as:

$$l_d = \frac{0.13\, d_b^2 f_y \gamma}{k f_m'} \quad (1)$$

Where

= diameter of reinforcement

f_y = specified yield stress of the reinforcement

f_m' = specified compressive strength of masonry

l_d = required splice length of reinforcement

k = lesser of the masonry cover, clear spacing between adjacent reinforcement or 5 times d_b

γ = Gamma Ratio (0.5, 1.0, 1.5 & 2.0)

The metric form of equation (1) above is therefore:

$$l_d = \frac{1.5\, d_b^2 f_y \gamma}{k f_m'} \quad (2)$$

It is these equations (1) and (2) that have been evaluated in a probabilistic setting to determine the safety of their provisions when used in reinforced concrete. The safety checking is carried out as suggested by Gollwitzer et al., (1988). This procedure is described in the next sector.

3. 0 Reliability analysis

Reliability is defined as the probability of a performance function $g(\mathbf{X})$ greater than zero i.e. $P\{g(\mathbf{X}) > 0\}$. In other words, reliability is the probability that the random variables

$X_i = (X_1, \ldots, X_n)$ are in the safe region that is defined by $g(\mathbf{X}) > 0$. The probability of failure is defined as the probability $P\{g(\mathbf{X}) < 0\}$ .Or it is the probability that the random variables $X_i=(X_1, \ldots, X_n)$ are in the failure region that is defined by $g(\mathbf{X}) < 0$.

In a mathematical sense, structural reliability can be defined as the probability that a structure will attain each specified limit state (ultimate or serviceability) during a specified reference period and set of conditions. The idea of a `reference period' comes into play because the majority of structural loads vary with times in an uncertain manner. Hence the probability that any selected load intensity or criterion will be exceeded in a fixed interval of time is a function of the length of that interval. Thus, in general, structural reliability is dependent on the time of exposure to the loading environment.

3.1 Concept of Structural Reliability Analysis

Assume that R and S are random variables whose statistical distributions are known very precisely as a result of a very long series of measurements. R is a variable representing the variations in strength between nominally identical structures, whereas S represents the maximum load effects in successive T-yr periods, then the probability that the structure will collapse during any reference period of duration T years is given by

$$P_f = P(R - S \le 0) = \int_{-\infty}^{\infty} F_R(x) f_s(x) d_x \qquad 1$$

Where, F_R is the probability distribution function of R and f_s the probability density function of S. Note that R and S are statistically independent and must necessarily have the same dimensions.

The reliability of the structure is the probability that it will survive when the load is applied, given by:

$$R = 1 - P_f = 1 - \int_{-\infty}^{\infty} F_R(x) f_s(x) d_x \qquad 2$$

3.2 Resistant and load Interaction

In basic reliability problems, consideration is given to the effect of a load S and the resistance R offered by the structure. Both the load and resistant S and R can be described by a known probability density function (Fs) and (Fr) respectively. S can be obtained from the applied load through a structural analysis making sure that R and S are expressed in the same unit.

Considering only safety of a structural element, It would be said that a structural element has failed if its resistance R, is less that stress resulting S acting on it. The probability of failure Pf of the structural element can be expressed in any of the following ways.

$P_f = P(R – S)$

Where R = strength (resistance) and S = loading in the structure. The failure in this case is defined in this region where R-S is less than zero or R is less than S i.e

$P_f = P((R - S) \leq 0)$ (3)

As an alternative approach to equation 3.2, the performance function can also be given as

$P_f = P\left(\frac{R}{S} \leq 1\right)$ (4)

Where in this case, the failure is defined in the region where P_f is less that one, or R is less that S, that is.

$P_f \leq 1$ or $R \leq S$

It could also be expressed as

$P_f = P(InR - InS \leq 1)$ (5)

Or in general,

$P_f = P[G(R, S) \leq 0]$ (6)

Where G(x) is the "limit state function" and the probability of failure is an identical with probability of the limit state violation.

