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SYSTEM PERFORMANCE MEASURES OF A MACHINE INTERFERENCE FUZZY QUEUE MODEL

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ABSTRACT:

Machine interference queue models play vital role in many manufacturing concern, textile industries, grinding machines, at service centers and Clint server computing problems. The system performance measures of these problems are breakdown rate, loss of production rate, service rate, etc. In real world situations these parameters may not be presented precisely due to uncontrollable factors i.e. the models behave fuzzy in nature. In this paper, a methodology for constructing system performance has been proposed in which break down rate and service rate are supposed trapezoidal fuzzy number. α- cut approach and fuzzy arithmetic operator are used to construct system characteristics membership function. The model finds suitable for practitioners. The validity of the proposed model has been presented through numerical illustration.

Keywords: Poisson process, Trapezoidal fuzzy number, Steady state, Effective arrival rate, α- cut.

INTRODUCTION:

The Machine interference problems are common in textile industries, where a number of automatic machines like thread cutting or wire drawing machines, are put under the change of a single operator. From time to time a machine may stop and require a corrective action by the operator, after which it again starts working. Another example of this type is that of workers, who from time to time have to stop their work in order to grind their tools at a grinding machine. If at any time, more than one machine need the operator's attention or more than one workman report for grinding tools, a queue develops. One may like to find the optimal number of machines to be assigned to an operator or the optimal number of workman assigned a grinding machine. The Machine interference problem are the important application of finite calling population variation of M/M/R : M/GD, M>R Model, in which one or more maintenance people (called Repairmen) are assigned the responsibility of maintaining in operational order from a certain group of M Machines by repairing each one that breakdown. The maintenance people are considered to be individual servers in queuing system if they work individually on different machines whereas the entire crew is considered to be single server, if crew members work together in each machine.

Assume that each member outside time (i.e. the elapsed time from leaving the system until returning for next time) has an exponential distribution with parameter λ. If at any instant out of total M machines, n are in queue system for repair, then (M-n) machines will be proportional to (M-n) with distribution exponential with parameter λ_n =(M-n) λ.

Multiple server case (R>1)

$$\lambda_n = \begin{cases} (M-n)\,\lambda & \text{for } n = 0,1,2,\dots.M \\ 0 & \text{for } n \geq M \end{cases}$$

$$\mu_n = \begin{cases} n\mu & \text{for } n = 0,1,2,\dots.R \\ R\mu & \text{for } n = R, R+1, R+2 \dots. \end{cases}$$

Most of the researchers and authors derived the system performance measure of machine interference problems and its variants when their parameters are not exact. But in real world situations these parameters may not be presented precisely due to uncontrollable factors. The breakdown and the service patterns are more suitably described by linguistic terms such as slow, medium and fast based on possibility theory. To deal with uncertain information in decision making, Zadeh (1965, 1978) introduced the concept of fuzziness. The concept of fuzzy logic has been applied by various researchers in different situations. Li & lee (1989) investigated the analytical result for two special queues M/F/1/∞ and FM/FM/1/∞ where F denotes the fuzzy time and FM denotes the fuzzified exponential distribution. Negi and Li (1992) proposed a procedure using α- cut. Chen's (1996) introduced functional principle in fuzzy inventory model. Robert & Ritha (2010) studied the machine interference model in trapezoidal fuzzy environment. W.Ritha etal (2011) proposed an inventory model with partial backordering in a fuzzy situation by employing trapezoidal fuzzy numbers. Recently Kusum and Singh T.P (2012) studied the machine repairing triangular fuzzy queue model under different parameters. We extend this model by taking trapezoidal fuzzy number and applying α- cut approach. Our work differs from Robert & Ritha in the sense that in their work, the fuzzy system performance measures have been calculated on the basis of chain function principle while in our proposed model fuzzy arithmetic & Yager's formula for defuzzification of the system has been applied.

FUZZY SET:

Fuzzy set theory was formally introduced by Zadeh in 1965; the theory is primarily concerned with quantifying the vagueness in human thoughts and perceptions. The theory also allows mathematical operators such as addition, subtraction, multiplication division etc. to be applied to the fuzzy domain. Hence, the fuzzy number can be employed in queue analysis to replace the single valued estimation for vague input or output data.

Fuzzy logic extends Boolean logic to handle the expression of vague concepts. To express impression quantitatively a set membership function maps elements to real values between 0 & 1. The value indicates the degree to which an element belongs to a set. The degree is not describing probabilities that the item is in the set, but instead describes to what extent the item is in the set.

In the universe of discourse X, a fuzzy subset $\widetilde{A}$ on X is defined by the membership function $\mu_{\tilde{A}}$ (X) Which maps each element x into X to a real number in the interval [0,1]. $\mu_{\tilde{A}}$ (X) Denotes the grade or degree of membership and it is usually denoted as

$$\mu_{\tilde{A}}(X) : X \to [0,1].$$

If a fuzzy set A is defined on X, for any $\alpha \in [0,1]$, the α-cuts ${}^{\alpha}A$ is represented by the following crisp set,

Strong α-cuts: ${}^{\alpha+}A = \{ x \in X \ / \ \mu_A(x) > \alpha\}; \alpha \in [0,1]$

Weak α-cuts: ${}^{\alpha}A = \{ x \in X \ / \ \mu_A(x) \geq \alpha\}; \alpha \in [0,1]$

Hence, the fuzzy set A can be treated as crisp set ${}^{\alpha}A$ in which all the members have their membership values greater than or at least equal to α. It is one of the most important concepts in fuzzy set theory.

