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# FIXED POINT THEOREMS ON *f*-RECIPROCAL CONTINUITY

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**ABSTRACT:**
***The study of common fixed points of different contractive mappings has concentrated around the continuous mappings for many years. However, in the last decade it has been shown that the study of common fixed points of mappings which are discontinuous at their fixed points is also fascinating. In this regard, we define f-reciprocal continuity, a generalization of continuity but independent of both reciprocal continuity and g-reciprocal continuity. Also, we have proved some common fixed point theorems which are illustrated by suitable examples.***
**Keywords:** ***Fixed point, f-reciprocal continuity, pseudo compatible maps, compatible maps, non compatible maps.***
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## 1. INTRODUCTION AND PRELIMINARIES

In 1980's, Rhoades [1] posed an open problem-"Whether there exists a contractive definition which is strong enough to generate a fixed point, but which does not force the map to be continuous at the fixed point". This problem has remained open for more than a decade. The work along these lines was initiated by Pant [2,4], who established the existence of fixed points for mappings which may be discontinuous at their fixed points and as well as for non compatible mappings. Interestingly, the best examples of non compatible mappings are found among the pairs of mappings which are discontinuous at their common fixed points. Later on, Pant [4] proved the common fixed point theorem without any continuity requirement by introducing the notion of reciprocal continuity, which is mainly applicable to the setting of compatible mappings. To extend the scope of study of fixed points from the class of compatible mappings to a wider class of non compatible and discontinuous mappings, Pant [5] generalized the concept of reciprocal continuity to weak reciprocal continuity. Many interesting works on weak reciprocal continuity have come through by many authors (see [8]-[11]).

Very recently, in [6] Pant et.al. have introduced two more generalized concepts. Firstly, $g$-reciprocal continuity which is a generalization of continuity, but independent of reciprocal continuity (see examples in[7]). Secondly, Pseudo compatible, a proper generalization of occasionally weakly compatible. By using these two newly introduced concepts Pant et.al.[6] have proved some common fixed point theorems. Motivated by these works of Pant [6] and following the lines of Pant, we define f-reciprocal continuity which is a generalization of continuity and independent of both reciprocal and g-reciprocal continuity. Also, we have proved some common fixed point theorems by using this new notion and pseudo compatibility. The suitable examples are demonstrated to exhibit the utility of the main results. Our results extend and generalize the results of Pant [6] and many more results in the literature.

Before proceeding to further, we recollect some basic definitions which are needed in our main results.

**Definition 1.1:[11]** Two self maps $f$ and $g$ of a metric space *(X, d)* are called compatible if $lim_{n\to\infty} d(fgx_n, gfx_n) = 0$, whenever $\{x_n\}$ is a sequence in $X$ such that $lim_{n\to\infty} fx_n = lim_{n\to\infty} gx_n = t$ for some $t$ in $X$. Thus the mappings $f$ and $g$ will be non compatible if there exists at least one sequence $\{x_n\}$ such that $lim_{n\to\infty} fx_n = lim_{n\to\infty} gx_n = t$ for some $t$ in $X$ but $lim_{n\to\infty} d(fgx_n, gfx_n)$ is either nonzero or nonexistent.

**Definition 1.2: [4]** Two self mappings $f$ and $g$ of a metric space *(X, d)* are called reciprocally continuous if $lim_{n\to\infty} fgx_n = ft$ and $lim_{n\to\infty} gfx_n = gt$ whenever $\{x_n\}$ is a sequence in $X$ such that $lim_{n\to\infty} fx_n = lim_{n\to\infty} gx_n = t$ for some $t$ in $X$.

**Definition 1.3:[5]** Two self mappings $f$ and $g$ of a metric space *(X, d)* are called weakly reciprocally continuous if $lim_{n\to\infty} fgx_n = ft$ or $lim_{n\to\infty} gfx_n = gt$ whenever $\{x_n\}$ is a sequence in $X$ such that $lim_{n\to\infty} fx_n = lim_{n\to\infty} gx_n = t$ for some $t$ in $X$.

