Optimal Solution For Fuzzy Transportation Problem Using Stepping-Stone Method

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Abstract — In this paper a new algorithm namely fuzzy stepping stone method is proposed for finding a fuzzy optimal solution for a fuzzy transportation problem where the transportation cost, supply and demand are trapezoidal fuzzy numbers. The initial solution of for the fuzzy transportation problem we used the fuzzy Vogel's Approximation method and determining the optimality of obtained solution fuzzy stepping stone method is used. The solution procedure is illustrated with numerical example. It can be seen that the proposed algorithm gives a better fuzzy optimal solution to the given fuzzy transportation problem**.**

Key-words: Fuzzy transportation problem, Trapezoidal fuzzy numbers, Robus't ranking optimal solution, fuzzy stepping stone Method

INTRODUCTION

The transportation problem is an important problem which has been widely studied in Operation Research domain. It has been often used to simulate different real life problems. The transportation problem is a special linear programming problem which arises in many practical applications. In this problem we determine optimal shipping patterns between origins or sources and destinations. Several methods are introduced for ranking of fuzzy numbers. Here we use Robut's ranking method [1] which satisfies the properties of compensation, linearity and additive. In fuzzy Transportation all parameters are fuzzy numbers. Fuzzy numbers may be normal or abnormal, Triangular or Trapezoidal numbers. On the basis of this idea the Robust Ranking method [4] has been adopted a transform the fuzzy transportation problem. The idea is to transform a problem with fuzzy parameters is change for fuzzy number into crisp form. The objective of fuzzy transportation problem is to determine the transportation schedule that minimizes the total fuzzy transportation cost while satisfying the availability and requirement limits. In order to solve a transportation problem, the decision parameters such as availability, requirement and the unit transportation cost of the model must be fixed at crisp values. But in real life applications supply, demand and unit transportation cost may be uncertain due to several factors. These imprecise data may be represented by fuzzy numbers. The idea of fuzzy set was introduced by Zadeh [17] in 1965. Bellmann and Zadeh [2] proposed the concept of decision making in fuzzy environment.

Chanas and Kuchta [2], proposed a method for solving fuzzy transportation problem. Chanas et al [4] developed a method for solving fuzzy transportation problems. Liu and Kao [8] proposed a new method for the solution of the fuzzy transportation problem by using the Zadeh's extension principle. Using parametric approach, Nagoorgani and Abdul Razak [10] obtained a fuzzy solution for a two stage Fuzzy Transportation problem with trapezoidal fuzzy numbers. Omar et. al [11] also proposed a parametric approach for solving transportation problem under fuzziness. Fegad et.al[12] Pandian and Natarajan [12] proposed a fuzzy zero point method to find the fuzzy optimal solution of fuzzy transportation problems. Narayana Murthy et.al [11] also proposed Russel's method for the solution of Fuzzy Transportation problem with trapezoidal fuzzy numbers.

. In this paper, we propose a new algorithm to find the optimal solution to a fuzzy transportation problem Robust's ranking is used to change for fuzzy number into crisp form. This method is very easy to understand and to apply .A t the end, the optimal solution of a problem can be obtain in a fuzzy number or a crisp. It is important to note that the proposed algorithm avoid degeneracy and provides the fuzzy optimal solution quickly for the given fuzzy transportation problem. The rest of the paper is organized as follows. In section 2, we recall the basic concepts and the results of trapezoidal fuzzy number and their arithmetic operations. In section 3, we introduce the fuzzy transportation problem with trapezoidal fuzzy numbers fuzzy numbers and related results. In section 4, we propose a new algorithm to find the initial fuzzy feasible solution for the given fuzzy transportation problem and obtained the fuzzy optimal solution by applying the fuzzy stepping stone method and section 5 is the discussion. A numerical example is also provided to illustrate the theory developed in this paper.

2. Preliminaries

2.1 Fuzzy Set: A fuzzy set is characterized by a membership function mapping element of a domain, space, or the universe of discourse X to the unit interval [0, 1] i.e. $A = \{(x, \mu_A(x)); x \in X.\}$ Here μ_A : X → [0, 1] is a mapping called the degree of membership function of the fuzzy set A and $\mu_A(x)$ is called the membership value of x∈ X in the fuzzy set A. These membership grades are often represented by real numbers ranging from [0, 1].

