ANALYTIC HIERARCHY PROCESS AND TOPSIS METHOD TO EVALUATE THE PERFORMANCE OF SCHOOLS

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Abstract-Evaluation of schools is a multi-criteria decision making (MCDM) problem. The Building up of good living status at the school level is the need of an hour.. As the honeybees fleck to the flower where there is honey, the best schools are glorified and demanded . In this paper a study has been made by applying Analytic Hierarchy Process (AHP) and Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) to evaluate the performance of schools. The proposed model yields the ranking of the schools for evaluating their performance.

Keywords-Multi-Criteria Decision Analysis, Analytical Hierarchy Process, TOPSIS model.

1. Introduction

School is an educational institution, which foster systematic learning in more or less prescribed manner. The social and psychological behavior of the child gradually transforms in a school as he interacts with his peers or teachers. This indicates that school improves and expands a child's ways of thinking in a number of ways. The school provides a structured education and promotes a child's mental and psychological growth. Aside from learning academics, a child will also learn other important life skills such as teamwork, good manners, unity, sharing, and responsibility. The present social set up is about multi focused competition. In this competitive world, our children should be thought provoking, creative and productive. It means that schools' performance evaluation has become one of the most significant actions not only in the long run of an organization but for the development of the society. This could be achieved by multi criteria decision making (MCDM).

MCDM constitutes an advanced field of operations research that is devoted to the development and implementation of decision support tools and methodologies to deal with complex decision problems involving multiple criteria, goals, or objective of conflicting nature. The most preferable situation for a MCDM problem is when all ratings of the criteria and their degree of importance are known precisely, which makes it possible to arrange them in a crisp ranking .However, many of the decision making problems in the real world take place in an environment in which the goals, the constraints and the consequence of possible actions are not known precisely. These situations imply that a real decision problems are very complicated and thus often seems to be little suited to mathematical modeling because there is no crisp definition. Consequently, the ideal condition for a classic MCDM problem may not be satisfied, in particular when the decision situation involves both fuzzy and crisp data.

In this paper, performance evaluation of schools is presented. The proposed approach is based on the two powerful MCDM techniques AHP and TOPSIS which are capable of handling linguistic information effectively for selecting the best school.

2. Literature Review

The importance of school performance evaluation and improvement has received increasing emphasis as of late,such as Anugerah Sekolah Berprestasi Tinggi, Anugerah Sekolah Bestari, Anugerah Sekolah Cemerlang, Anugerah Sekolah Harapan Negara, Anugerah Menteri Pendidikan and Anugerah 3K. This position has a considerable impact on the school as it is a form of measurement to describe the degree of excellence in school performance, school quality assessment as well as information to community [12]. This situation also creates indirect competition between students [6], and parents had the option to send their children to schools that meet the required criteria. The importance of school ranking process can show the school performance. School ranking is not implemented to punish ,but to identify which schools need help in terms of infrastructure, financial or additional teachers to develop a conducive environment for teaching and learning process. School ranking should be viewed in a positive sight so that strategic planning for each school can be done. Evaluation of best technical institutions fuzzy analytical hierarchy process was developed to tolerate vagueness and uncertainty of human judgment [3]. S. Mahmoodzadeh and et.al proposed a new methodology to provide a simple approach to assess alternative projects and help decision makers to select the best one with the help of fuzzy AHP and TOPSIS technique [13]. A number of new aggregation techniques have been presented for application to the faculty selection [2].

Though many performance evaluations had been done in various fields only a few researchers concentrated on performance evaluation of schools. Suhaina Musani[14] in their study will focus on standardized examination results. They emphasized that Academic performance evaluation is the most practical way to rank the schools because it will give us a quick and cost effective of determining the performance of the school involved. They gave the importance only to academic achievements. The holistic developments of the children depends on some additional requirements such as quality of the teachers,Board, Co-scholastic area etc.

The rest of this study is structured as follows: The first part describes important aspects for the assessment of the performance of schools and presents the evaluation framework and methodology. Next part discusses the procedure and the results of empirical studies. The final results of the empirical study are presented and discussed in the final section.

3. Evaluation framework and methods of evaluating the performance of schools

The First step of the proposed methodology is to identify important criteria affecting the performance of the school and develop best possible alternatives.Four schools were selected at the Tanjore district for the evaluation process. Nine top rated criteria were chosen which included education quality, academic achievements, school infrastructure, method of teaching, fees, the quantity& quality of the teacher, board, co-scholastic area, sophistication for the development of alternatives for selection of the best school , extensive study of decision problem is required.