**Definition 1.4:[6]** Two self mappings $f$ and $g$ of a metric space *(X, d)* are called $g$- reciprocally continuous iff $lim_{n\to\infty} ffx_n = ft$ and $lim_{n\to\infty} gfx_n = gt$ whenever $\{x_n\}$ is a sequence such that $lim_{n\to\infty} fx_n = lim_{n\to\infty} gx_n = t$ for some $t$ in $X$.

**Definition 1.5:** Let $f$ and $g$ be self mappings of a metric space *(X,d)*. Then for a sequence $\{y_n\}$ in $X$ satisfying $lim_{n\to\infty} fy_n = lim_{n\to\infty} gy_n$, a sequence $\{z_n\}$ will be called an associated sequence if $fy_n = gz_n$ or $gy_n = fz_n$ and $lim_{n\to\infty} fz_n = lim_{n\to\infty} gz_n$.

**Definition 1.6:[6]** Two self mappings $f$ and $g$ of a metric space *(X,d)* will be defined to be pseudo compatible if and only if whenever the set of sequences $\{x_n\}$ satisfying $lim_{n\to\infty} fx_n = lim_{n\to\infty} gx_n$ is nonempty, there exists a sequence $\{y_n\}$ such that $lim_{n\to\infty} fy_n = lim_{n\to\infty} gy_n = t$ (say), $lim_{n\to\infty} d(fgy_n, gfy_n) = 0$ and $lim_{n\to\infty} d(fgz_n, gfz_n) = 0$ for any associated sequence $\{z_n\}$ of $\{y_n\}$.

**Theorem 1.7:[6]** *Let f and g be g-reciprocally continuous self mappings of a complete metric space (X,d) such that*

*(i)* $fX \subseteq gX$

*(ii)* $d(fx, fy) \leq k\, d(gx, gy)$, $k \in [0,1)$.

*If f and g are pseudo compatible, then f and g have a unique common fixed point.*

**Theorem 1.8:[6]** *Let f and g be g-reciprocally continuous non compatible self mappings of a metric space (X,d) such that*

*(i)* $fX \subseteq gX$

*(ii)* $d(fx, fy) < max\{ d(gx, gy), \frac{k[d(fx,gx)+d(fy,gy)]}{2}, \frac{[d(fx,gy)+d(fy,gx)]}{2}\}$, $1 \leq k < 2$.

*(iii)* $d(x, fx) \neq max(d(x, gx), d(fx, gx))$,

*whenever right-hand side is nonzero. If f and g are pseudo compatible, then f and g have a unique common fixed point.*

## 2. MAIN RESULT

First we present a new notion, $f$-reciprocal continuity which is a generalization of continuity.

**Definition 2.1:** Two self mappings $f$ and $g$ of a metric space *(X, d)* are called $f$-reciprocally continuous iff $lim_{n\to\infty} fgx_n = ft$ and $lim_{n\to\infty} ggx_n = gt$ whenever $\{x_n\}$ is a sequence in $X$ such that $lim_{n\to\infty} fx_n = lim_{n\to\infty} gx_n = t$ for some $t$ in $X$.

If $f$ and $g$ are continuous, then they are obviously $f$-reciprocally continuous but, the converse is not true. Also, it may be observed that $f$-reciprocal continuity is independent of both reciprocal continuity and $g$-reciprocal continuity. The following three examples clearly demonstrate this fact.

**Example 2.2:** Let $X = [2,10]$ and $d$ be the usual metric on $X$. Define $f, g : X \to X$ by

$f2 = 2,\ fx = 9$ if $2 < x \leq 5$ *and* $fx = \frac{x+1}{3}$ if $x > 5$

$g2 = 2$ if $x = 2$ *and* $x > 5,\ gx = 4$ if $2 < x \leq 5$.