2.2. Normal fuzzy set: A fuzzy set A of the universe of discourse X is called a normal fuzzy set Implying that there exist at least one x ∈ X such that $\mu_A(x) = 1$

2.3 Definition: (***α-cut of a trapezoidal fuzzy number***): The α-cut of a fuzzy number A(x) is defined as A(α) = { x : μ(x) ≥ α,

2.4 convex: fuzzy set $\tilde{A}$ is convex if $\mu_{\tilde{A}}$ $(\lambda x_1 + (1 - \lambda\ x_2)$ ≥(min($\mu_{\tilde{A}}(x_1)$, ($\mu_{\tilde{A}}(x_2)$))

2.5 Fuzzy Number: A real fuzzy number $\tilde{a}$ is a fuzzy subset of the real number R with membership function $\mu_{\tilde{a}}$ satisfying the following conditions,

(i) $\mu_{\tilde{a}}$ is continuous from R to the closed interval [0,1]

(ii) $\mu_{\tilde{a}}$ is strictly increasing and continuous on $[a_1, a_2]$

(iii) $\mu_{\tilde{a}}$ is strictly decreasing and continuous on $[a_3, a_4]$

2.6 Trapezoidal fuzzy number: A fuzzy number A=(a_1, a_2, a_3, a_4) is a trapezoidal if

$$\mu_{\tilde{A}}(x) = \begin{cases} \dfrac{x - a_1}{a_2 - a_1} & , a_1 \le x \le a_2 \\ 1 & , a_2 \le x \le a_3 \\ \dfrac{x - a_4}{a_3 - a_4} & , a_3 \le x \le a_4 \end{cases}$$

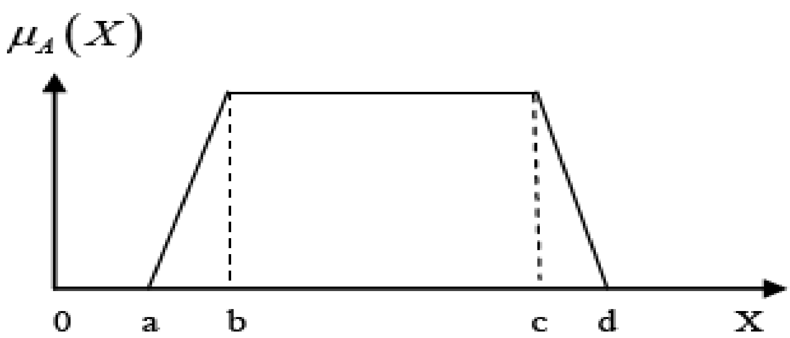

Fig. 2.1: Trapezoidal Fuzzy number

2.7 Arithmetic operation on A trapezoidal fuzzy numbers:

we define Let (a_1, a_2, a_3, a_4) and (b_1, b_2, b, b_4) be two trapezoidal fuzzy numbers. Then

(i) $\tilde{a} + \tilde{b} = (a_1, a_2, a_3, a_4) \oplus (b_1, b_2, b_3, b_4) = (a_1 + b_1, a_2 + b_2, a_3 + b_3, a_4 + b_4)$

(ii) $\tilde{a} - \tilde{b} = (a_1, a_2, a_3, a_4) \ominus (b_1, b_2, b_3, b_4) = (a_1 - b_1, a_2 - b_3, a_3 - b_2, a_4 - b_1)$

(iii) $k(a_1, a_2, a_3, a_4) = (ka_1, ka_2, ka_3, ka_4)$, for $k \geq 0$.

(iv) $k(a_1, a_2, a_3, a_4) = (ka_4, ka_3, ka_2, k a_1)$, for $k < 0$.

(v) $(a_1, a_2, a_3, a_4) \times (b_1, b_2, b_3, b_4) = (t_1, t_2, t_3, t_4)$

where t_1 = minimum $\{a_1 b_1, a_2 b_4, a_4 b_2, a_4 b_4\}$

where t_2 = minimum $\{a_2 b_2, a_2 b_3, a_3 b_2, a_3 b_3\}$

where t_3 = maximum $\{a_2 b_2, a_2 b_3, a_3 b_2, a_3 b_3\}$

where t_4 = maximum $\{a_1 b_1, a_2 b_4, a_4 b_2, a_4 b_4\}$

2.8 Properties of Trapezoidal Fuzzy Number:

1. Trapezoidal fuzzy number $\tilde{A} = (a_1, a_2, a_3, a_4)$ is said to be non negative trapezoidal fuzzy number if and only if $a_1 - a_3 \geq 0$.