The hierarchy for evaluating the performance of the Schools through the Parent's opinion is shown in Fig 1

Hierarchy of Decision

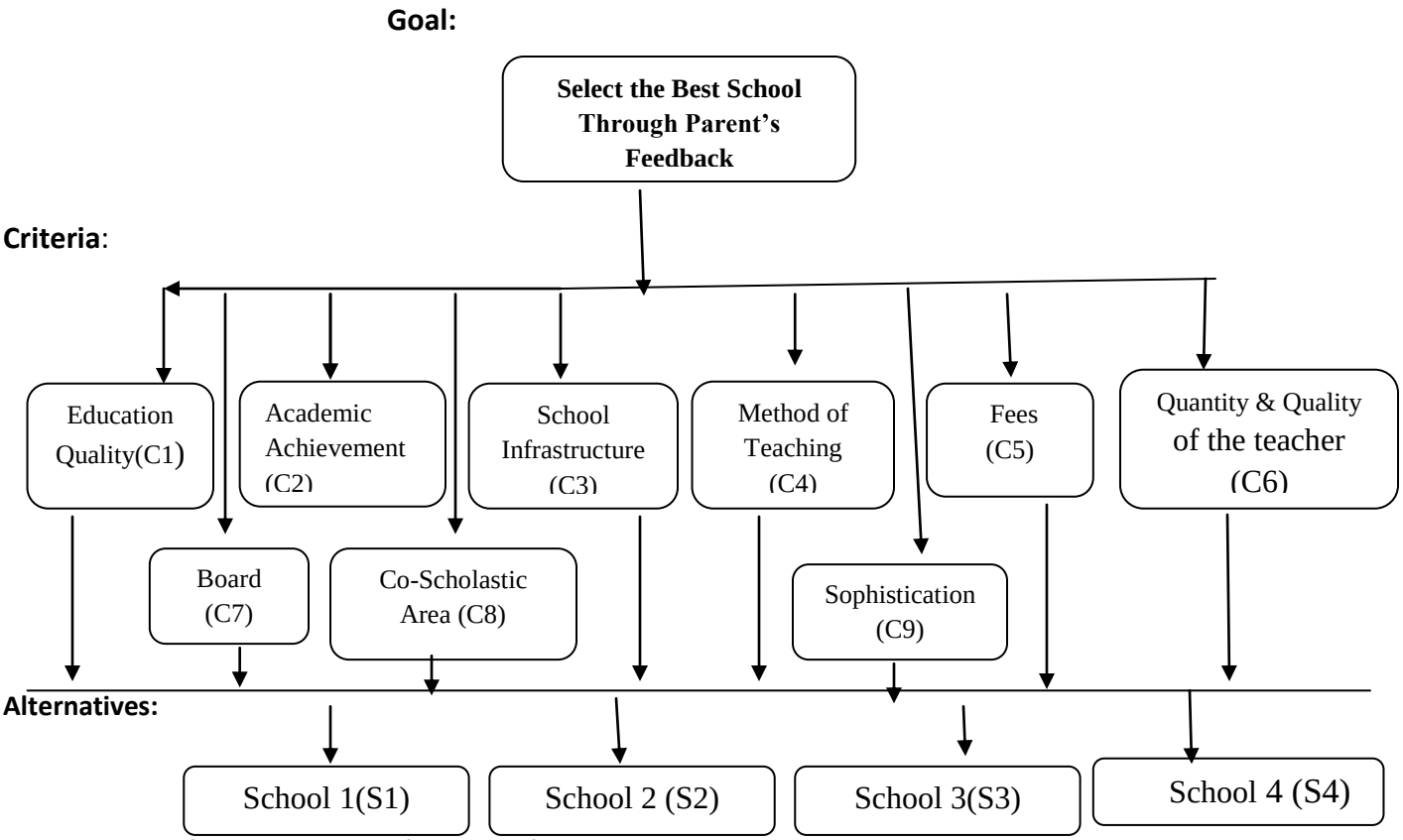

Fig1: Hierarchy of evaluating the performance of the schools

3.1. Analytic Hierarchy Process (AHP)

The pairwise comparison method and the hierarchical model were developed in 1980 by T.L.Saaty in the context of the Analytical Hierarchy Process (AHP) [15, 16]. It is one of the best and most widely used MCDM approaches. AHP is an approach to decision making that involves structuring multiple choice criteria into a hierarchy, assessing the relative importance of these criteria, comparing alternatives for each criterion and determining an overall ranking of the alternatives [1]. AHP helps to capture both subjective and objective evaluation measures, providing a useful mechanism for checking the consistency of the evaluation measures and alternatives suggested by the team thus reducing bias in decision making [17]. Some of its applications include technology choice [18], vendor selection of a telecommunications system [9], project selection, budget allocation. The construction of the Pairwise Comparison matrices using Satty's 9-point scale are illustrated [4] as follows:

Table 1: SAATY'S 9-POINT SCALE OF PAIR-WISE COMPARISON

Scale	Compare factor of i and j
1	Equally Important
3	Weakly Important
5	Strongly Important
7	Very Important
9	Extremely Important
2,4,6,8	Intermediate value between adjacent scales

3.2. TECHNIQUE FOR ORDER PREFERENCE BY SIMILARITY TO IDEAL SOLUTION (TOPSIS)

TOPSIS, known as one of the most classical MCDM methods, was first developed by Hwang and Yoon [2], is based on the idea that the chosen alternative should have the shortest distance from the Positive Ideal Solution (PIS) and on the other side the farthest distance of the Negative Ideal Solution (NIS). The Positive Ideal Solution maximizes the benefit criteria and minimizes the cost criteria, whereas the Negative Ideal Solution maximizes the cost criteria and minimizes the benefit criteria [19, 20]. In the process of TOPSIS, the performance ratings and the weights of the criteria are given as exact values. Abo-sinna and Amer [7] extend TOPSIS approach to solve multi-objective nonlinear programming problems. Jahanshahloo et al. [5] extends the concept of TOPSIS to develop a methodology for solving multi-criteria decision-making problems with interval data.

3.3. STEPS AHP & TOPSIS METHOD

In this paper ,we use AHP and TOPSIS methods to evaluate the performance of the schools

The steps of AHP are as follows

Step1: Selection of Experts
Step2: Identify the Attributes/Criteria
Step3: Identify the Alternatives
Step4: Design the Hierarchy
Step5: Establish the pairwise comparison of the of the Alternatives with respect to each Criteria.
Step6: Calculate the priority vectors for each of them
Step7: Establish the pairwise comparison of the Criteria
Step8: Calculate the priority vectors for each of them
Step9:calculate the overall priority vectors for the criteria and the alternatives with respect to each Criteria.
Step10:Finally, calculate the AHP ranking by multiplying them.

The following are the steps of TOPSIS

Step1:Calculate the normalized decision matrix.
Step2:Calculate the weighted normalized decision matrix.
Step3:Determine the Positive Ideal solution and Negative Ideal Solution.
Step4:Calculate Separation measures for each alternative from the Positive and Negative ideal Solution.
Step5:Calculate the relative closeness to the ideal Solution for each alternative.
Step6:Rank the preference order.

4. The Empirical study of performance evaluation of schools

After the identification of criteria and development of alternatives, evaluation of criteria and alternatives is done by the selected parents. Parents are asked to give opinions in the form of verbal judgments. Based on the type of evaluation, verbal judgments are developed for the study. Verbal judgments from different parents are going to be used as an input for the AHP & TOPSIS algorithm. The questioners are filled through direct communication with them. Parents give their own remarks for each school for each criterion which is shown in the Table 2

Table 2-Parents opinion against each school

Criteria	Education Quality(C1)					
School	P1	P2	P3	P4	P5	P6
S1	VSI	EXP	SVS	VSP	VSE	VSP
S2	EXP	VSP	EXP	VSE	VSP	VSE
S3	SVS	VSE	SVS	SP	VSE	VSP
S4	SP	VSP	SVS	SP	VSP	MS

Criteria	Academic Achievements(C2)					
School	P1	P2	P3	P4	P5	P6
S1	SP	VSP	VSE	SP	SVS	EXP
S2	VSP	SVS	VSP	MS	EXP	EXP
S3	SVS	SP	VSP	SVS	SP	EXP
S4	MS	SVS	VSP	MS	SP	SVS

Criteria	School Infrastructure(C3)					
School	P1	P2	P3	P4	P5	P6
S1	MS	VSE	EM	VSP	SP	SVS
S2	MS	SVS	SP	SVS	MS	EM
S3	MS	SVS	SP	MP	SP	MS
S4	VSE	SP	SVS	EXP	SVS	VSE