Let $\{x_n\} = \left\{5 + \frac{1}{n}\right\}$ be a sequence in *X*. Then $fx_n = 2 + \frac{1}{3n} \to 2$ and $gx_n \to 2$. $fgx_n = f(2) = 2$, $ffx_n = f\left(2 + \frac{1}{3n}\right) \to 9$, $gfx_n = g\left(2 + \frac{1}{3n}\right) \to 4$ and $ggx_n = g(2) = 2$. Thus $lim_{n\to\infty} fgx_n = f2$ and $lim_{n\to\infty} ggx_n = g2$. Hence *f* and *g* are *f*-reciprocally continuous mappings but neither *g*-reciprocally continuous nor reciprocally continuous.

**Example 2.3:** Let $X = [2,10]$ and *d* be the usual metric on *X*. Define $f, g : X \to X$ by

$fx = 2$ if $x = 2 \; and \; x > 5$, $fx = \frac{x+7}{3}$ if $2 < x \leq 5$

$g2 = 2$ , $gx = 6$ if $2 < x \leq 5$, $gx = \frac{x+5}{5}$ if $x > 5$ .

Let $\{x_n\} = \left\{5 + \frac{1}{n}\right\}$ be a sequence in *X*. Then $fx_n \to 2$ and $gx_n = 2 + \frac{1}{5n} \to 2$. $fgx_n = f\left(2 + \frac{1}{5n}\right) \to 3$, $ffx_n = f(2) = 2$, $gfx_n = g(2) = 2$ and $ggx_n = g\left(2 + \frac{1}{5n}\right) \to 6$. Thus $lim_{n\to\infty} ffx_n = f2$ and $lim_{n\to\infty} gfx_n = g2$. Hence *f* and *g* are *g*-reciprocally continuous mappings but neither *f*-reciprocally continuous nor reciprocally continuous.

**Example 2.4:** Let $X = [1,10]$ and *d* be the usual metric on *X*. Define $f, g : X \to X$ by

$fx = \frac{x+3}{2}$ if $x < 6$ and $fx = 3$ if $x \geq 6$

$g3 = 3$ , $gx = 8$ if $x < 3$ and $3 < x < 6$, $gx = \frac{x}{2}$ if $x \geq 6$ .

Let $\{x_n\} = \left\{6 + \frac{1}{n}\right\}$ be a sequence in *X*. Then $fx_n \to 3$ and $gx_n = 3 + \frac{1}{2n} \to 3$. $fgx_n = f\left(3 + \frac{1}{2n}\right) \to 3$, $gfx_n = g(3) = 3$ and $ggx_n = g\left(3 + \frac{1}{2n}\right) \to 8$. Thus $lim_{n\to\infty} fgx_n = f3$ and $lim_{n\to\infty} gfx_n = g3$. Hence *f* and *g* are reciprocally continuous mappings but not *f*-reciprocally continuous.

We now state and prove our first main result.

**Theorem 2.5:** *Let f and g be f-reciprocally continuous self mappings of a metric space (X,d) such that*

*(i)* $fX \subseteq gX$ *and fX is complete*

*(ii)* $d(fx, fy) \leq a\, d(gx, gy) + b\, d(fx, gx) + c\, d(fy, gy)$ *with* $a, b, c \in [0,1)$ *and* $a+b+c<1$.

*If f and g are pseudo compatible, then f and g have a unique common fixed point.*

**Proof**:

Let $x_o$ be any point in *X*. Since $fX \subseteq gX$, there exists a sequence of points $x_o, x_1, x_2, \ldots, x_n, \ldots$ such that $x_{n+1}$ is in the preimage under *g* of $fx_n$ .

i.e. $fx_o = gx_1,\; fx_1 = gx_2, \ldots, fx_n = gx_{n+1}, \ldots$

Define a sequence $\{S_n\}$ in *X* by

$$S_n = fx_n = gx_{n+1} \quad \text{for } n = 0,1,2,\ldots$$

Clearly $\{S_n\}$ is a sequence in *fX*.