2. A trapezoidal fuzzy number $\tilde{A} = (a_1, a_2, a_3, a_4)$ is said to be zero trapezoidal fuzzy number if and only if $a_1 = 0$, $a_2 = 0$, $a_3 = 0$, $a_4 = 0$.

3. Two trapezoidal fuzzy numbers $\tilde{A}_1 = (a_1, a_2, a_3, a_4)$ and $\tilde{A}_2 = (b_1, b_2, b_3, b_4)$ are said be equal i.e., if and only if $a_1 = b_1$, $a_2 = b_2$, $a_3 = b_3$, $a_4 = b_4$

4 .For any two triangular fuzzynumbers $\tilde{a} = (a_1, a_2, a_3, a_4)$ & $\tilde{b} = (b_1, b_2, b_3, b_4)$ in F(R) we define the ranking of $\tilde{a}$ and $\tilde{b}$ by comparing the R $(\tilde{a})$ and R$(\tilde{b})$ on R as follows:

(i) $\tilde{a} \geq \tilde{b}$ if and only if R$(\tilde{a}) \geq$ R$(\tilde{b})$

(ii) $\tilde{a} \leq \tilde{b}$ if and only if R$(\tilde{a}) \leq$ R$(\tilde{b})$

(iii) $\tilde{a} = \tilde{b}$ if and only if R$(\tilde{a})$ = R$(\tilde{b})$

2.9 Robus't Ranking Technique:

Robus't ranking technique which satisfy compensation, linearity, and additive properties and provides results which are consist human intuition. If ã is a fuzzy number then the Robus't Ranking is defined by

R($\tilde{a}$)= $\int_0^1 0.5\ (a^l_\alpha\ a^U_\alpha)\ d\alpha$ where ($a^L_\alpha\ a^U_\alpha$) is the α level cut of the fuzzy numbers

Where ($a^L_{\alpha,}$ a^U_α) ={ [(b-a α +a] , [d-(d-c α]}

.FUZZY TRANSPORTATION PROBLEM

Consider a fuzzy transportation with m sources and n destinations with triangular fuzzy numbers. Let $\tilde{a}_i$($\tilde{a}_i$≥)be the fuzzy availability at source i and $\tilde{b}_j$ ($\tilde{b}_j \geq \tilde{0}$)be the fuzzy requirement at destination j. Let c_{ij} be the fuzzy unit transportation cost from source i to destination j. Let $\tilde{x}_{ij}$ denote the number of fuzzy units to be transported from source i to destination j. Then the problem is to determine a feasible way of transporting the available amount at each source to satisfy the demand at each destination so that the total transportation cost is minimized. The mathematical formulation of the fuzzy transportation problem whose parameters are trapezoidal fuzzy numbers under the case that the total supply is equivalent to the total demand is given by

Mathematical formation of Fuzzy Transportation Table

				Supply
1	c_{11}	……	c_{1n}	$\tilde{a}_1$
:	:		:	:
	c_{m1}		c_{mn}	$\tilde{a}_m$
Demand	$\tilde{b}_1$	……	$\tilde{b}_n$	

Definition: 3.1 A set of non-negative allocations $\tilde{x}_{ij}$ which satisfies (in the sense equivalent) the row and the column restrictions is known as fuzzy feasible solution

Definition: 3.2 A fuzzy feasible solution to a fuzzy transportation problem with m sources and n destinations is said to be a fuzzy basic feasible solution if the number of positive allocations are (m+n-1). If the number of allocations in a fuzzy basic solution is less than (m+n-1), it is called fuzzy degenerate basic feasible solution.

Definition: 3.3 Any fuzzy feasible solution to the transportation problem containing origins and n destinations is said to be fuzzy non– degenerate, if it contains exactly (m + n - 1) occupied it is called fuzzy non-degenerate basic feasible solution

Definition: 3.4 A fuzzy feasible solution is said to be fuzzy optimal solution if it minimizes the total fuzzy transportation cost

Theorem:3.6 (Existence of fuzzy feasible solution)

The necessary and sufficient condition for the existence of a fuzzy feasible solution to the fuzzy transportation problem is $\sum_{i=1}^{m} \tilde{a}_i = \sum_{i=1}^{m} \tilde{b}_j$

Proof: (Necessary condition) Let there exist a fuzzy feasible solution to the fuzzy transportation problem

(FTP) Minimize z= $\sum_{i=1}^{m} \sum_{j=1}^{n} \tilde{C}_{ij} \otimes \tilde{x}_{ij}$