For any random variable X, the cumulative distribution function Fx(x) is given by

$F_x(\chi) = P(X \leq \chi) = \int_{-\infty}^{x} fx(y)\,dy$

(7)

Provided that x ≥ y

It follows for the common, but special case where R and S are independent, the expression for the probability of failure is

$P_f = P(R - S \leq 0) = \int_{-\infty}^{\infty} F_R(\chi) f_s(\chi)dx$

(8)

Expression (3.8) as known as the "convolution integral" and $F_R(X)$ is the probability that R ≤ x, or the probability that the actual Resistance R of the member is less than some value x. Fs(x) represents probability that the load effect S acting in the member has a value x and x+Δx in the limit as Δx → 0.

Considering all possible value of x, total failure probability is obtained as follows:

$$P_f = \int_{-\infty}^{\infty} [1 - F_s(x)] F_R(x)\,dx \quad (9)$$

Interpretation of what is considered to be an acceptable failure probability is made with consideration of the sequences of failure, which is predetermined.

The limit state "g(x) = R-S" is a function of material properties, loads and dimensions. The state of the performance function g(x) of a structure or its components at a given limit state is usually modeled in terms of infinite uncertain basic random variable x = (x1, x2, ……….,xn) with joint distribution function gives as

$$F_s(x) = P\{\prod_{i=1}^{n} x_1 \leq x_2)\} \quad (10)$$

And

(11)

i.e. sum of all the cases of resistance for which the load exceed the resistance.

3.3 Limit State / Performance function

The performance function g(x) is sometimes called the limit state function. It relates the random variables for the limit-state of interest. The limit state function, gives a discretised assessment of the state of a structural element as being either failed or safe. It is obtained from traditional deterministic analysis, but uncertain input parameters are identified and quantified.

Where $f_x(x)$ is the joint probability

distribution function of x.

The region of integration of the function g(x) is stated below.

g(x) > 0 : represents safety

g(x) = 0 : represents attainment of the limit state

g(x) < 0 : represents failure.

The probability of failure is given by P(g(x)<0) and therefore the reliability index β can be related to probability of failure by the following equation

$P_f = 1 - \Phi(\beta)$

(12)

3.4 Computation of Reliability Index

The basic approach to develop a structural reliability base strength standard is to determine the relative reliability of the design. In order to do this, reliability assessment of existing structural components is needed to estimate a representative value of the reliability index β. This first order reliability method is very

$$P_f = P\{g(x) \leq 0\} = \int f_x(x)\, dx$$

the limit state function in that point and by estimating the failure probability using the standard normal integral.

The reliability index,β, is then defined by

$$\beta = \frac{\mu_m}{\sigma_m}$$

Where μ_m = mean of M

And σ_m = Standard deviation of M

If R and S are uncorrelated and with M = R-S,

$\mu_m = \mu_R - \mu_s$

$and\ \sigma_m^2 = \sigma_R^2 + \sigma_s^2$

3.5 Method of Reliability Safety Checking

All methods are approximate and the problems becomes more difficult as the number of random variables and the complexity of the limit state function increases and when statistical dependence between random variable is present.

Of the two broad classes of methods of structural reliability analysis (level 2 and level 3 method of safety checking), level 2 method shall be employed. Level 2 is known as second moment, First Order Reliability Method (FORM). The random variables are defined as terms of means and variance and are considered to be normally distributed. The measure of reliability is based on the reliability index. It involves use of certain iteration correlation procedure to obtain an approximation to the probability of failure of a structure or structural system. This generally requires an idealization of failure domain and it is often associated with a simplified representation of the joint probability distribution of a variable.

The necessity to have a method of reliability analysis which is computationally fast and efficient and which produces result with degree of accuracy prompted the use of this level 2 method.

4. Results and Analysis

The stochastic models generated are analysed using the First Order Reliability Method to give values for safety index,β, for the various diameter of reinforced concrete bars. Three algorithms developed into FORTRAN modules were designed for failure mode in relation to the three building codes, BS8110 (1997); CP110 (1972), & EC2 (2008). The diameter and compressive strength of masonry were varied for all yield stresses in the three algorithms to get the various safety indexes. These safety indexes were plotted against the various respective diameters of reinforced concrete bars. Some of the results of the safety index values against their respective diameters are shown below.