Now, we claim that $\{S_n\}$is a cauchy sequence in *fX*. Consider

$$\begin{aligned} d(S_n, S_{n+1}) &= d(fx_n, fx_{n+1}) \\ &\leq a\, d(gx_n, gx_{n+1}) + b\, d(fx_n, gx_n) + c\, d(fx_{n+1}, gx_{n+1}) \\ &= a\, d(S_{n-1}, S_n) + b\, d(S_n, S_{n-1}) + c\, d(S_{n+1}, S_n) \end{aligned}$$

$$i.e.\, d(S_n, S_{n+1}) \leq k\, d(S_{n-1}, S_n) \leq k^n\, d(S_0, S_1), \qquad where \;\; k = \left(\frac{a+b}{1-c}\right) < 1.$$

Also for every integer $p > 0$, we have

$$\begin{aligned} d(S_n, S_{n+p}) &\leq d(S_n, S_{n+1}) + d(S_{n+1}, S_{n+2}) + \ldots + d(S_{n+p-1}, S_{n+p}) \\ &\leq k^n\,(1 + k + k^2 + \ldots + k^{p-1})\, d(S_0, S_1) \\ &\leq \left(\frac{1}{1-k}\right) k^n\, d(S_0, S_1) \end{aligned}$$

That is $d(S_n, S_{n+p}) \to 0$ as $n \to \infty$. Therefore $\{S_n\}$ is a cauchy sequence in $fX$.

Since $fX$ is complete, there exists a point $t \in fX$ such that $S_n \to t$ as $n \to \infty$.

Moreover, $S_n = fx_n = gx_{n+1} \to t$.

Now $f$ and $g$ are pseudo compatible implies there exists a sequence $\{y_n\}$ such that $fy_n \to u$, $gy_n \to u$ and $lim_{n\to\infty} d(fgy_n, gfy_n) = 0$ .

Since $fX \subseteq gX$, for each $y_n$ there exists a $z_n$ in $X$ such that $fy_n = gz_n$ $\forall n$.

Now we prove that $fz_n \to u$. Consider

$$d(fy_n, fz_n) \le a\, d(gy_n, gz_n) + b\, d(fy_n, gy_n) + c\, d(fz_n, gz_n)$$

on letting $n \to \infty$ we get $(1-c)d(u, fz_n) \le 0$, which gives $fz_n \to u$ since $c < 1$.

Therefore $\{y_n\}$ and $\{z_n\}$ are associated sequences and $lim_{n\to\infty} d(fgz_n, gfz_n) = 0$ .

$$i.e. lim_{n\to\infty} fy_n = lim_{n\to\infty} gy_n = lim_{n\to\infty} fz_n = lim_{n\to\infty} gz_n = u.$$

Further, $f$- reciprocal continuity of $f$ and $g$ implies that $fgy_n \to fu$ and $ggy_n \to gu$.

Since $lim_{n\to\infty} d(fgy_n, gfy_n) = 0$ , we have $gfy_n = ggz_n \to fu$. Similarly, $fgz_n \to fu$ and $ggz_n \to gu$. Hence $fu = gu$. Now we prove that $fu = u$. Consider

$$d(fu, fz_n) \le a\, d(gu, gz_n) + b\, d(fu, gu) + c\, d(fz_n, gz_n)$$

on letting $n \to \infty$ we get $(1-a)d(fu, u) \le 0$, which gives $u = fu = gu$ since $a < 1$.

Therefore $u$ is a common fixed point of $f$ and $g$. To prove the uniqueness, let $u$ and $v$ be two common fixed points of $f$ and $g$. Then $u = fu = gu$ and $v = fv = gv$. Consider

$$d(u, v) = d(fu, fv) \le a\, d(gu, gv) + b\, d(fu, gu) + c\, d(fv, gv)$$

on letting $n \to \infty$ we get $d(u, v) \le ad(u, v)$, which gives $u = v$ since $a < 1$.

