# RELIABILITY AND PROFIT EVALUATION OF A TWO-UNIT COLD STANDBY SYSTEM WITH INSPECTION AND CHANCES OF REPLACEMENT

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# **ABSTRACT:**

The present paper discusses a two-unit cold standby system with inspection and replacement. Initially one unit is operative and the other is cold standby. On the failure of operative unit, standby takes some time (called activation time) to become operative. After the completion of activation time, the operative unit is undertaken for inspection. The inspection is carried out to detect the reparability of the unit. If it is repairable; it is repaired by the ordinary repairman. But if it is not repairable then expert opinion is taken to confirm whether the unit is actually not repairable and then is repaired or replaced according as it is found repairable or irreparable. Various measures of the system effectiveness are obtained. using semi-Markov processes and regenerative point technique. Profit incurred to the system is evaluated. Graphical study is also made.

Keywords: Cold Standby, activation time, Semi-markov Process, Regenerative Point Technique.

# **INTRODUCTION**

Two-unit cold standby systems have been widely studied by a large number of researchers in the field of reliability under various assumptions and concepts. Most of these researchers assumed that the unit on failure is repairable. These researchers and the others, who considered the possibility of unit being found irreparable on its failure and hence needed to be replaced, assumed negligible time for standby unit to become operative.

There exists the situations in the industries, firms etc. where we may observe an inspection to be carried out on the failure of a unit which reveals whether the unit is repairable or needs to be replaced. Also, the cold standby unit takes time to become operative, whenever required. Observing such a practical situation in the industries like Steel plants and Biscuit manufacturing companies where the programmable Logic Controllers are used as two-unit cold standby, the present paper is an attempt to analyse various aspects including the reliability and the profit.

Thus, in the present paper, we discuss a two-unit cold standby system with inspection and replacement. Initially one unit is operative and the other is cold standby. On the failure of operative unit, standby takes some time (called activation time) to become operative. After the completion of activation time, the operative unit is undertaken for inspection. The inspection is carried out to detect the reparability of the unit. If it is repairable; it is repaired by the ordinary repairman. But if it not repairable then expert opinion is taken to confirm whether the unit is actually not repairable and then is repaired or replaced according as it is found repairable or irreparable.

Following measures of the system effectiveness are obtained. by making use of semi-Markov processes and regenerative point technique:

- Mean time to system failure (MTSF)
- Steady-state availability of the system
- Expected busy period (for repair only) per unit time by ordinary/expert repairman

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- Expected busy period (for inspection only) per unit time by ordinary/expert repairman
- Expected busy period (for replacement only) per unit time by ordinary repairman
- Expected number of visits per unit time by ordinary/expert repairman
- Expected number of replacements per unit time
- Expected activation time
- Expected profit incurred to the system

# NOTATIONS

λ	:	constant failure rate of operative unit			
$p_1$	:	probability that unit is repairable			
$p_2$	:	probability that unit is irreparable			
$h_1(t), H_1(t)$ :		p.d.f. and c.d.f. of time to inspection for detecting reparability of a failed unit			
$h_2(t), H_2(t)$ :		p.d.f. and c.d.f. of replacement time			
w(t), W(t) :		p.d.f. and c.d.f. of activation time			
g(t),G(t) :		p.d.f. anc c.df. of time to repair of ordinary repairman			
h <sub>e</sub> (t),I	$H_e(t)$ :	p.d.f. and c.d.f. of time to inspection of expert repairman			
$g_e(t), G_e(t)$ :		p.d.f. and c.d.f. of time to repair of expert repairman			
Symbols for the states of the system are					
0		operative unit			
cs		cold standby unit			
Fui		failed unit under inspection of ordinary repairman			
Fur		failed unit under repair of ordinary repairman			
Frep		failed unit under replacement of ordinary repairman			
Fuie		failed unit under inspection of expert repairman			
Fre		failed unit under repair of expert repairman			
Wi		failed unit waiting for repair			
$F_{\text{UR}}$		repair of failed unit is continuing by the ordinary repairman from the previous state.			
$F_{UI}/F_{I}$	UIe	inspection of the failed unit is continuing by the ordinary/ expert repairman from the previous state			
$F_{RE}$		repair of the failed unit is continuing by the expert repairman from the previous state.			