Subject to $\sum_{j=1}^{n} \tilde{x}_{ij} \approx \tilde{a}_i$, for i=1,2,.........m

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$\sum_{i=1}^{m} \tilde{x}_{ij} \approx \tilde{b}_j$, for j=1,2,……,n

$\tilde{x}_{ij} \succeq 0$, for i=1,2,….m and j=1,2,……n

where m = the number of supply points; n = the number of demand points;

from $\sum_{j=1}^{n} \tilde{x}_{ij} \approx \tilde{a}_i$ for i=1,2,….m we have

$\sum_{i=1}^{m} \sum_{j=1}^{n} \tilde{X}_{ij} \approx \sum_{i=1}^{m} \tilde{a}_i$

Also from $\sum_{i=1}^{m} \tilde{x}_{ij} \approx \tilde{b}_j$ for j=1,2,……,n we have

$\sum_{i=1}^{m} \sum_{j=1}^{n} \tilde{X}_{ij} \approx \sum_{i=1}^{m} \tilde{b}_j$

therefore $\sum_{i=1}^{m} \tilde{a}_i \approx \sum_{i=1}^{m} \tilde{b}_j$

(Sufficient condition) Since all $\tilde{a}_i$ and $\tilde{b}_j$ are positive, $\tilde{x}_{ij}$ must be all positive. Therefore equation (3.2) yields a feasible solution given below.

Theorem3.7: The values of the basic variables in a basic feasible solution to the transportation problem are given by the expressions of the for where, (in $\pm$ and $\mp$) the upper signs apply to some basic variables and the lower signs apply to the remaining basic variables
.

Theorem3.8: The solution of the transportation problem is never unbounded
.

Theorem3.9: A subset of the columns of the coefficient matrix of a transportation problem is linearly dependent, if and only if, the corresponding cells or a subset of them can be sequenced to form a loop

Solution of the Transportation Problem using Fuzzy Trapezoidal Numbers.

The solution of the fuzzy transportation problem is generally obtained in following two stages:

a) Initial basic feasible solution

b) Test of Optimal solution

The initial basic feasible solution can be easily obtained using the methods like North West Corner Rule, Least Cost Method or Matrix Minima Method, Vogel's Approximation Method (VAM) etc. VAM is preferred over the other methods, since the initial basic feasible solution obtained by this method is either optimal or very close to the optimal solution. Here, we discuss only VAM and using the fuzzy trapezoidal numbers. The steps involved in FVAM for finding the fuzzy initial solution are briefly enumerated below12:

Step 1: The penalty cost is found by considering the difference the smallest and next smallest costs in each row and column.

Step 2: Among the penalties calculated in step 1, the maximum penalty is chosen. If the maximum penalty occurs more than once then any one can be chosen arbitrarily.

Step 3: In the selected row or column found in step 2, the cell having the least cost is considered. An allocation is made to this cell by taking the minimum of the supply and demand values.

Step 4: Finally, the row or column is deleted which is fully fuzzy exhausted. Now, considering the reduced transportation tables repeat steps 1 - 3 until all the requirements are fulfilled.

NEW ALGORITHM TO FIND THE OPTIMAL SOLUTION TO FUZZY TRANSPORTATION PROBLEM

The Stepping stone method can be summarised as follows:

STEP 1: Determine an initial basic feasible solution using any of the three methods discussed earlier

2. Make sure that the number of occupied cells is exactly equal to m+n+-1 where m is number of rows and n is number of columns.

3. Evaluate the cost -effectiveness of shipping goods via transportation routes not currently in solution.This testing of each unoccupied cell is conducted by the following five steps as follows:
(a)Select an unoccupied cell, where a shipment should be made.

(b) Beginning at this cell, trace a closed path using the most direct route through at least three only horizontal and vertical moves. Further, since only the cell at the turning point are occupied cells used in the solution and then back to the original occupied cell and moving with considered to be on the closed path, both unoccupied and occupied boxes may be skipped over. The cells at the turning points are called Stepping stones on the path.

(c) Assingning plus(+) and minus(-)signs alternatively on each corner cell of the closed path just traced, starting with plus sign at the un occupied cell to be evaluated.

(d) Compute the 'net change in the cost' along the closed path by adding together the unit cost in each square containing the minus sign.

(e) Repeat the sub-stept (a) through sub-step (b) until 'net change' in cost has been calculated for all unoccupied cells of the transportation table.