Therefore $u$ is the unique common fixed point of $f$ and $g$.

The above theorem is illustrated by the following example.

**Example 2.6:** Let $X = [1,10]$ and $d$ be the usual metric on $X$. Define $f, g : X \to X$ by

$fx = \frac{x+3}{2}$ if $x \le 3$, $\quad fx = 2$ if $x > 3$

$gx = \frac{2x+3}{3}$ if $x \le 3$, $\quad gx = 9$ if $x > 3$

Then $f$ and $g$ satisfy all the conditions of Theorem 2.5 and have a unique common fixed point at $x = 3$. Further, $f$ and $g$ satisfy the contraction condition (ii) for $a = \frac{1}{3}, b = \frac{1}{3}, c = \frac{1}{4}$. The mappings $f$ and $g$ are $f$-reciprocally continuous. To see this, let $\{x_n\}$ be a sequence in $X$ such that $fx_n \to t$ and $gx_n \to t$ for some $t$. Then $t = 3$ and either $x_n = 3$ for each $n$ or $x_n = 3 - \frac{1}{n}$ . If $x_n = 3$ for each $n$ then $fx_n = 3$, $gx_n = 3$, $fgx_n = f3 = 3$ and $ggx_n = g3 = 3$. If $x_n = 3 - \frac{1}{n}$ then $fx_n = 3$, $gx_n = 3 - \frac{2}{3n} \to 3$, $fgx_n = f\left(3 - \frac{2}{3n}\right) = 3 - \frac{1}{3n} \to 3 = f3$ and $ggx_n = g\left(3 - \frac{2}{3n}\right) = 3 - \frac{4}{9n} \to 3 = g3$. Thus $lim_{n\to\infty} fgx_n = f3$ and $lim_{n\to\infty} ggx_n = g3$. Hence $f$ and $g$ are $f$-reciprocally continuous mappings. Also $f$ and $g$ are pseudo compatible. To see this consider the sequence $\{x_n\} = \left\{3 - \frac{1}{n}\right\}$. Then $fx_n \to 3$ and $gx_n \to 3$. Consider another sequence $\{y_n\} = 3$ for all $n$. Then $fy_n \to 3$, $gy_n \to 3$ and $lim_{n\to\infty} d(fgy_n, gfy_n) = 0$ .

If $\{z_n\}$ is an associated sequence of $\{y_n\}$ such that $fy_n = gz_n$ $\forall n$ and $lim_{n\to\infty} fz_n = lim_{n\to\infty} gz_n$, then $z_n = 3$ $\forall n$ and $lim_{n\to\infty} d(fgz_n, gfz_n) = 0$. Therefore $f$ and $g$ are pseudo compatible.

It is well known that strict contractive conditions do not ensure the existence of fixed points unless very strong conditions like compactness are assumed. But the next result demonstrates that the generalized strict contractive condition ensure the existence of common fixed point under the notion of *f*-reciprocal continuity.

**Theorem 2.7:** *Let f and g be f-reciprocally continuous non compatible self mappings of a metric space (X, d) such that*

*(i)* $fX \subseteq gX$

*(ii)* $d(fx, fy) < max\left\{kd(gx, gy), \frac{k}{2}[d(fx, gx) + d(fy, gy)], \frac{k}{2}[\, d(fx, gy) + d(fy, gx)]\right\}, \forall x \neq y$

*where* $0 < k < 1.$

*If f and g are pseudo compatible, then f and g have a unique common fixed point.*

**Proof**:

Since f and g are non compatible mappings, there exists a sequence $\{x_n\}$ in $X$ such that $lim_{n\to\infty} fx_n = lim_{n\to\infty} gx_n = t$ for some *t* in *X* but either $lim_{n\to\infty}\, d(fgx_n, gfx_n) \neq 0$ or the limit does not exist.