Notations:

$\tilde{\lambda}$	**:**	Fuzzy arrival rate
$\tilde{\mu}$	**:**	Fuzzy service rate
$\tilde{\rho}$	**:**	Fuzzy busy time of the server = $\frac{\tilde{\lambda}}{\tilde{\mu}}$
$\widetilde{P_{ij}}$ (t)	**:**	Possibility that the system is in state (I, j) at any time t.
$\widetilde{L_s}$	:	Expected number of customers in the system.
$\widetilde{W_s}$	**:**	Expected waiting time in the system.
$\widetilde{A}$ **&** $\widetilde{B}$	**:**	Trapezoidal Fuzzy Number

TRAPEZOIDAL FUZZY NUMBER:

$$\widetilde{A} = \begin{cases} \frac{x-a_1}{a_2-a_1}, & a_1 \le x \le a_2, \\ 1 & a_2 \le x \le a_3, \\ \frac{a_4-x}{a_4-a_3}, & a_3 \le x \le a_4, \\ 0, & \text{otherwise} \end{cases}$$

TRAPEZOIDAL FUZZY NUMBER OPERATION:

Let $\widetilde{A} = (a_1,a_2,a_3,a_4)$ and $\tilde{B} = (b_1,b_2,b_3,b_4)$ be two trapezoidal fuzzy number, then the arithmetic operation on $\widetilde{A}$ and $\tilde{B}$ are given as follows :

Addition $\widetilde{A} + \tilde{B} = [a_1+b_1, a_2+b_2, a_3+b_3, a_4+b_4]$

Subtraction $\widetilde{A} - \tilde{B} = [a_1-b_4, a_2-b_3, a_3-b_2, a_4-b_1]$

Multiplication $\widetilde{A} * \tilde{B} = [a_1b_1, a_2b_2, a_3b_3, a_4b_4]$

Division $\widetilde{A} / \tilde{B} = [a_1/b_4, a_2/b_3, a_3/b_2, a_4/b_1]$

Provided $\widetilde{A}$ *and* $\tilde{B}$ are all non-zero positive numbers.

DEFUZZIFICATION OF TRAPEZOIDAL FUZZY NUMBER:

If $\widetilde{A} = (a_1,a_2,a_3,a_4)$ is a trapezoidal fuzzy number, then its associated crisp number is given by Yager's formula as follows:

$$A = \frac{a_1+2a_2+2a_3+a_4}{6}$$

PERFORMANCE MEASURES:

In this paper, we have made an attempt to find the following fuzzy performance measures that are commonly used in traditional queuing theory

1. Operator utilisation i.e. $\tilde{\rho}$.
2. Average number of machine waiting for service i.e. $\widetilde{W_q}$.
3. Average number of machine out of action i.e. breakdown i.e. $\widetilde{L_q}$
4. Effective arrival rate i.e. $\widetilde{\lambda_{eff}}$

MODEL DESCRIPTION & NOTATION:

Consider, there are M machines which are serviced by R repairmen. Whenever a machine breaks down, resulting a loss of production unit it is repaired. Consequently, a broken machine cannot generate new calls while in service. This is equivalent to the finite calling source with maximum limit of M potential customers.

In the model, the approximate probability of a single service during an instant Δt is $n\tilde{\mu}\Delta t$ for $n \le R$, and $R\tilde{\mu}\Delta t$ for $n \ge R$. On the other hand, the probability of a single arrival during Δt is approximately $(M-n)\tilde{\lambda}\Delta t$ for $n \le M$, where $\tilde{\lambda}$ is defined as the fuzzy breakdown rate per machine in fuzzy number.

Once a machine is to be repaired it turns to good condition and again susceptible to breakdown. The length of time that machine is in good condition follows an exponential rate with breakdown rate $\tilde{\lambda}$ and repair rate $\tilde{\mu}$ which are in trapezoidal fuzzy number.

In transient state:

Define $\widetilde{P_n}(t)$ be the possibility measure of 'n' customers at any time't' the differential difference equation can be modelled as:

$$\widetilde{P_0}(t+\Delta t) = \widetilde{P_0}(t)\left(1 - M\,\tilde{\lambda}\Delta t\right) + \widetilde{P_1}(t)\tilde{\mu}\Delta t\{1-(M-1)\,\tilde{\lambda}\,\Delta t\}, \text{for } n=0 \quad (1)$$

$$\widetilde{P_n}(t+\Delta t) = \widetilde{P_n}(t)\{1-(M-n)\tilde{\lambda}\Delta t\}(1-n\tilde{\mu}\Delta t) + \widetilde{P_{n-1}}(t)\{(M-n+1)\tilde{\lambda}\Delta t\}\{1-(n-1)\tilde{\mu}\Delta t\} + \widetilde{P_{n+1}}(t)\{1- M-n-1\lambda\Delta t n+1\mu\Delta t, \text{for } 0<n<R, \quad (2)$$

$$\widetilde{P_R}(t+\Delta t) = \widetilde{P_R}(t)\{1-(M-R)\tilde{\lambda}\Delta t\}(1-R\tilde{\mu}\Delta t) + \widetilde{P_{R-1}}(t)\{(M-R+1)\tilde{\lambda}\Delta t\}\{1-(R-1)\tilde{\mu}\Delta t\} + \widetilde{P_{R+1}}(t)\{1-(M-R-1)\tilde{\lambda}\Delta t\}\{R\tilde{\mu}\Delta t\}, \text{for } n=R, \quad (3)$$