Criteria	Method of Teaching(C4)					
School	P1	P2	P3	P4	P5	P6
S1	SP	SVS	MS	EP	VSP	SP
S2	VSP	SP	SVS	VSP	EXP	VSE
S3	SP	SVS	EP	MS	VSP	VSP
S4	SP	MP	SVS	MS	EP	SP
Criteria	Fees(C5)					
School	P1	P2	P3	P4	P5	P6
S1	SP	MS	EM	MP	EM	MP
S2	EXP	VSE	EXP	VSP	SVS	SVS
S3	EM	MS	MP	SP	MP	EP
S4	SP	SVS	MP	EM	MS	MS
Criteria	Quantity& Quality of the Teacher(C6)					
School	P1	P2	P3	P4	P5	P6
S1	VSE	SVS	VSP	VSE	VSP	EXP
S2	VSE	SVS	VSP	SVS	VSE	VSP
S3	VSP	SVS	VSE	SVS	MS	SP
S4	MS	SP	MS	EM	SVS	SP

Criteria	Board(C7)					
School	P1	P2	P3	P4	P5	P6
S1	MS	SP	SVS	MP	VSE	SVS
S2	VSE	SVS	SP	SVS	EM	SP
S3	MS	SP	EM	EP	EM	MS
S4	SVS	EM	SVS	SP	EP	MS

Criteria	Co-Scholastic area(C8)					
School	P1	P2	P3	P4	P5	P6
S1	VSP	SP	EM	SVS	SVS	MS
S2	VSE	SP	SVS	MS	EXP	MS
S3	EM	EP	MS	SVS	MP	SP
S4	VSP	MP	SVS	VSE	EM	MS

Criteria	Sophistication (C9)					
School	P1	P2	P3	P4	P5	P6
S1	SP	SVS	SP	MS	EP	MS
S2	SP	EP	MS	SV	EM	SVS
S3	MP	SVS	EM	VSP	SP	EP
S4	EP	EM	SP	MS	MP	SVS

According to the verbal judgements of the parents the final ranking is calculated by AHP as follows

Step-1:

From the Table 2, the pairwise comparison matrix according to Satty's scale mentioned in Table1 of parent -1 for the criteria Education Quality(C1) is as follows:

Table 3: Pairwise comparison matrix for C1

Education Quality	S1	S2	S3	S4
S1	1	4	4	5
S2	0.25	1	0.333	0.25
S3	0.25	3	1	2
S4	0.2	4	0.5	1

Step-2:

Calculate the column sum $\sum_i C_{ij}$ for each column in table.4.

Table 4:. Column sum for C1

Education Quality	S1	S2	S3	S4
S1	1	4	4	5
S2	0.25	1	0.333	0.25
S3	0.25	3	1	2
S4	0. 2	4	0.5	1
Sum	1.7	12	5.833	8.25

Step-3:

Standardized each cell $X_{ij} = \frac{C_{ij}}{\sum_i C_{ij}}$

Table 5: Standardized matrix for C1

Education Quality	S1	S2	S3	S4
S1	0.588	0.333	0.686	0.606
S2	0.147	0.083	0.057	0.030
S3	0.147	0.25	0.171	0.242
S4	0.118	0.333	0.086	0.121

Step-4:

Calculate the Priority Vector by adding the sum of the rows:

Table 6: Priority Vector for each alternative with respect to C1

Education Quality	S1	S2	S3	S4	Row Sum	Average
S1	0.588	0.333	0.686	0.606	0.553	
S2	0.147	0.083	0.057	0.030	0.079	
S3	0.147	0.25	0.171	0.242	0.203	
S4	0.118	0.333	0.086	0.121	0.165	

Step-5:

Table 7. Shows the priority vector for Education Quality for the Parents P1 to P6

Table 7

P/S	S1	S2	S3	S4
P1	**0.553**	0.079	0.203	0.165
P2	**0.103**	0.207	0.426	0.265
P3	**0.082**	0.184	0.199	0.535
P4	**0.447**	0.108	0.161	0.284
P5	**0.054**	0.670	0.162	0.116
P6	**0.070**	0.619	0.185	0.126
Final Priority	**0.218**	0.311	0.223	0.249

Similarly, the priority vectors for the remaining criteria are calculated.An overall priority vector of the alternatives w.r.t. each criteria are given in the table below