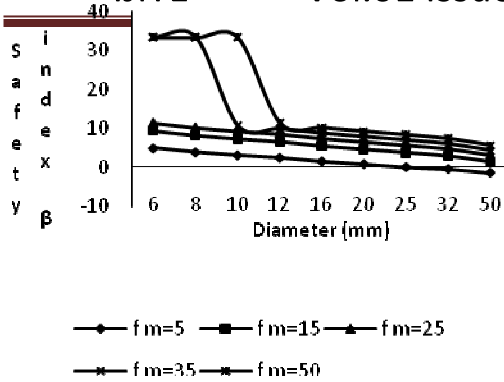


Fig 1: Effect of Bar Size on Safety of Lap Lengths
at f_y = 250 & ∝ = 0.5 (BS8110)

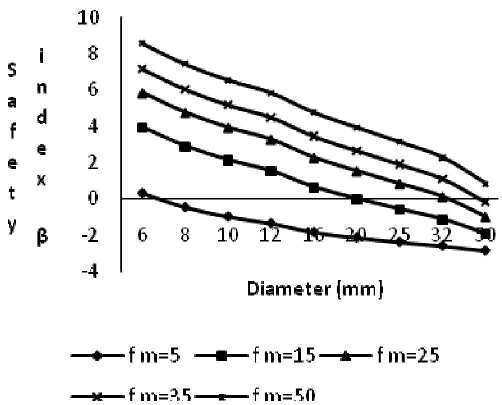


Fig 2: Effect of Bar Size on Safety of Lap Lengths
at f_y = 250 & ∝ = 2 (BS8110)

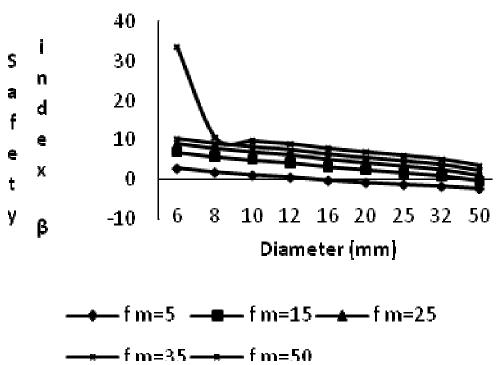


Fig 3: Effect of Bar Size on Safety of Lap Lengths
at f_y = 460 & ∝ = 0.5 (BS8110)

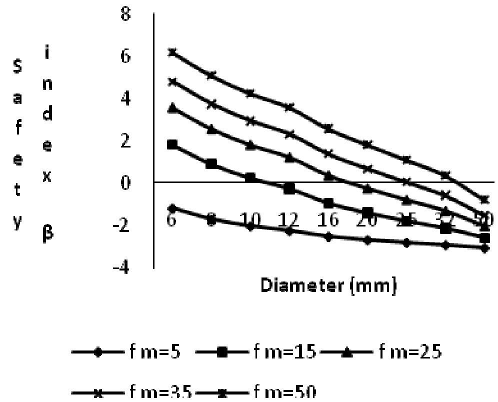


Fig 4: Effect of Bar Size on Safety of Lap Lengths
at f_y = 460 & ∝ = 2 (BS8110)

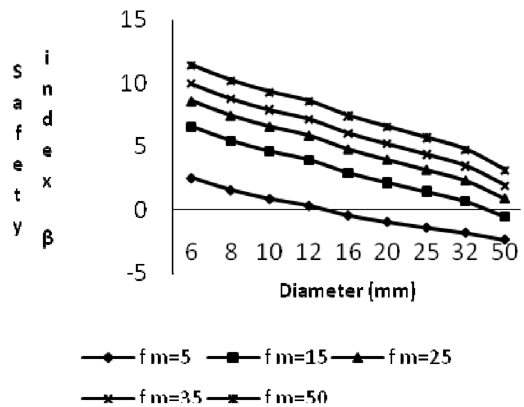


Fig 5: Effect of Bar Size on Safety of Lap Lengths
at f_y = 500 & ∝ = 0.5 (BS8110)