4. Check the sign of each of the net changes. If all net changes computed are grater than or equal to zero, an optimal solution has been reached. If not, it is possible to improve the current solution and decrease total shipping costs.

5. Select the unoccupied cells having the highest negative net cost change and determine the maximum number of units that can be assigned to a cell marked with a minus sign on the closed path corresponding to this cell. Add this number to the unoccupied cell and to all other cells on the path marked with a plus sign. Subtract this number from cells on the closed path marked with a minus sign.

6. Go to step 2 and repeat the procedure until we get an optimal solution.

4 NUMERICAL EXAMPLE

Consider an example given in [6], a balanced fuzzy transportation problem in which all the decision parameters are trapozodial fuzzy numbers of the form =(a_1, a_2, a_3, a_4)

Table 2: Balanced Fuzzy Transportation problem

	1	2	3	4	supply
1	(1,2,3,4)	(1,3,4,6)	(9,11,12,14)	(5,7,8,11)	(1,6,7,12)
2	(0,1,2,4)	(-1,0,1,2)	(5,6,7,8)	(0,1,2,3)	(0,1,2,3)
3	(3,5,6,8)	(5,8,9,12)	(12,15,16,19)	(7,9,10,12)	(5,10,12,15)
Demand	(5,7,8,10)	(1,5,6,10)	(1,3,4,6)	(1,2,3,4)	(6,17,21,30)

Therefore $\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$, the problem is balanced fuzzy transportation problem. There exists a fuzzy initial basic feasible solution. It is important to note that the solution obtained by our algorithm is better than the solution obtained by them.

In this paper we use this method for ranking the objective values. The Robust ranking index R(ã) gives the representative value of fuzzy number

R($\tilde{a}$)= $\int_0^1 0.5\ (a_\alpha^l\ a_\alpha^U)$ dα where (a^L_α a_α^U) is the α level cut of the fuzzy numbers

Where ($a^L_{\alpha,}$ a^U_α) ={ [(b-a α +a] , [d-(d-c α]}

R(1,2,3,4)= R($\tilde{a}$)= $\int_0^1 0.5$ (α+1,4- α) dα= $\int_0^1 0.5$ (5) dα=2.5

R(1,3,4,6)=3.5. R(9,11,12,14)=11.5, R(5,7,8,11)=7.75 R(0,1,2,4)=1.75
R(-1,0,1,2)=0.5 R(5,6,7,8)=6.5 R(0,1,2,3)=1.5 R(3,5,6,8)=5.5
R(5,8,9,12)=8.5 R(12,15,16,19)=15.5 R(7,9,10,12)=9.5

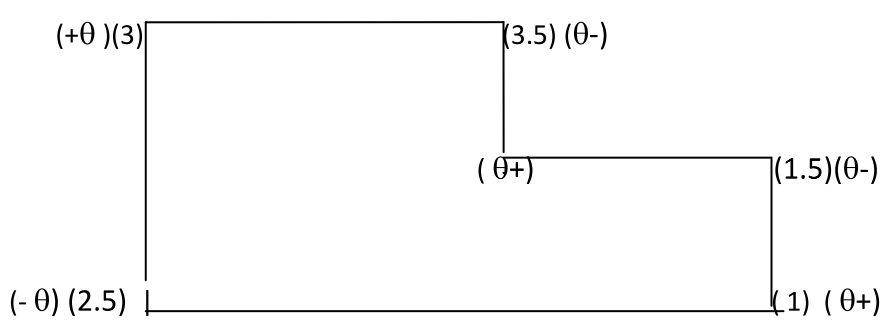
2.5	3.5 **3**	11.5 **3.5**	7.75	6.5
1.75	0.5	6.5	1.5 **1.5**	1.5
5.5 **7.5**	8.5 **2.5**	15.5	9.5 **1**	11
7.5	5.5	3.5	2.5	

The intial solution has 4+3-1=6 occupied cells and involving transportation cost Max Z=131

To determine next cost change, let us list down the changes as shown below.

unoccupaid	Closed path	Net cost change	Remarks
(1,1)	(1,1)→(3,1)→(3,2)→(1,2)	2.5-5.5+8.5-3.5=2	Cost increase
(1,4)	(1,4)→((1,2)→(3,2)→(3,4)	7.75-35+8.5-9.5=	Cost increase
(2,1)	(3,2)→(3,4)→(2,4)→(2,2)	1.75-5.5+9.5-1.5=4.25	Cost increase
(2,2)	(2,2)→ (3,2)→(3,4)→(2,4)	0.5-8.5+9.5-1.5=0	No change
(2,3)	(1,3)→(1,2)→(3,2)→(3,4)→(2,4)	6.5-11.5+3.5-8.5+9.5-1.5=-2	Cost decrease
(3,3)	(3.3)→(3,2)→(1,2)→(1,3)	15.5-8.5+3.5-11.5=-1	Cost decrease