Table 8

School	S1	S2	S3	S4
Criteria				
C1	0.218	0.311	0.223	0.249
C2	0.297	0.265	0.176	0.261
C3	0.385	0.356	0.126	0.145
C4	0.196	0.374	0.209	0.226
C5	0.182	0.416	0.138	0.264
C6	0.273	0.334	0.207	0.188

C7	0.373	0.319	0.136	0.186
C8	0.351	0.338	0.164	0.149
C9	0.299	0.259	0.225	0.217

Step6:

Then construct the pairwise comparison matrix for the nine criteria in the school selection problem, as below

Table 9

Criteria	C1	C2	C3	C4	C5	C6	C7	C8	C9
C1	1.000	9.000	5.000	6.000	5.000	1.000	6.000	7.00	5.000
C2	0.111	1.000	7.000	5.000	8.000	1.000	5.000	4.000	6.000
C3	0.200	0.143	1.000	2.000	1.000	3.000	4.000	5.000	2.000
C4	0.167	0.200	0.500	1.000	2.000	4.000	5.000	6.000	5.000
C5	0.200	0.125	1.000	0.500	1.000	5.000	1.000	4.000	3.000
C6	1.000	1.000	0.333	0.250	0.200	1.000	2.000	4.000	8.000
C7	0.167	0.200	0.250	0.200	1.000	0.500	1.000	6.000	5.000
C8	0.143	0.250	0.200	0.167	0.250	0.200	0.167	1.000	0.333
C9	0.200	0.167	0.500	0.200	0.333	0.125	0.200	3.000	1.000
Column Sum	3.188	12.085	15.783	15.317	18.783	15.825	24.367	41.000	35.333

Step7:

The following Table 10 gives the overall Priorities for each criteria

Table 10

Criteria	Overall Priority
C1	0.295
C2	0.205
C3	0.095
C4	0.133
C5	0.085
C6	0.104
C7	0.057
C8	0.017
C9	0.029

Step9:

Finally, the AHP ranking of alternatives is calculated by step10, which is presented in Table 11

Table 11

S1	0.263
S2	0.325
S3	0.188
S4	0.228

The above table suggests, **the school S2** as the best school from the parents' opinion.

Now, let us consider the solution by TOPSIS method. According to the parents' opinion described in Table 2 the weight for each criteria are shown in the following table

Table 12

Criteria	P1	P2	P3	P4	P5	P6	Attribute Weights
C1	8	6	7	8	7	9	7.500
C2	7	9	4	9	5	5	6.500
C3	3	2	6	4	5	5	4.167
C4	2	4	6	5	3	6	4.333
C5	5	6	8	7	6	4	6.000
C6	7	8	6	7	5	9	7.000
C7	5	8	5	4	2	6	5.000
C8	5	2	1	4	7	6	4.167
C9	4	2	3	5	3	1	3.000

Then the weight for **school 1** according to the given criterion is shown in Table 21

Table 13: (S1)

Criteria	P1	P2	P3	P4	P5	P6	Attribute weight
C1	7	9	6	7	8	7	7.333
C2	5	7	8	5	6	9	6.667
C3	4	8	2	7	5	6	5.333
C4	5	6	4	1	7	5	4.667
C5	5	4	2	3	2	3	3.167
C6	8	6	7	8	7	9	7.500
C7	4	5	6	3	8	6	5.333
C8	7	5	2	6	6	4	5.000
C9	5	6	5	4	1	4	4.167

Similarly the weights for other schools are calculated.Using the calculated attribute weights, the decision matrix for the proposed model is given in Table 14

Table 14

Criteria	S1	S2	S3	S4
C1	7.333	8.000	6.667	5.667
C2	6.667	7.000	6.333	5.333
C3	5.333	4.500	4.500	7.000
C4	4.667	7.000	5.000	4.000
C5	3.167	7.500	3.000	4.000
C6	7.500	7.000	6.000	4.333
C7	5.333	5.333	3.000	4.000
C8	5.000	6.000	3.500	5.000
C9	4.167	4.000	4.000	3.500

The stepwise analysis of the TOPSIS method is given below,

Step1: construct Normalized decision matrix by, $r_{ij}=\frac{x_{ij}}{(\sum_i x_{ij}^2)^{\frac{1}{2}}}$ for i=1,…,m; j=1,…,n

Table 15: The Normalized decision matrix

Criteria	S1	S2	S3	S4
C1	0.526	0.574	0.478	0.407
C2	0.524	0.549	0.497	0.419
C3	0.491	0.414	0.414	0.645
C4	0.442	0.662	0.473	0.378
C5	0.332	0.785	0.314	0.419
C6	0.593	0.553	0.474	0.343
C7	0.589	0.589	0.331	0.442
C8	0.505	0.605	0.353	0.505
C9	0.531	0.509	0.509	0.446

Step2: Construct the weighted normalized decision matrix, by multiplying each row of the normalized decision matrix by its associated weight.