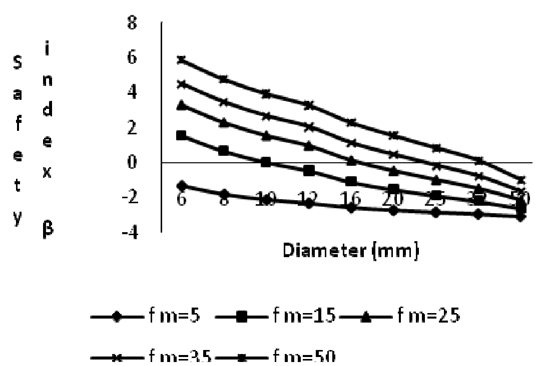


Fig 6: Effect of Bar Size on Safety of Lap Lengths
at f_y = 500 & ∝ = 2 (BS8110)

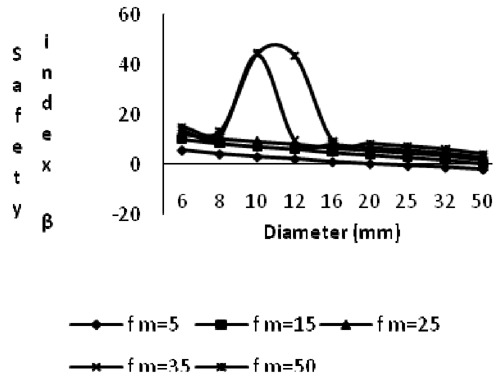


Fig 7: Effect of Bar Size on Safety of Lap Lengths
at f_y = 250 & ∝ = 0.5 (CP110)

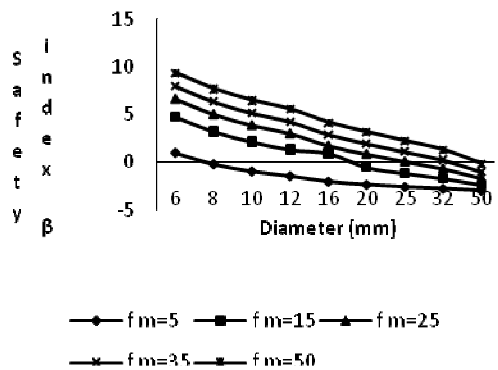


Fig 8: Effect of Bar Size on Safety of Lap Lengths

at f_y = 250 & ∝ = 2 (CP110)

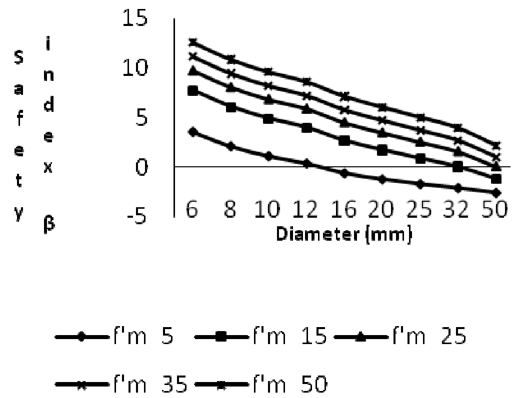


Fig 9: Effect of Bar Size on Safety of Lap Lengths
at f_y = 460 & ∝ = 0.5 (CP110)

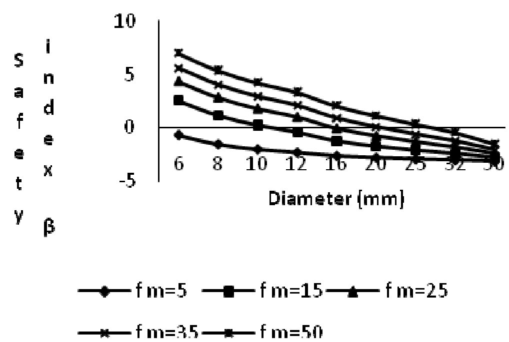


Fig 10: Effect of Bar Size on Safety of Lap Lengths at f_y = 460 & ∝ = 2 (CP110)