$$\widetilde{P_n}(t+\Delta t) = \widetilde{P_n}(t)\{1-(M-n)\tilde{\lambda}\Delta t\}(1-R\tilde{\mu}\Delta t) + \widetilde{P_{n-1}}(t)\{(M-n+1)\tilde{\lambda}\Delta t\}\{1-R\tilde{\mu}\Delta t\} + \widetilde{P_{n+1}}(t)\{1-(M-n-1)\tilde{\lambda}\Delta t\}\{R\tilde{\mu}\Delta t\}, \text{for } R<n\le K-1, \quad (4)$$

$$\widetilde{P_M}(t+\Delta t) = \widetilde{P_M}(t)(1-R\tilde{\mu}\Delta t) + \widetilde{P_{M-1}}(t)\{\tilde{\lambda}\Delta t\}\{1-R\tilde{\mu}\Delta t\}, \text{for } n=M, \quad (5)$$

In steady state:

The steady state condition is reached when the behavior of the system becomes independent of the time. Since in machine repairing model the initial start-ups and ending stages do not change the arrival rate and service rate of the queue and queue ultimately settle down with values of the parameters oscillating around their predictable averages hence the queue system is taken as the steady state. The steady state equations are $t\to\infty$ and $\Delta t\to 0$ we have

$M\tilde{\rho}\widetilde{P_0}=\widetilde{P_1}$, for n=0 (6)

$\{(M-n)\tilde{\rho}+n\}\widetilde{P_n}= (M-n+1)\tilde{\rho}\widetilde{P_{n-1}}+(n+1)\widetilde{P_{n+1}}$, for $0< n < R$, (7)

$\{(M-n)\tilde{\rho}+R\}\widetilde{P_n}= (M-n+1)\tilde{\rho}\widetilde{P_{n-1}}+R\widetilde{P_{n+1}}$, for $R< n \le M-1$, (8)

$R\widetilde{P_K}=\tilde{\rho}\widetilde{P_{K-1}}$, for n=K, (9)

To solve the steady state equations:

From (6), $\widetilde{P_1} = M\tilde{\rho}\widetilde{P_0}$

Substituting n=1 in (7), $2\widetilde{P_2} = (M-1)\tilde{\rho}\widetilde{P_1}$

By induction, it can be shown that, $(n+1)\widetilde{P_{n+1}} = (M-n)\,\tilde{\rho}\widetilde{P_n}$, $0\le n\le R$

Similarly, from equation (8) and (9) we get $R\widetilde{P_{n+1}} = (M-n)\,\tilde{\rho}\widetilde{P_n}$, for $R\le n\le M$

After solving the equations we get,

$$\widetilde{P_n} = \begin{cases} \binom{M}{n}\widetilde{\rho^n}\,\widetilde{P_0}, & 0\le n\le R, \\ \binom{M}{n}\dfrac{n!\widetilde{\rho^n}}{R!R^{n-R}}\widetilde{P_0}, & R\le n\le M. \end{cases}$$

$$\text{where } \widetilde{P_0} = \left\{\sum_{n=0}^{R}\binom{M}{n}\widetilde{\rho^n} + \sum_{n=R+1}^{M}\binom{M}{n}\frac{n!\,\widetilde{\rho^n}}{R!\,R^{n-R}}\right\}^{-1}$$

The other measures are obtained as follows:

$$\widetilde{L_q} = \sum_{n=R+1}^{M}(n-R)\widetilde{P_n} = \sum_{n=0}^{M}n\widetilde{P_n} - \left\{R-\sum_{n=0}^{R}(R-n)\widetilde{P_n}\right\} = \widetilde{L_s} - (R-\bar{R})$$

$\widetilde{L_s} = \widetilde{L_q} + (\bar{R})$ where $\bar{R}$ = expected number of idle repairmen = $\sum_{n=0}^{R}\widetilde{P_n}$

Suppose $\widetilde{\lambda_{eff}}$ define the effective arrival rate, value of $\widetilde{\lambda_{eff}}$ can be conveniently determined from

$\widetilde{L_q} = \widetilde{L_s} - \frac{\widetilde{\lambda_{eff}}}{\tilde{\mu}}$ or $\tilde{\lambda}_{eff} = \mu[\widetilde{L_s} - \widetilde{L_q}]$

$\widetilde{W_q} = \frac{\widetilde{L_q}}{\widetilde{\lambda_{eff}}}$, $\widetilde{W_s} = \frac{\widetilde{L_s}}{\widetilde{\lambda_{eff}}}$

NUMERICAL: Consider there are four machines on running, each suffers breakdown at the fuzzy rate of (1,2,3,4) there are two service man and only one man can work on machine at a time. If n machines are out of order when n>2 then (n-2) of them wait until a service man free the time to complete the repair follows the exponential distribution with fuzzy departure rate (5,6,7,8). Our objective is to find out the first distribution of number of machines out of action at a given time, average time out of action machine has to spend waiting for the repairs to start i.e. $W_{q.}$

Take $\tilde{\lambda}$= (1, 2, 3, 4) $\tilde{\mu}$= (5, 6, 7, 8)