We observe that only un occupied cell (2,3) for which the large reduction in cost change being-2 will decrease the total transportation cost by Rs.2 for unit therefore the unoccupied cell (2,3) will be consider further reduction in the cost making a closed paths to (2,3) is exactly the minimum quantity of cells with the minus sign in closed path as shown in this case minimum of –ve{3,5.2.5,1.5}=1.5



Subtract negative sign of these value (1.5) and add positive sign in above loop

unoccupaid	Closed path	Net cost change	Remarks
(1,1)	(1,1)→(1,2)→(3,2)→(3,1)	2.5-5.5+8.5-3.5=2	Cost increase
(1,4)	(1,4)→(1,2)→(3,2)→(3,4)	7.75-3.5+8.5-9.5=3.25	Cost increase
(2,1)	(2,1)→(3,1)→(3,2)→(1,2) →(1,2)→(1,3)	1.7-5.5+8.5-3.5+11.5-6.5=6.25	Cost increase
(2,2)	(2,2)→(1,2)→(1,3)→(2,3)	0.5-3.5+11.5-6.5=2	Cost increase
(2,4)	(2,4)→(3,4)→(3,2)→(1,2)→(1,3)→(2,3)	1.5-9.5+8.5-3.5+11.5-6.5=2	Cost increase
(3,3)	(3.3)→(3,2)→(1 ,2)→(1,3)	15.5-8.5+3.5-11.5=-1	Cost decrease

We observe that only un occupied cell (3,3) for which the large reduction in cost change being-1 will decrease the total transportation cost by Rs.1 for unit therefore the unoccupied cell (3,3) will be consider further reduction in the cost making a closed paths to (3,3) is exactly the minimum quantity of cells with the minus sign in closed path as shown in this case minimum of –ve{1,2}=1

2.5	3.5 **5.5**	11.5 **1**	7.75	6.5
1.75	0.5	6.5 **1.5**	1.5	1.5
5. 5 **7.5**	8.5	15.5 **1**	9.5 **2.5**	11
7.5	5.5	3.5	2.5	

The total transportation cost of the improved solution is 121

unoccupaid	Closed path	Net cost change	Remarks
(1,1)	(1,1)→(1,3)→(3,3)→(3,1)	2.5-11.5+15.5-5.5=1	Cost increase
(1,4)	(1,4)→(3,4)→(2,3)→(2,1)	7.75-9.5+15.5-11.5=2.25	Cost increase
(2,1)	(2,1)→(3,1)→(3,3)→(2,3)	1.75-5.5+15.5-6.5=5.25	Cost increase
(2,2)	(2,2)→(1,2)→(1,3)→(2,3)	0.5-3.5+11.5-6.5=2	Cost increase
(2,4)	(2,4)→→(3,4)→(3,3)→(2,3)	1.5-9.5+15.5-6.5=1	Cost increase
(3,2)	(3.2)→(3,3)→(1 ,3)→(1,2)	8.5-15.5+11.5-3.5=1	Cost increase

All loops are positive so optimal solution of fuzzy transportation problem is

2.5	3.5 5.5	11.5 **1**	7.75	6.5
1.75	0.5	6.5 **1.5**	1.5	1.5
5. 5 **7.5**	8.5	15.5 **1**	9.5 **2.5**	11
7.5	5.5	3.5	2.5	

The total transportation cost of the optimal solution is Rs.121

CONCLUSION:

In This paper the balanced fuzzy transportation problem are consider We have proposed a new algorithm for the fuzzy optimal solution to the given fuzzy transportation problem with trapezoidal fuzzy number converted into crisp transportation problem using Robust ranking indices and Stepping stone method has been applied to find an optimal solution of fuzzy numbers. First The initial solution for the fuzzy transportation problem we can use the fuzzy Vogel's Approximation method with trapezoidal fuzzy number and obtained an initial fuzzy feasible solution. By applying the fuzzy version of Stepping stone method we have tested the optimality of the fuzzy feasible solution Further, the fuzzy optimal solution obtained by the stepping stone methods is better than the fuzzy Zero point method and Russell's method.

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