# TRANSITION PROBABILITIES AND MEAN SOJOURN TIMES

The epochs of entry into states 0, 1, 2, 3, 4, 7, 8, 9, 10, 13 and 14 are regeneration points and thus are the regenerative states. States 5, 6, 8, 9, 10, 11, 12, 13, 14 and 15 are failed states. State 1 is down state. The transition probabilities are given by :

 $\begin{array}{ll} q_{01}(t) = \lambda e^{-\lambda t} & ; & q_{12}(t) = w(t) & ; & q_{23}(t) = p_1 e^{-\lambda t} h_1(t) \\ q_{24}(t) = p_2 e^{-\lambda t} h_1(t) & ; & q_{25}(t) = \lambda e^{-\lambda t} \ \overline{H}_1(t) : & q_{29}^{(5)}(t) = p_1 [\lambda e^{-\lambda t} \mathbb{C} \ 1] \ h_1(t) = p_1(1 - e^{-\lambda t}) \ h_1(t) \\ q_{2,10}^{(5)}(t) = p_2 [\lambda e^{-\lambda t} \ \mathbb{C} \ 1] \ h_1(t) = p_2(1 - e^{-\lambda t}) \ h_1(t) : & q_{3,0}(t) = e^{-\lambda t} \ g(t) \ ; & q_{36}(t) = \lambda e^{-\lambda t} \ \overline{G}(t); \\ q_{32}^{(6)}(t) = [\lambda \ e^{-\lambda t} \ \mathbb{C} \ 1] \ g(t) : & q_{4,15}(t) = \lambda e^{-\lambda t} \ \overline{H}_e(t) \quad ; & q_{4,13}^{(15)}(t) = p_1 [\lambda \ e^{-\lambda t} \ \mathbb{C} \ 1] \ h_e(t) \\ q_{4,14}^{(15)}(t) = p_2 [\lambda e^{-\lambda t} \ \mathbb{C} \ 1] \ h_e(t) : & q_{47}(t) = p_1 e^{-\lambda t} h_e(t) : & q_{48}(t) = p_2 e^{-\lambda t} h_e(t); & q_{70}(t) = e^{-\lambda t} \ g_e(t) ; \\ q_{72}^{(11)}(t) = [\lambda \ e^{-\lambda t} \ \mathbb{C} \ 1] \ g_e(t) = [1 - e^{-\lambda t} \ \mathbb{C} \ 1] \ g_e(t) : & q_{7,11}(t) = \lambda e^{-\lambda t} \ \overline{G}_e(t) ; & q_{80}(t) = e^{-\lambda t} h_2(t); \\ q_{82}^{(12)}(t) = [\lambda \ e^{-\lambda t} \ \mathbb{C} \ 1] \ h_2(t) : & q_{8,11}(t) = \lambda e^{-\lambda t} \ \overline{H}_2(t) & : & q_{92}(t) = g(t) \ ; & q_{10,13}(t) = p_1 h_e(t); \end{array}$ 





<u>p2he(t)</u> h2(t)

g<sub>a</sub>(t)

Fig. 1

By these transition probabilities, it can be verified that

 $p_{01} = 1, \quad p_{12} = 1, \quad p_{23} + p_{24} + p_{25} = 1, \quad p_{23} + p_{24} + p_{29}^{(5)} + p_{2,10}^{(5)} = 1; \quad p_{30} + p_{36} = 1, \quad p_{30} + p_{32}^{(6)} = 1, \\ p_{47} + p_{48} + p_{4,15} = 1, \quad p_{47} + p_{48} + p_{4,13}^{(15)} + p_{4,14}^{(15)} = 1, \quad p_{70} + p_{7,11} = 1, \quad p_{70} + p_{72}^{(11)} = 1, \\ p_{80} + p_{82}^{(11)} = 1, \quad p_{80} + p_{8,12} = 1, \quad p_{10,13} + p_{10,14} = 1, \quad p_{13,2} = 1, \quad p_{14,2} = 1 \quad (51-65) \\ \text{The mean sojourn times } (\mu_i) \text{ in state (i) are :}$ 

$$\mu_{0} = \frac{1}{\lambda} , \qquad \qquad \mu_{1} = \int_{0}^{\infty} \overline{W}(t) dt = \int_{0}^{\infty} tw(t) dt$$
$$\mu_{2} = \int_{0}^{\infty} e^{-\lambda t} \overline{H}_{1}(t) dt = \frac{[1 - h_{1}^{*}(\lambda)]}{\lambda} , \qquad \mu_{3} = \int_{0}^{\infty} e^{-\lambda t} \overline{G}(t) dt = \frac{1 - g^{*}(\lambda)}{\lambda}$$