M = total number of machines in the system = 4,

R= number of servicemen = 2

Let n= number of machines out of order

$$\tilde{\rho} = \left(\frac{1}{8}, \frac{2}{7}, \frac{1}{2}, \frac{4}{5}\right)$$

$$\widetilde{P_n} = \begin{cases} \binom{4}{n}\left(\frac{1}{8}\right)^n \widetilde{P_0}, & 0 \le n < 2 \\ \binom{4}{n}\left(\frac{2}{7}\right)^n \widetilde{P_0}, & 0 \le n < 2 \\ \binom{4}{n}\left(\frac{1}{2}\right)^n \widetilde{P_0}, & 0 \le n < 2 \\ \binom{4}{n}\left(\frac{1}{2}\right)^n \widetilde{P_0}, & 0 \le n < 2 \end{cases}$$

$$\widetilde{P_n} = \begin{cases} \binom{4}{n}\frac{n!}{2!2^{n-2}}\left(\frac{1}{8}\right)^n \widetilde{P_0}, & 2 \le n \le 4 \\ \binom{4}{n}\frac{n!}{2!2^{n-2}}\left(\frac{2}{7}\right)^n \widetilde{P_0}, & 2 \le n \le 4 \\ \binom{4}{n}\frac{n!}{2!2^{n-2}}\left(\frac{1}{2}\right)^n \widetilde{P_0}, & 2 \le n \le 4 \\ \binom{4}{n}\frac{n!}{2!2^{n-2}}\left(\frac{1}{2}\right)^n \widetilde{P_0}, & 2 \le n < 4 \end{cases} \quad \text{(A)}$$

$$\widetilde{P_0} = \left(\sum_{n=0}^{1}\binom{4}{n}\widetilde{\rho^n} + \sum_{n=2}^{4}\binom{4}{n}\widetilde{\rho^n}\frac{n!}{2!2^{n-2}}\right)^{-1}$$

Using this we get

$$\widetilde{P_0} = \left(\frac{4096}{6579}, \frac{2401}{4647}, \frac{16}{87}, \frac{625}{7713}\right) = (.62,.51,.18,.08)$$

Putting this value in (A) we get

$$\widetilde{P_n} = \begin{cases} \binom{4}{n}\left(\frac{1}{8}\right)^n 0.62, & 0 \le n < 2 \\ \binom{4}{n}\left(\frac{2}{7}\right)^n 0.51, & 0 \le n < 2 \\ \binom{4}{n}\left(\frac{1}{2}\right)^n 0.18, & 0 \le n < 2 \\ \binom{4}{n}\left(\frac{1}{2}\right)^n 0.08, & 0 \le n < 2 \end{cases} \quad \text{(B)}$$

$$\widetilde{\mathbf{P_n}} = \begin{cases} \binom{4}{n}\frac{n!}{2!2^{n-2}}\left(\frac{1}{8}\right)^n \widetilde{P_0}, & 2\le n \le 4 \\ \binom{4}{n}\frac{n!}{2!2^{n-2}}\left(\frac{\mathbf{2}}{\mathbf{7}}\right)^n \widetilde{P_0}, & 2\le n \le 4 \\ \binom{4}{n}\frac{n!}{2!2^{n-2}}\left(\frac{\mathbf{1}}{\mathbf{2}}\right)^n \widetilde{P_0}, & 2\le n \le 4 \\ \binom{4}{n}\frac{n!}{2!2^{n-2}}\left(\frac{\mathbf{1}}{\mathbf{2}}\right)^n \widetilde{P_0}, & 2\le n < 4 \end{cases} \qquad \text{(B)}$$

Average number of machines out of action is given by

$$\widetilde{L_q} = \sum_{n=2+1}^{4}(n-2)\widetilde{P_n} = \sum_{n=3}^{4}(n-2)\widetilde{P_n} = \widetilde{P_3} + 2\widetilde{P_4}$$

Putting the value in (B) for finding $\widetilde{P_3}, \widetilde{P_4}$ then we get

$$\widetilde{P_3} = \begin{cases} (.005,..05,.09,.163), & 0 \le n < 2, \\ (.0075,.075,.135,.244), & 2\le n \le 4 \end{cases}$$

$$2\widetilde{P_4} = \begin{cases} (.00030,.0066,.0112,.12), & 0 \le n < 2, \\ (.00090,.0198,.0336,.36), & 2\le n \le 4 \end{cases}$$

$$\widetilde{L_q} = \widetilde{P_3} + 2\widetilde{P_4}$$

$$= \begin{cases} (.0053,.0566,.1012,.283), & 0 \le n < 2, \\ (.0084,.0948,.1686,.424), & 2\le n \le 4 \end{cases}$$

Average time out of action machine has to spend waiting for the repairs to start is

$$\widetilde{W_q} = \frac{\widetilde{L_q}}{\widetilde{\lambda_{eff}}}$$

but $\widetilde{\lambda_{eff}} = \tilde{\lambda}\sum_{n=0}^{4}(4-n)\widetilde{P_n} = \tilde{\lambda}\left(4\widetilde{P_0} + 3\widetilde{P_1} + 2\widetilde{P_2} + \widetilde{P_3}\right)$ (C)

$$4\widetilde{P_0} = \begin{cases} (2.48, 2.04, .72, .32), & 0 \le n < 2, \\ (4.86, 4.08, 1.44, .64), & 2\le n \le 4 \end{cases}$$

$$3\widetilde{P_1} = \begin{cases} (.93, 1.74, 1.08, .75), & 0 \le n < 2, \\ (.93, 1.74, 1.08, .75), & 2\le n \le 4 \end{cases}$$

$$2\widetilde{P_2} = \begin{cases} (.10, .48, .54, .60), & 0 \le n < 2, \\ (.10, .48, .54, .60), & 2\le n \le 4 \end{cases}$$

$$\widetilde{P_3} = \begin{cases} (.005,..05,.09,.163), & 0 \le n < 2, \\ (.0075,.075,.135,.244), & 2\le n \le 4 \end{cases}$$