Now $f$ and $g$ are pseudo compatible implies there exists a sequence $\{y_n\}$ such that $fy_n \to u$, $gy_n \to u$ and $lim_{n\to\infty}\, d(fgy_n, gfy_n) = 0$ .

Since $fX \subseteq gX$, for each $y_n$ there exists a $z_n$ in *X* such that $fy_n = gz_n \ \ \forall n$.

Now we prove that $fz_n \to u$. Consider,

$$d(fy_n, fz_n) < max\left\{kd(gy_n, gz_n), \frac{k}{2}[d(fy_n, gy_n) + d(fz_n, gz_n)], \frac{k}{2}[\, d(fy_n, gz_n) + d(fz_n, gy_n)]\right\}$$

on letting $n \to \infty$ we get $\left(1 - \frac{k}{2}\right) d(u, fz_n) \leq 0,$ which gives $fz_n \to u$ since $k < 1$.

Therefore $\{y_n\}$ and $\{z_n\}$ are associated sequences and $lim_{n\to\infty}\, d(fgz_n, gfz_n) = 0$ .

$$i.e.\, lim_{n\to\infty}\, fy_n = lim_{n\to\infty}\, gy_n = lim_{n\to\infty}\, fz_n = lim_{n\to\infty}\, gz_n = u.$$

Further, *f*- reciprocal continuity of *f* and *g* implies that $fgy_n \to fu$ and $ggy_n \to gu$.

Since $lim_{n\to\infty}\, d(fgy_n, gfy_n) = 0$ , we have $gfy_n = ggz_n \to fu$. Similarly, $fgz_n \to fu$ and $ggz_n \to gu$. Hence $fu = gu$. Now we prove that $fu = u$. Consider

$$d(fu, fz_n) < max\left\{kd(gu, gz_n), \frac{k}{2}[d(fu, gu) + d(fz_n, gz_n)], \frac{k}{2}[\, d(fu, gz_n) + d(fz_n, gu)]\right\}$$

on letting $n \to \infty$ we get $(1 - k)d(fu, u) \leq 0,$ which gives $u = fu = gu$ since $k < 1$.

Therefore *u* is a common fixed point of *f* and *g*. To prove the uniqueness, let *u* and *v* be two common fixed points of *f* and *g*. Then $u = fu = gu$ and $v = fv = gv$.

Now we prove that $u = v$. Suppose that $u \neq v$, then

$$d(u, v) = d(fu, fv) < max\left\{kd(gu, gv), \frac{k}{2}[d(fu, gu) + d(fv, gv)], \frac{k}{2}[\, d(fu, gv) + d(fv, gu)]\right\}$$

on letting $n \to \infty$ we get $d(u, v) < kd(u, v) < d(u, v),$ a contradiction. Therefore $u = v$ .

Hence u is the unique common fixed point of *f* and *g*.

Now we present an example to illustrate Theorem 2.7.