$$\mu_{4} = \int_{0}^{\infty} e^{-\lambda t} \overline{H}_{e}(t) dt = \frac{1 - h_{e}^{*}(\lambda)}{\lambda}, \qquad \mu_{7} = \int_{0}^{\infty} e^{-\lambda t} \overline{G}_{e}(t) dt = \frac{1 - g_{e}^{*}(\lambda)}{\lambda}$$

$$\mu_{8} = \int_{0}^{\infty} e^{-\lambda t} \overline{H}_{2}(t) dt = \frac{1 - h_{2}^{*}(\lambda)}{\lambda}, \qquad \mu_{9} = \int_{0}^{\infty} t g(t) dt$$

$$\mu_{10} = \int_{0}^{\infty} t h_{e}(t) dt, \qquad \mu_{13} = \int_{0}^{\infty} t g_{e}(t) dt, \qquad \mu_{14} = \int_{0}^{\infty} t h_{2}(t) dt \qquad (66-76)$$

The unconditional mean time taken by the system to transit for any state 'j' when it is counted from epoch of entrance into state 'i' is mathematically stated as :

$$m_{ij} = \int_{0}^{\infty} tq_{ij}(t)dt = -q_{ij}*'(0)$$
(77)

Thus,

# MEAN TIME TO SYSTEM FAILURE

By probabilistic arguments, we obtain the following recursive relations for  $\phi_i(t)$ :

$$\begin{split} \phi_{0}(t) &= Q_{01}(t) \ (S) \ \phi_{1}(t) \\ \phi_{1}(t) &= Q_{12}(t) \ (S) \ \phi_{2}(t) \\ \phi_{2}(t) &= Q_{23}(t) \ (S) \ Q_{3}(t) + Q_{24}(t) \ (S) \ \phi_{4}(t) + Q_{25}(t) \\ \phi_{3}(t) &= Q_{30}(t) \ (S) \ \phi_{0}(t) + Q_{36}(t) \\ \phi_{4}(t) &= Q_{47}(t) \ (S) \ \phi_{7}(t) + Q_{48}(t) \ (S) \ \phi_{8}(t) + Q_{4,15}(t) \\ \phi_{7}(t) &= Q_{70}(t) \ (S) \ \phi_{0}(t) + Q_{7,11}(t) \\ \phi_{8}(t) &= Q_{80}(t) \ (S) \ \phi_{0}(t) + Q_{8,12}(t) \end{split}$$
(94-100)  
the mean time to gutter foilure (MTSE) when the system starts from the state (0) is

the mean time to system failure (MTSF) when the system starts from the state '0' is

$$T_{0} = \lim_{s \to 0} \frac{1 - \phi^{**}(s)}{s}$$
(101)

Using L'Hospital rule and putting the value of  $\phi_0^{**}(s)$ , we have

 $T_0$ 

$$= N/D \tag{102}$$

where  $N = \mu_0 + \mu_1 + \mu_2 + p_{23}\mu_3 + p_{24}(\mu_4 + p_{47}\mu_7 + p_{48}\mu_8)$  (103)

$$D = 1 - p_{23}p_{30} + p_{24}(p_{47}p_{70} + p_{48}p_{80})$$
(104)

# **AVAILABILITY ANALYSIS**

In steady-state, the availability of the system is given by

$$A_0 = \lim_{s \to 0} \{s A_0^*(s)\} = \frac{N_1}{D_1}$$
(105)

where  $N_1 = \mu_0(p_{23}p_{30} + p_{24}p_{47}p_{70} + p_{24}p_{48}p_{80}) + \mu_2 + \mu_{23}p_3 + p_{24}(\mu_4 + p_{47}\mu_7 + p_{48}\mu_8)$ 

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and  $D_{1} = (\mu_{0} + \mu_{1}) (p_{23}p_{30} + p_{24}p_{47}p_{70} + p_{24}p_{48}p_{80}) + k_{1} + (p_{24} + p_{2,10}^{(5)})\mu_{10} + (p_{24}p_{47} + p_{2,10}^{(5)}p_{10,13} + p_{24}p_{4,13}^{(15)})\mu_{13} + (p_{23} + p_{29}^{(5)})\mu_{9} + (p_{24}p_{48} + p_{24}p_{4,14}^{(15)} + p_{2,10}^{(5)} + p_{10,14})\mu_{14}$ (106-107)

## BUSY PERIOD ANALYSIS OF ORDINARY REPAIRMAN (Repair Time Only)

In steady-state, the total fraction of time for which the system is under repair of ordinary repairman is given by

$$B_0 = \lim_{s \to 0} \{s B_0^*(s)\} = \frac{N_2}{D_1}$$
(108)

where

$$N_2 = (p_{23} + p_{29}{}^{(5)})\mu_9 \tag{109}$$

 $D_1$  is already specified.