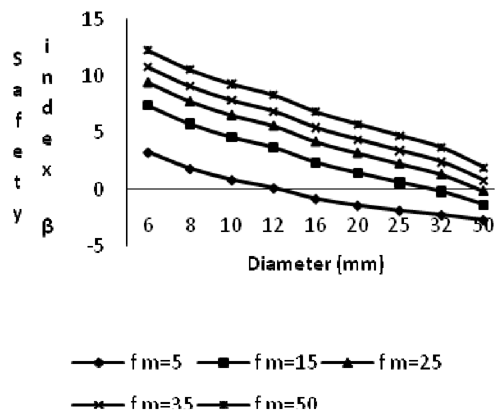


Fig 11: **Effect of Bar Size on Safety of Lap Lengths = 500 & = 0.5 (CP110)**

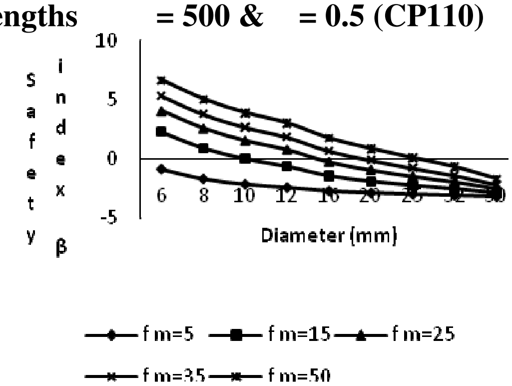
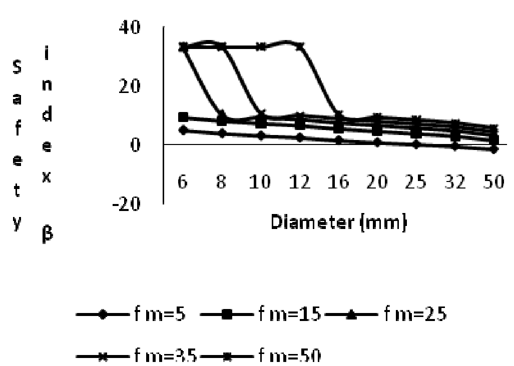


Fig 12: **Effect of Bar Size on Safety of Lap Lengths at f_y = 500 & $\propto$ = 2 (CP110)**



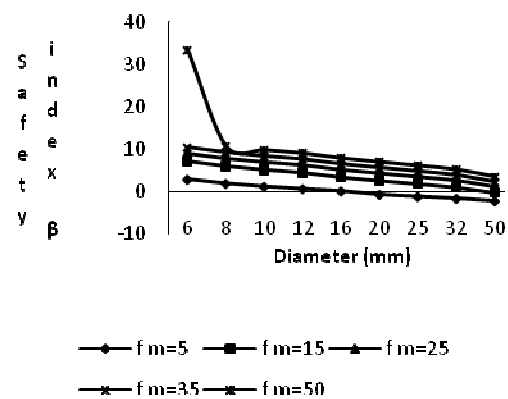


Fig 15: **Effect of Bar Size on Safety of Lap Lengths at f_y = 460 & $\propto$ = 0.5 (EC2)**

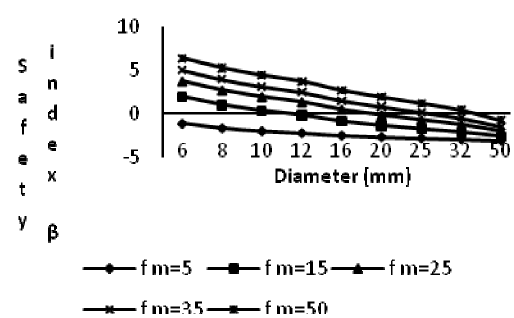


Fig 13: Effect of Bar Size on Safety of Lap Lengths at f_y = 250 & ∝ = 0.5 (EC2)

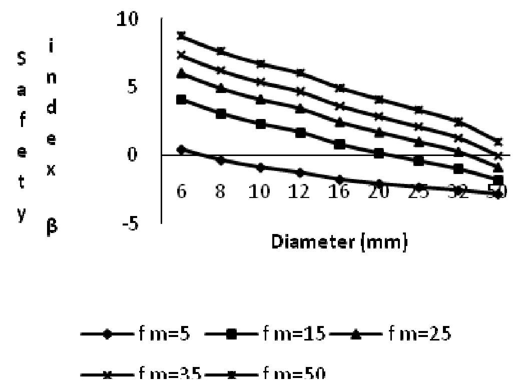


Fig 14: Effect of Bar Size on Safety of Lap Lengths at f_y = 250 & ∝ = 2 (EC2)