Putting all values in (C) we get

$$\widetilde{\lambda_{eff}} = \tilde{\lambda} \begin{cases} (3.515, 4.31, 2.43, 1.833), & 0 \le n < 2, \\ (5.99, 6.37, 3.195, 2.394), & 2 \le n \le 4 \end{cases}$$

$$= (1,2,3,4) \begin{cases} (3.515, 4.31, 2.43, 1.833), & 0 \le n < 2, \\ (5.99, 6.37, 3.195, 2.394), & 2 \le n \le 4 \end{cases}$$

Solve the above expression with the help of fuzzy operation, we get

$$= \begin{cases} (3.515, 4.31, 7.29, 7.332), & 0 \le n < 2, \\ (5.99, 12.74, 9.585, 9.56), & 2 \le n \le 4 \end{cases}$$

Similarly we can find out $\widetilde{W_q} = \frac{\widetilde{L_q}}{\widetilde{\lambda_{eff}}}$

$$\widetilde{W_q} = \begin{cases} (.00072, .0077, .011, .038), & 0 \le n < 2 \\ (.00087, .0098, .013, .07), & 2 \le n \le 4 \end{cases}$$

FUZZY QUEUE MODEL THROUGH α – CUT:

In order to reduce the range expansion of fuzzy set, decision makers can apply α -cut to reduce the final fuzzy set. The approach is to decrease the range of the final fuzzy set by increasing the degree of membership. In normal fuzzy sets, the degree of membership is from 0 to 1. If it is desirable to reduce the range of final fuzzy set, the queue analyst can increase the degree of membership from 0 to α, $\alpha \in [0, 1]$.

Then using α-cut

Given $\tilde{\lambda} = (1, 2, 3, 4)$ $\qquad \tilde{\mu} = (5, 6, 7, 8)$

$\tilde{\lambda}(\alpha) = (\alpha + 1, 4 - \alpha)$, $\qquad \tilde{\mu}(\alpha) = (\alpha + 5, 8 - \alpha)$

$$\tilde{\rho}(\alpha) = \left(\frac{\alpha+1}{8-\alpha}, \frac{4-\alpha}{\alpha+5}\right)$$

$$\widetilde{P_0} = \left(\sum_{n=0}^{1} \binom{4}{n} \widetilde{\rho^n} + \sum_{n=2}^{4} \binom{4}{n} \widetilde{\rho^n} \frac{n!}{2!2^{n-2}}\right)^{-1}$$

$$\widetilde{P_0}(\alpha) = \begin{cases} \left(\frac{-174\alpha^3 + 1203\alpha^2 + 438\alpha + 6579}{(8-\alpha)^4}\right)^{-1}, \\ \left(\frac{3\alpha^4 - 34\alpha^3 + 276\alpha^2 - 336\alpha + 6753}{(5+\alpha)^4}\right)^{-1} \end{cases}$$

$$\widetilde{P_n}(\alpha) = \begin{cases} \binom{M}{n} \widetilde{\rho^n}\, \widetilde{P_0}(\alpha), & 0 \le n \le R, \\ \binom{M}{n} \frac{n!\widetilde{\rho^n}}{R!R^{n-R}} \widetilde{P_0}(\alpha), & R \le n \le M. \end{cases}$$

For finding $\widetilde{L_q}(\alpha)$ firstly we have to find out,

$$\widetilde{L_q}(\alpha) = \widetilde{P_3}(\alpha) + 2\widetilde{P_4}(\alpha)$$

Where,

$$\widetilde{P_3}(\alpha) = \begin{cases} \binom{4}{3}\left(\frac{\alpha+1}{8-\alpha}\right)^3 \left(\frac{(8-\alpha)^4}{-174\alpha^3+1203\alpha^2+438\alpha+6579}\right), \\ \binom{4}{3}\left(\frac{4-\alpha}{\alpha+5}\right)^3 \left(\frac{(5+\alpha)^4}{3\alpha^4-34\alpha^3+276\alpha^2-336\alpha+6753}\right), \end{cases} \quad \text{for } 0 \leq n < 2$$

$$\widetilde{P_3}(\alpha) = \begin{cases} \binom{4}{3}\left(\frac{\alpha+1}{8-\alpha}\right)^3 \frac{3!}{2!2} \left(\frac{(8-\alpha)^4}{-174\alpha^3+1203\alpha^2+438\alpha+6579}\right), \\ \binom{4}{3}\left(\frac{4-\alpha}{\alpha+5}\right)^3 \frac{3!}{2!2} \left(\frac{(5+\alpha)^4}{3\alpha^4-34\alpha^3+276\alpha^2-336\alpha+6753}\right) \end{cases} \quad \text{for } 2 \leq n \leq 4.$$