#### BUSY PERIOD ANALYSIS OF ORDINARY REPAIRMAN (Inspection Time Only)

In steady-state, the total fraction of the time for which the system is under inspection of ordinary repairman is given

$$BI_0 = \frac{N_3}{D_1} \tag{110}$$

by

$$\mathbf{N}_3 = \mathbf{k}_1 \tag{111}$$

and  $D_1$  is already specified.

# BUSY PERIOD ANALYSIS OF ORDINARY REPAIRMAN (Replacement Time Only)

In steady-state, the total fraction of the time for which the system is under replacement of ordinary repairman is given by

$$BR_0 = \frac{N_4}{D_1} \tag{112}$$

where  $N_4 = (p_{24}p_{48} + p_{24}p_{4,14}^{(15)} + p_{2,10}^{(5)}p_{10,14})\mu_{14}$  (113)

and  $D_1$  is already specified.

#### EXPECTED NUMBER OF VISITS BY THE REPAIRMAN

In steady-state, the number of visits per unit time by the ordinary repairman is given by

$$V_0 = \frac{N_5}{D_1}$$
 (114)

(115)

where  $N_5 = p_{24}p_{48} + p_{24}p_{4,14}^{(15)} + p_{2,10}^{(5)}p_{10,14}$ 

#### **EXPECTED NUMBER OF REPLACEMENTS**

In steady-state, the expected the number of replacements is the system is given by

$$\mathbf{R}_0 = \frac{\mathbf{N}_6}{\mathbf{D}_1} \tag{116}$$

where 
$$N_6 = p_{24} (p_{48} + p_{4,14}^{(15)}) + p_{2,10}^{(5)} + p_{10,14}$$
 (117)

## BUSY PERIOD ANALYSIS OF EXPERT REPAIRMAN (Repair Time Only)

In steady-state, the total fraction of time for which the system is under repair of expert repairman is given by

$$B_0^e = \frac{N_7}{D_1}$$
(118)

where  $N_7 = p_{24}p_{47}\mu_7 + (p_{24}p_{4,13}^{(15)} + p_{2,10}^{(5)}p_{10,13})\mu_{13}.$  (119) PUSY DEDIOD ANALYSIS OF EXPERT DEPAIDMAN (Inspection Time Only)

BUSY PERIOD ANALYSIS OF EXPERT REPAIRMAN (Inspection Time Only)

In steady-state, the total fraction of the time for which the system is under inspection of expert repairman is given

by

$$BI_0^{e} = \frac{N_8}{D_1}$$
 (120)

where  $N_8 = (p_{24} + p_{2,10}^{(5)})\mu_{10}$  (121)

# EXPECTED NUMBER OF VISITS BY THE EXPERT REPAIRMAN

In steady-state, the total number of visits of the expert on the system is given by

$$V_0^e = \frac{N_9}{D_1}$$
 (122)

where

$$N_9 = p_{24} + p_{2,10}^{(5)}$$
(123)

(125)

## ANALYSIS OF ACTIVATION TIME

In steady-state, the total activation time of the system is given by

$$AT_{0} = \frac{N_{10}}{D_{1}}$$
(124)

where  $N_{10} = [p_{23}p_{30} + p_{24}(p_{47}p_{70} + p_{48}p_{80})]\mu_1$ and  $D_1$  is already specified.

# **PROFIT ANALYSIS**

The expected total profit incurred to the system is steady-state is given by

$$P_{32} = C_0 A_0 - C_1 B_0 - C_2 B I_0 - C_3 B R_0 - C_4 V_0 - C_5 R_0 - C_6 B_0^e - C_7 B I_0^e - C_8 V_0^e - C_9 A T_0$$
(126)

where  $C_0 =$  revenue per unit up time of the system

 $C_1 = cost per unit time for which ordinary repairman is busy in repair$ 

 $C_2 = cost per unit time for which ordinary repairman is busy in inspection$ 

 $C_3 = cost per unit time for which ordinary repairman is busy in replacement$ 

 $C_4 = cost per visit of ordinary repairman$ 

- $C_5 = Cost$  per replacement of the unit
- $C_6$  = Cost per unit time for which expert repairman is busy in repair