Fig 16: Effect of Bar Size on Safety of Lap Lengths at f_y = 460 & ∝ = 2 (EC2)

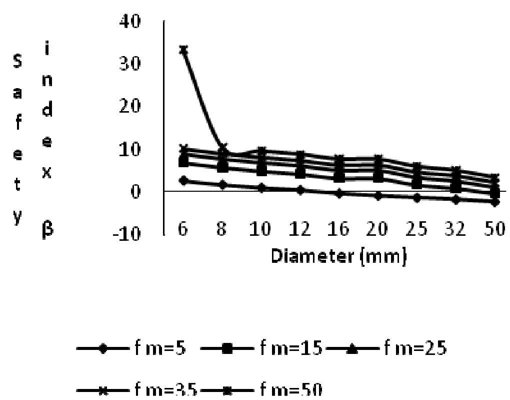


Fig 17: Effect of Bar Size on Safety of Lap Lengths at f_y = 500 & ∝ = 0.5 (EC2)

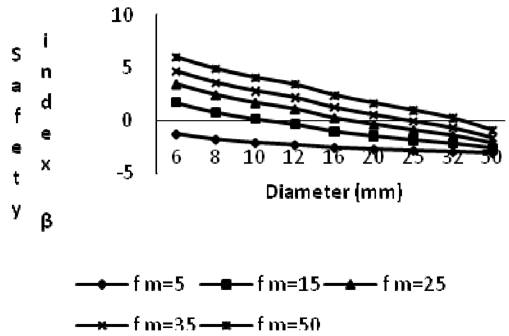


Fig 18: Effect of Bar Size on Safety of Lap Lengths f_y = 500 & = 2 (EC2)

5.0 Discussion of Results

With regards to the β values in the figures 1-18 as shown also above, lap lengths in bars can be said to be safest at the lowest possible values of the variables (that is bar diameter, yield stress and gamma ratio). From thorough analysis of these values and thorough examination of the graphs plotted it would be observed that at a constant yield strength and gamma ratio, the best safety index value (i.e. the maximum β value) is mostly ascertained at the minimum value of diameter of reinforced concrete bars and the maximum value of the specified compressive strength of concrete. These values of the safety indices diminish as the bar diameter increases and the compressive strength of concrete decreases simultaneously. Therefore it could be seen that the safety value of lap lengths of reinforced concrete bars is inversely proportional to the diameter of reinforced concrete bars and directly proportional to the specified compressive strength of concrete at constant yield stress of reinforcement bars and gamma ratio.

By comparison, it can be observed that the three algorithms designed relating to the entire building codes (BS8110 (1985, 1997); CP110 (1972) & E2 (2008)) share the relationship above.

Referencing the three building codes (i.e. CP110, EC2, and BS110) the best results were obtained from the safety index values from the CP110 (1972) code. This implies that the CP110 (1972), which stipulates lap lengths of reinforced concrete bars to be equal to $(25d_b + 150)$, gives the best provision for lap lengths of steel in tension.

6.0 Conclusion and

Recommendation

The aim of this study is to achieve safety. From the analysis and results obtained it can be concluded that the code which gives the best provision for lap lengths is the code that gives the highest β value (i.e safety index value). From the results obtained, the code that gives the best of this provision for lap length is the (CP110 (1972)).

From examination on the safety index values it would be observed that the safety index value at the same yield stress(f_y), gamma ratio(α) and compressive strength of concrete (f'_m) is at the best provision only in the code that has the largest lap length value at varying diameters of reinforced concrete bars.

It is therefore recommended that for every diameter of reinforced concrete bar, the lap length could be 50 times the diameter of the reinforcement bar irrespective of the compressive strength and yield stress; that is;

Lap Length $= 50d_b$

where d_b = diameter of reinforcement bar.

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