$$\widetilde{P_4}(\alpha) = \begin{cases} \binom{4}{4}\left(\frac{\alpha+1}{8-\alpha}\right)^4 \left(\frac{(8-\alpha)^4}{-174\alpha^3+1203\alpha^2+438\alpha+6579}\right), \\ \binom{4}{4}\left(\frac{4-\alpha}{\alpha+5}\right)^4 \left(\frac{(5+\alpha)^4}{3\alpha^4-34\alpha^3+276\alpha^2-336\alpha+6753}\right), \end{cases} \quad \text{for } 0 \leq n < 2$$

$$\widetilde{P_4}(\alpha) = \begin{cases} \binom{4}{4}\left(\frac{\alpha+1}{8-\alpha}\right)^4 \frac{4!}{2!2^2} \left(\frac{(8-\alpha)^4}{-174\alpha^3+1203\alpha^2+438\alpha+6579}\right), \\ \binom{4}{4}\left(\frac{4-\alpha}{\alpha+5}\right)^4 \frac{4!}{2!2^2} \left(\frac{(5+\alpha)^4}{3\alpha^4-34\alpha^3+276\alpha^2-336\alpha+6753}\right) \end{cases} \quad \text{for } 2 \leq n \leq 4.$$

Now

$\widetilde{L_q}(\alpha) = \widetilde{P_3}(\alpha) + 2\widetilde{P_4}(\alpha)$

$$\widetilde{L_q}(\alpha) = \begin{cases} \frac{(\alpha+1)^3}{(8-\alpha)^4}(-2\alpha+34)\left(\frac{(8-\alpha)^4}{-174\alpha^3+1203\alpha^2+438\alpha+6579}\right) \\ \frac{(4-\alpha)^3}{(\alpha+5)^4}(2\alpha+28)\left(\frac{(5+\alpha)^4}{3\alpha^4-34\alpha^3+276\alpha^2-336\alpha+6753}\right) \end{cases} \quad \text{for } 0 \leq n < 2$$

$$\widetilde{L_q}(\alpha) = \begin{cases} \frac{(\alpha+1)^3}{(8-\alpha)^4} 54 \left(\frac{(8-\alpha)^4}{-174\alpha^3+1203\alpha^2+438\alpha+6579}\right) \\ \frac{(4-\alpha)^3}{(\alpha+5)^4} 54 \left(\frac{(5+\alpha)^4}{3\alpha^4-34\alpha^3+276\alpha^2-336\alpha+6753}\right) \end{cases} \quad \text{for } 2 \leq n \leq 4$$

For $\alpha = 0$, we get

$$\widetilde{L_q}(\alpha) = \begin{cases} (.0051, .265) & 0 \leq n < 2 \\ (.0082, .511) & 2 \leq n \leq 4 \end{cases}$$

Similarly, for different values of α, the fuzzy queue length of the system can be depicted below:

For $\alpha = .2$, we get

$$\widetilde{L_q}(\alpha) = \begin{cases} (.0086, .23) & 0 \leq n < 2 \\ (.0138, .44) & 2 \leq n \leq 4 \end{cases}$$

For $\alpha = .4$, we get

$$\widetilde{L_q}(\alpha) = \begin{cases} (.013, .20) & 0 \le n < 2 \\ (.022, .37) & 2 \le n \le 4 \end{cases}$$

For $\alpha = .6$, we get

$$\widetilde{L_q}(\alpha) = \begin{cases} (.018, .172) & 0 \le n < 2 \\ (.030, .31) & 2 \le n \le 4 \end{cases}$$

For $\alpha = .8$, we get

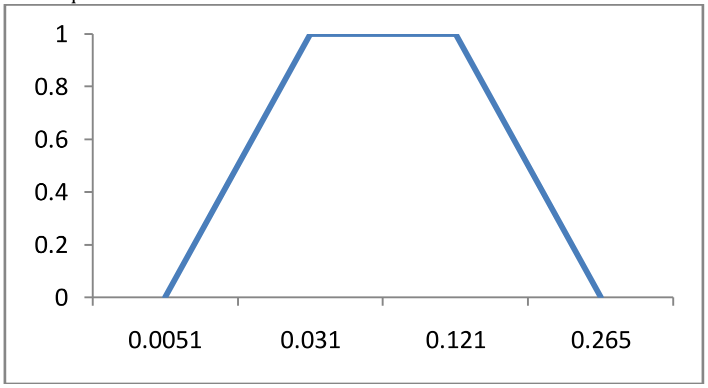
$$\widetilde{L_q}(\alpha) = \begin{cases} (.024, .14) & 0 \le n < 2 \\ (.014, .26) & 2 \le n \le 4 \end{cases}$$

For $\alpha = 1$, we get

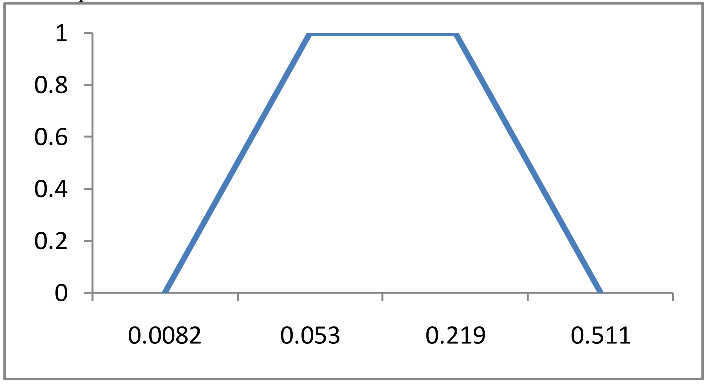
$$\widetilde{L_q} = \begin{cases} (.031, .121) & 0 \le n < 2 \\ (.053, .219) & 2 \le n \le 4 \end{cases}$$

$$\widetilde{L_q} = \begin{cases} (.0051, .031, .121, .265) & 0 \le n < 2 \\ (.0082, .053, .219, .511) & 2 \le n \le 4 \end{cases}$$

