Aryabhatta Journal of Mathematics & Informatics Vol. 6, No. 2, July-Dec, 2014 ISSN : 0975-7139
Journal Impact Factor (2013) : 0.489

# FUZZY MODELING OF CORTISOL SECRETION OF JOB STRAIN DUE TO STRESS USING EXTENDED HAUSDROFF DISTANCES FOR INTUITIONISTIC FUZZY SETS

**P. Senthil Kumar* & B. Mohamed Harif ****
*Assistant professor of Mathematics, Rajah Serofji Government College.
Thanjavur.(T.N): E-mail: senthilscas@yahoo.com
**Assistant professor of Mathematics, Rajah Serofji Government College.
Thanjavur.(T.N): E-mail: harif1984@gmail.com

**ABSTRACT :**
***In this paper, Fuzzy model of extended hausdroff distance for intuitonistic fuzzy sets and interval-valued fuzzy sets based on the hausdroff metric (four inputs–one output and two inputs–one output) were developed to test the hypothesis that high job demands and low job control (job strain) are associated with elevated free cortisol levels early in the working day and with reduced variability across the day and to evaluate the contribution of anger expression to this pattern. The quality of the model was determined by comparing predicted and actual fuzzy classification and defuzzification of the predicted outputs to get crisp values for correlating estimates with published values. A modified form of the Hamming distance and Euclidian distance measure is proposed to compare predicted and actual fuzzy classification. An entropy measure is used to describe the ambiguity associated with the predicted fuzzy outputs. The two inputs (high and low) predicted over 40% of the test data within one-half of a fuzzy class of the published data. Comparison of the model (men and women) shows that the hausdroff hamming distances exhibited less entropy than the hausdroff Euclidian distance.***

**Key words:** ***job strain, cortisol, anger, work stress, teaching, Intuitionistic fuzzy set, Mamdani fuzzy modeling, Hamming distance, Hausdroff Hamming distance, Hausdroff Euclidean distance, Extended Hausdroff distance***
***2000 Mathematics Subject Classification: Primary 90B22 Secondary 90B05; 60K30***

## 1. INTRODUCTION

Fuzzy set was proposed by Zadeh in 1965 as a frame work to encounter uncertainty, vagueness and partial truth. It represents a degree of membership for each member of the universe of discourse to a subset of it. Intuitionistic fuzzy set was proposed by Attanassov, [2] in 1986 which looks more accurate to uncertainty quantification and provides the opportunity to precisely model the problem based on the existing knowledge and observations. The Intuitionistic fuzzy set theory has been applied in different areas.

Fuzzy model has proved valuable in understanding the work characteristics associated with coronary heart disease risk, hypertension, mental health, quality of life, and other outcomes. This model proposes that people working in highly demanding jobs who also have low control and limited opportunities to use skills will experience high job strain. The HPA axis is one of the principal pathways activated as part of the physiological stress response. Using the concept of an intuitionistic fuzzy set that makes it possible to express many new aspects of imperfect information. For instance, in many cases information obtained cannot be classified due to lack of knowledge, discriminating power of measuring tools, etc. In such a case the use of a degree of membership and non-membership can be an adequate knowledge representation solution.

The hausdroff distances play an important role in practical application, notably in image matching, image analysis, motion tracking, visual navigation of robots, computer-assisted surgery and so on. We therefore, obtained measures of salivary cortisol throughout the working day, evening, assessed differences between day and evening as well as early morning levels. We hypothesized that job strain would be associated with elevated cortisol early in the morning together with heightened cortisol later in the day. Such a pattern might lead to reduced variability in cortisol output over the working day. An additional aim of this study was to investigate possible interactions between Job **Strain and Anger Expression.**

## 2. MAMDANI-TYPE FUZZY MODELING

This paper presents a Mamdani fuzzy modeling scheme where rules are derived from multiple knowledge sources such as previously published databases and models, existing literature, intuition and solicitation of expert opinion to verify the gathered information. The output or consequence of a Mamdani-type model is represented by a fuzzy set. To assess model performance, a crisp estimate of the consequence is usually made by defuzzification methods such as the centroid, weighted average, maximum membership principle and mean membership principle [3]. Depending on the shape of the output fuzzy set, defuzzification methods do not effectively characterize the output with the corresponding ambiguity associated with the prediction. An alternative strategy could be implemented such that the actual values of the output infer an ordinal set representing a three point fuzzy classification (low and high) that could be compared to the actual fuzzy classification using distance measures. In addition, the ambiguity associated with the predicted fuzzy sets can be quantified by calculating entropy [4].