 $C_7 = Cost$  per unit time for which expert repairman is busy in inspection

 $C_8 = Cost per visit of the expert repairman$ 

 $C_9 = Cost per unit activation time.$ 

#### PARTICULAR CASE

Let as assume

$g(t) = \alpha_1 e^{-\alpha l t}$	;	$g_e(t) = \alpha_2 e^{-\alpha 2t}$	;	$W(t) = \beta e^{-\beta t}$
$h_1(t) = \gamma_1 e^{-\gamma_1 t}$	;	$h_e(t) = \gamma_2 e^{-\gamma_2 t}$	;	$h_2(t) = \gamma e^{-\gamma t}$

and the remaining distribution are same as in general case. Therefore, we get

$$p_{01} = p_{12} = 1$$
 ;  $p_{23} = \frac{p_1 \gamma_1}{\lambda + \gamma_1}$  ;  $p_{24} = \frac{p_2 \gamma_1}{\lambda + \gamma_1}$ 

$p_{25}=\frac{\lambda}{\lambda+\gamma_1}$	;	$p_{29}{}^{(5)}=\frac{p_1\lambda}{\lambda+\gamma_1}$	;	$p_{2,10}{}^{(5)}=\frac{p_2\lambda}{\lambda+\gamma_1}$
$p_{30}=\frac{\alpha_1}{\lambda+\alpha_1}$	;	$p_{36} = \frac{\lambda}{\lambda + \alpha_1}$	;	$p_{32}{}^{(6)}=\frac{\lambda}{\lambda+\alpha_1}$
$p_{47}=\frac{p_1\gamma_2}{\lambda+\gamma_2}$	;	$p_{48}=\frac{p_2\gamma_2}{\lambda+\gamma_2}$	;	$p_{4,15}=\frac{\lambda}{\lambda+\gamma_2}$
$p_{4,13}{}^{(15)}=\frac{p_1\lambda}{\lambda+\gamma_2}$	;	$p_{4,14}{}^{(15)}=\frac{p_2\lambda}{\lambda+\gamma_2}$	;	$p_{70}=\frac{\alpha_2}{\lambda+\alpha_2}$
$p_{72}{}^{(11)} = \frac{\lambda}{\lambda + \alpha_2}$	;	$p_{80}=\frac{\gamma}{\lambda+\gamma}$	;	$p_{82}{}^{(12)}=\frac{\lambda}{\lambda+\gamma}$
$p_{92} = 1$	;	$p_{10,13} = p_1$	;	$p_{10,14} = p_2$ :
$p_{13,2} = 1$	:	$p_{14,2} = 1$	;	$p_{7,11} = \frac{\lambda}{\lambda + \alpha_2}$
$:\!\mu_0=\frac{1}{\lambda}$	;	$\mu_1=\frac{1}{\beta}$	:	$\mu_2=\frac{1}{\lambda+\gamma_1}$
$\mu_3=\frac{1}{\lambda+\alpha_1}$	;	$\mu_4=\frac{1}{\lambda+\gamma_2}$	;	$\mu_7 = \frac{1}{\lambda + \alpha_2}$
$\mu_8=\frac{1}{\lambda+\gamma}$	;	$\mu_9=\frac{1}{\alpha_1}$	;	$\mu_{10}=\frac{1}{\gamma_2}$
$\mu_{13} = \frac{1}{\alpha_2}$	;	$\mu_{14}=\frac{1}{\gamma}$	;	$k_1 = \frac{1}{\gamma_1}$

Using the above equations and the equations (102), (105), (108), (110), (112), (114), (116), (118), (120), (122), (124) and (126). We can have MTSF, availability and profit for this particular case.

## **GRAPHICAL INTERPRETATION**

For the graphical interpretation, the mentioned particular case is considered. The behaviour of MTSF and the availability (A<sub>0</sub>) with respect to failure rate ( $\lambda$ ) for different values of repair rate ( $\alpha_1$ ) is shown as in Fig. 2 and 3 respectively. It is clear from the graphs that the MTSF and availability decrease with increase in the values of failure rate. However, their values become higher for higher values of repair rate ( $\alpha_1$ ).

Fig. 4 shows the behaviour of profit (P) with respect to cost per replacement ( $C_5$ ) for different values of probability ( $p_2$ ) that the unit is not repairable. Following conclusions can be drawn:

- (i) If  $p_2 = 0.9$ ,  $p_2 = 0.5$ , then profit will become negative for a very high value of cost (C<sub>5</sub>) and hence it can be concluded that if there are less chances of replacement. Then the system is more profitable.
- (ii) If  $p_2 = 0.9$  then P > or = or < 0 according as  $C_5 < or = or > 855$  i.e. the system is profitable if  $C_5 \le 855$ . Hence the cost per replacement should not exceed.

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