For $\widetilde{L_q}$ i.e fuzzy average queue length in interval $0 \le n < 2$,



For $\widetilde{L_q}$ i. e fuzzy average queue length in interval $2 \le n \le 4$



FUZZY QUEUE MODEL THROUGH α – CUT:

For finding $\widetilde{W}_q(\alpha) = \frac{\widetilde{L_q}(\alpha)}{\widetilde{\lambda_{eff}(\alpha)}}$ then we have:

$$\widetilde{\lambda_{eff}} = \begin{cases} (3.515, 8.62, 7.29, 7.332), & 0 \le n < 2, \\ (5.99, 12.74, 9.58, 9.56), & 2 \le n \le 4 \end{cases}$$

$$\widetilde{\lambda_{eff}}(\alpha) == \begin{cases} (5.105\alpha + 3.151, 7.33 - .04\alpha), & 0 \le n < 2, \\ (6.75\alpha + 5.99, 9.56 - .02\alpha), & 2 \le n \le 4 \end{cases}$$

$$\widetilde{W_q}(\alpha) = \left\{ \begin{array}{l} \frac{1}{(5.105\alpha+3.151)} \frac{(\alpha+1)^3}{(8-\alpha)^4} (-2\alpha + 34) \left(\frac{(8-\alpha)^4}{-174\alpha^3+1203\alpha^2+438\alpha+6579} \right) \\ \frac{1}{(7.33-.04\alpha)} \frac{(4-\alpha)^3}{(\alpha+5)^4} (2\alpha + 28) \left(\frac{(5+\alpha)^4}{3\alpha^4-34\alpha^3+276\alpha^2-336\alpha+6753} \right) \end{array} \right\} \quad \text{for } 0 \le n < 2$$

$$\widetilde{W}_q(\alpha) = \left\{ \begin{array}{l} \frac{1}{(6.75\alpha+5.99)} \frac{(\alpha+1)^3}{(8-\alpha)^4} 54 \left(\frac{(8-\alpha)^4}{-174\alpha^3+1203\alpha^2+438\alpha+6579} \right) \\ \frac{1}{(9.56-.02\alpha)} \frac{(4-\alpha)^3}{(\alpha+5)^4} 54 \left(\frac{(5+\alpha)^4}{3\alpha^4-34\alpha^3+276\alpha^2-336\alpha+6753} \right) \end{array} \right\} \quad \text{for } 2 \le n \le 4$$

For α = 0, we get

$$\widetilde{W}_q(\alpha) = \begin{cases} (.0014, .036) & 0 \le n < 2 \\ (.0013, .053) & 2 \le n \le 4 \end{cases}$$

Similarly, for different values of α, the Average fuzzy waiting time for the repairs to start in the system can be depicted below:

For α = .2, we get

$$\widetilde{W}_q(\alpha) = \begin{cases} (.0018, .031) & 0 \le n < 2 \\ (.0017, .046) & 2 \le n \le 4 \end{cases}$$

For α = .4, we get

$$\widetilde{W}_q(\alpha) = \begin{cases} (.0025, .027) & 0 \le n < 2 \\ (.0023, .038) & 2 \le n \le 4 \end{cases}$$

For α = .6, we get

$$\widetilde{W}_q(\alpha) = \begin{cases} (.0028, 0.023) & 0 \le n < 2 \\ (.0029, .032) & 2 \le n \le 4 \end{cases}$$

For α = .8, we get

$$\widetilde{W}_q(\alpha) = \begin{cases} (.0033, .019) & 0 \le n < 2 \\ (.0035, .027) & 2 \le n \le 4 \end{cases}$$