Table- 16: Weighted Normalized Decision Matrix

Criteria	S1	S2	S3	S4
C1	3.945	4.305	3.585	3.053
C2	3.406	3.569	3.231	2.724
C3	2.046	1.725	1.725	2.688
C4	1.915	2.868	2.049	1.638
C5	1.992	4.71	1.884	2.514
C6	4.151	3.871	3.318	2.401
C7	2.945	2.945	1.655	2.21

C8	2.104	2.521	1.471	2.104
C9	1.593	1.527	1.527	1.338

Step3:

Determining the Positive Ideal solution & Negative Ideal Solution

A* = {v1* , …, vn*}, where $v_j^* = \{\max_i(v_{ij}) if \ j \in J \ \min_i(v_{ij}) if \ j \in J'\}$

PIS d* = {4.305, 3.569, 2.688, 2.868, 4.71, 4.151, 2.945, 2.521, 1.593}

Negative ideal solution:-

A' = {v1' , …, vn' }, where $v_{j'} = \{\min_i(v_{ij}) if \ j \in J \ \max_i(v_{ij}) if \ j \in J'\}$

NIS d^- = {3.053, 2.724, 1.725, 1.638, 1.884, 2.401, 1.655, 1.471, 1.338}

Step -4: Calculate the separation measures for each alternative**.** The separation from the ideal alternative is:

$S_i^* = [\sum_j (v_j * - v_{ij}^2)]^{1/2}$ i = 1, …, m

Table 17: separation measurefrom Positive Idea Solution

Criteria	**S1**	**S2**	**S3**	**S4**
C1	0.129	0	0.518	1.568
C2	0.027	0	0.114	0.714
C3	0.412	0.927	0.927	0
C4	0.908	0	0.671	1.513
C5	7.388	0	7.986	4.822
C6	0	0.078	0.694	3.063
C7	0	0	1.664	0.540
C8	0.174	0	1.103	0.174
C9	0	0.004	0.004	0.065
Total weight s_i*	4.519	0.505	6.841	6.229

Similarly, the separation from the negative ideal alternative is:

$S_i' = [\sum_j (v_j' - v_{ij}^2)]^{1/2}$ i = 1, …, m

Table 18: separation from Negative Ideal Solution

Criteria	S1	S2	S3	S4
C1	0.796	1.568	0.283	0
C2	0.465	0.714	0.257	0
C3	0.103	0	0	0.927
C4	0.077	1.513	0.169	0
C5	0.012	7.986	0	0.397
C6	0.063	2.161	0.841	0
C7	1.664	1.664	0	0.308
C8	0.401	1.103	0	0.401
C9	0.065	0.036	0.036	0
Total weight S_i	3.323	8.373	0.793	1.017

Step -5: Calculate the relative closeness to the ideal solution Ci* and the corresponding rank of the school

$$C_i^{'} = \frac{S_i^i}{s_i^* + S_i^i} \quad 0 < C_i^* < 1$$

Table3.11. Relative Closeness & Rank of School

School	Result	Rank
S1	0.424	2
S2	0.943	1
S3	0.104	4
S4	0.140	3

From the above table, the **School S2** has been selected as the best school according to the parent's opinion.

Ranking of schools by AHP & TOPSIS

School	Rank	
	AHP	**TOPSIS**
S1	2	2
S2	1	1
S3	4	4
S4	3	3

5.Conclusion

This paper concludes that the school (s2) is the best in their performance and followed by school(s1) and school(s4). The overall performance of the school (s3) is not good enough with respect to different criteria among all other schools. It is notable that the Academic Achievements of the school (s1) are better than the schools (s2), (s3) & (s4). So it can also be concluded that in spite of having the Academic Achievements of a school may not be the best school in the parent's opinion. Every study done in this research will guide the schools to upgrade themselves with unique ideas.

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