**Example 2.8:** Let $X = [1, 10]$ and *d* be the usual metric on *X*. Define $f, g: X \to X$ by

$fx = 4 - \frac{x}{3}$ if $x \leq 3$, $fx = 2$ if $x > 3$

$gx = \frac{4x+3}{5}$ if $x \leq 3$, $gx = x - \frac{3}{x}$ if $x > 3$

Then $f$ and $g$ satisfy all the conditions of Theorem 2.7 and have a unique common fixed point at $x = 3$. Further, $f$ and $g$ satisfy the contraction condition (ii) for $k = \frac{1}{2}$. The mappings $f$ and $g$ are $f$-reciprocally continuous. To see this, let $\{x_n\}$ be a sequence in $X$ such that $fx_n \to t$ and $gx_n \to t$ for some $t$. Then $t = 3$ and either $x_n = 3$ for each $n$ or $x_n = 3 - \frac{1}{n}$. If $x_n = 3$ for each $n$, then $fx_n = 3,\ gx_n = 3,\ fgx_n = f3 = 3$ and $ggx_n = g3 = 3$. If $x_n = 3 - \frac{1}{n}$ then $fx_n \to 3,\ gx_n \to 3, fgx_n = f\left(3 - \frac{4}{5n}\right) \to 3 = f3,\ ggx_n = g\left(3 - \frac{4}{5n}\right) \to 3 = g3$ and $gfx_n = g\left(3 + \frac{1}{3n}\right) \to 2 \neq g3$. Thus $lim_{n\to\infty} fgx_n = f3$ and $lim_{n\to\infty} ggx_n = g3$ but $lim_{n\to\infty} d(fgx_n, gfx_n) \neq 0$. .

Hence $f$ and $g$ are $f$-reciprocally continuous and non compatible mappings. Also $f$ and $g$ are pseudo compatible. To see this, consider the sequence $\{x_n\} = \left\{3 - \frac{1}{n}\right\}$. Then $fx_n \to 3$ and $gx_n \to 3$. Consider another sequence $\{y_n\} = 3\ \ \forall\, n$. Then $fy_n \to 3,\ gy_n \to 3$ and $lim_{n\to\infty} d(fgy_n, gfy_n) = 0$ .

If $\{z_n\}$ is an associated sequence of $\{y_n\}$ such that $fy_n = gz_n\ \ \forall n$ and $lim_{n\to\infty} fz_n = lim_{n\to\infty} gz_n$, then $z_n = 3\ \ \forall n$ and $lim_{n\to\infty} d(fgz_n, gfz_n) = 0$. Therefore $f$ and $g$ are pseudo compatible.

**Remark 2.9:** The results established in this paper ensure the existence of common fixed points without assuming the continuity condition. Thus we provide more answers to the open problem posed by Rhoades[1].

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## REFERENCES

1. Rhoades B.E., *Contractive definitions and continuity,* Contemporary Math., 72(1982), 233-245.
2. Pant R.P., *Discontinuity and fixed points*, J.Math.Anal.Appl., 240(1999), 284-289.
3. Pant R.P., *Common fixed point theorems for contractive maps*, J.Math.Anal.Appl., 226(1998), 251-258.
4. Pant R.P., *Common fixed points for four mappings*, Bull.Cal.Math.Soc., 90(1998), 281-286.
5. Pant R.P.,Bisht R.K.,Arora D., *Weak reciprocal continuity and fixed point theorems*, Ann. Univ.Ferrara, 57(1)(2011), 181-190.
6. Pant R.P.,Bisht R.K., *Common fixed points of pseudo compatible mappings*, RACSAM, doi:10.1007/S13398-013-0119-5.
7. Joshi R.U.,Bisht R.K.,Bhatt A., *Common fixed points under strict contractive condition and g-reciprocal continuity* , J.Int.Acad.Phys.Sci., vol.16, no.1(2012), 1-6.
8. Bisht R.K.,Joshi R.U., *Common fixed point theorems for weakly reciprocally continuous maps*, J.Ind.Math.Soc., vol.79, no.1-4(2012), 1-12.
9. Manro S.,Kumam P., *Common fixed point theorems for expansion mappings in various abstract spaces using the concept of weak reciprocal continuity*, Fixed point theory and Applications, 2012, 2012:221.
10. Rao K.P.R., Kishore & Raju VCC (2010) "A Common Fixed Point Theorem for three maps on a cone metric space." Aryabhatta J. of Maths. & Info. Vol. 2 (1) pp 15-20.
11. Rao K.P.R., etal (2010) "A Unique common fixed point theorem for two pairs of Hybrid mappings." Aryabhatta J. of Maths. & Info. Vol. 2 (2) pp 251-25.
12. Giniswamy, Jeyanthi C.,Maheshwari P.G., *Fixed point theorems for compatible and noncompatible self maps*, J.Int.Acad.Phys.Sci., vol.16, no.1(2012), 13-26.