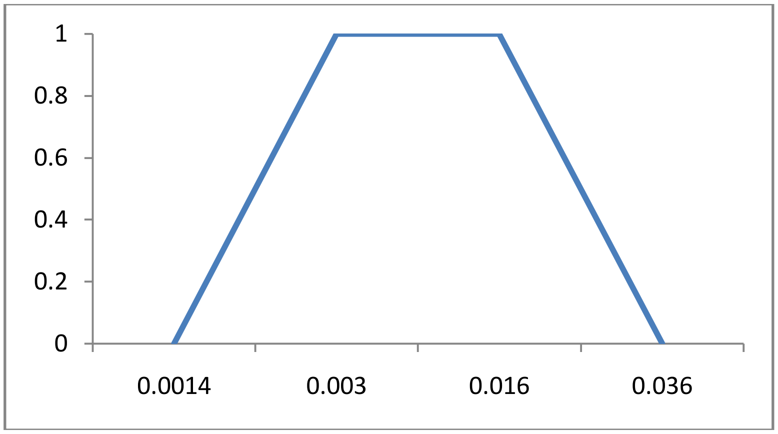
For α = 1, we get

$\widetilde{W_q}(\alpha) =$ (.0037, .016) $\quad 0 \leq n < 2$

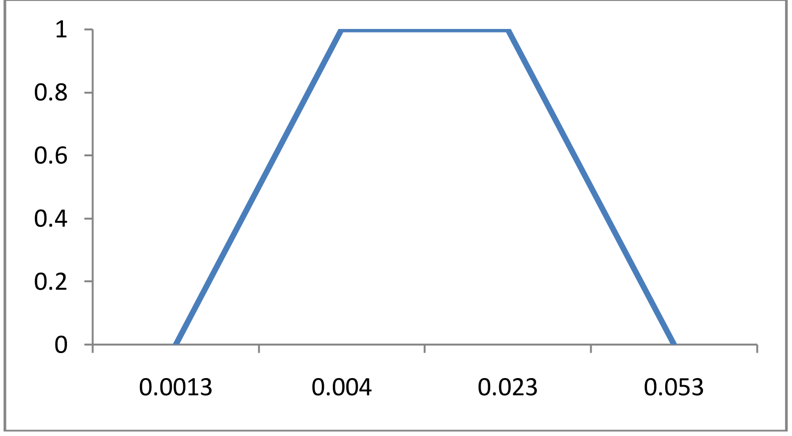
(.0041, .023) $\quad 2 \leq n \leq 4$

$$\widetilde{W_q} = \begin{cases} (.0014, .003, .016, .036) & 0 \leq n < 2 \\ (.0013, .004, .023, .053) & 2 \leq n \leq 4 \end{cases}$$

For $\widetilde{W_q}$ i.e Average fuzzy waiting time in *interval* $0 \leq n < 2$,



For $\widetilde{W_q}$ i. e Average fuzzy waiting time in interval $2 \leq n \leq 4$



CONCLUSION:

It has been observed that when the breakdown rate $\tilde{\lambda}$ and service rate $\tilde{\mu}$ are in fuzzy numbers, the performance measures in machine interfering system are expressed by fuzzy number that completely conserve the fuzziness of input information . By applying the fuzzy arithmetic operators and Yager's defuzzification, we derived the system performance measures as clear by numerical illustration. Fuzzy average number of queue system breakdown is

$$\widetilde{W_q} = \begin{cases} (.0014, .003, .016, .036) & 0 \leq n < 2 \\ (.0013, .004, .023, .053) & 2 \leq n \leq 4 \end{cases}$$

and the fuzzy average queue length for the queue system gives the

$$\widetilde{L_q} = \begin{cases} (.0051, .031, .121, .265) & 0 \leq n < 2 \\ (.0082, .053, .219, .511) & 2 \leq n \leq 4 \end{cases}$$

From the graph, it is clear that expected fuzzy waiting time cannot be below .0014 or never exceed above .036 in [0,2). Similarly, the expected fuzzy waiting time cannot be below .0013 or never exceed above .053 in [2,4] this gives the lower bound and upper bounds of the fuzzy waiting time for the said problem.

Similarly, it is clear that expected fuzzy queue length cannot be below .0051 or never exceed above .265 in [0,2). Also, the expected fuzzy queue length cannot be below .0082 or never exceed above .511 in [2,4] this gives the lower bound and upper bounds of the fuzzy queue length for the said problem.

The result obtained as Average length queue, Average waiting time in queue obtained from the proposed approach maintain the fuzziness of input data which describes the machine interfering model is more appropriate and practically better. The proposed model can be used in designing machines, repair system under fuzzy environment, since it deals with incomplete information. $\alpha -$ cut approach and fuzzy arithmetic operators which are used to construct system characteristic membership function find more suitable for practitioners.

REFRENCES:

1. D Gross, C.Harris (1998) "Fundamental of queuing theory" John Valley New York.
2. D.GrossJ.F.INC (1981) "The Machine Repairing Problem With Heterogeneous Population" Operation Research Vol.29.pp 532-549.
3. L .A. Zadeh (1978) "Fuzzy sets as a basis for theory of possibility" Fuzzy sets and system, pp.3-28.
4. R .R. Yager (1986) "A Characterisation of extension principle" Fuzzy sets and system pp.205-217.
5. Chen.S.H,Wang C.C&Ramer.A (1996) "Back order fuzzy inventory model under functional principle" Information Sciences and international Journal, pp.71-79.
6. A Nagoor Gani & Ritha .W (2006) " Fuzzy Tandem Queue" Acta Ciencia Indica Vol.xxxii M, No.1 pp257-262.
7. W.Ritha & L.Robert (2009) "Applications of fuzzy set theory to retrial queues" Inter. Journ. of algorithm , computing and mathematics Vol.2,No.4 pp 9-18.
8. Robert.L& Ritha.W (2010) "Machine interference problem with fuzzy environment" INT.J.Contemporary . MATH.Science Vol.5,39 pp.1905-1912.
9. Singh T.P & Kusum (2011) "Trapezoidal fuzzy network queue model with blocking" Arya Bhatta Journal of Mathematics and Informatics, Vol.3,No.1,pp185-192.
10. W.Ritha & Sr.Sagaya Rani Deepa,(2011) " Fuzzy EOQ Model with partial back ordering Cost" Arya Bhatta Journal Of Mathematics & Informatics Vol.3 ISSNE 2,pp 375-384.
11. Kusum & Singh T. P.(2012) " Machine Repairing Queue Model under Fuzzy Environment" Aryabhatta J. Of Mathematics & Informatics", Vol.4, No.2, pp 189-204.