A stochastic model for psychological effect of **Compassion & Anger** was explored by [11]. The purpose of this study was to develop generalized rule based fuzzy models from multiple knowledge sources to test the hypothesis that high job demands and low job control (job strain) are associated with elevated free cortisol levels early in the working day and with reduced variability across the day and to evaluate the contribution of **Anger Expression** to this pattern and subsequently test its performance by comparing defuzzified outputs to actual values from test data and comparing predicted and actual fuzzy classifications. The overall approach followed in this study is illustrated in Figure 1. The process begins with knowledge acquisition, continues to model building and then finally testing the model performance. In the context of fuzzy modeling, the proposed approach of converting the predicted fuzzy output and the actual crisp value into fuzzy classification sets is not well defined in literature.

Each row of membership functions constitutes an IF– THEN rule, also defined by the user. Depending on the values used, the input membership functions are activated to a certain degree. The contributed output from each rule reflects this degree of activation. The final output is a fuzzy set created by the superposition of individual rule actions (Figure 1).

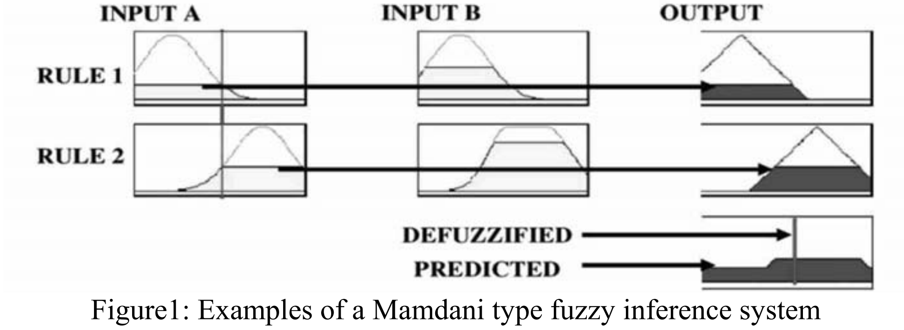



Figure1: Examples of a Mamdani type fuzzy inference system

### 2.1 DEFUZZIFICATION METHODS

The fuzzy output is obtained from aggregating the outputs from the firing of the rules. Subsequent defuzzification methods on the fuzzy output produce a crisp value. Two common techniques for defuzzification are the maxima methods and area-based methods, which are briefly explained. Several such methods are explained by Ross (1995).

## 3. DISTANCE MEASURES BETWEEN FUZZY SETS

For two fuzzy sets A and B in the same universe, the Hamming distance (HD) [5] is an ordinal measure of dissimilarity and is defined as:

$$\mathrm{HD(A,B)} = \sum_{i=1}^{n} \left| \mu_A(x_i) - \mu_B(x_i) \right|$$

where n is the number of points that define the fuzzy sets A and B, $\mu_A(x_i)$ the membership of point $x_i$ in A and $\mu_B(x_i)$ is the membership of point $x_i$ in B. The Hamming distance is smaller for fuzzy sets that are more alike than those that are less similar.

### 3.1 ENTROPY OF A FUZZY SET

Entropy is a measure of fuzziness associated with a fuzzy set. The degree of fuzziness can be described in terms of a lack of distinction between a fuzzy set and its complement. For a fuzzy set A, entropy [6] is calculated as:

$$\mathrm{E(A)} = 1 - \frac{1}{n} \sum_{i=1}^{n} \left| 2\mu_A(x_i) - 1 \right| \qquad \text{………………(1)}$$

where n is the number of points that define A, and $\mu_A(x_i)$ is the membership of point $x_i$ in A. In this study, the concept of entropy was used to quantify the ambiguity associated with the predicted fuzzy outputs. In the absence of actual values, entropy values are essentially a measure of confidence in outputs predicted by a fuzzy model.

### 3.2 PROPOSED DISTANCE MEASURE

As indicated in the theory section, a modified form of the Hamming distance is proposed which enables better distinction between different levels of classification (see Table1 and 2).

The proposed distance measure D(A, P) is defined as:

$$\mathrm{D(A,B)} = \frac{1}{4}\left(\sum_{i=1}^{n} \left| \mu_A(x_i) - \mu_B(x_i) \right| + \sum_{i,k=1(i \neq k)}^{n} (2(2|i-k|-1)\mu_A(x_i)\mu_P(x_k)\right) \quad \text{……(2)}$$

where A is the actual fuzzy classification, P the predicted fuzzy classification, n the number of classes that define A and P, $\mu_A(x_i)$ is the membership of point $x_i$ in A and $\mu_P(x_k)$ is the membership of point $x_k$ in P.

### 3.3 COMPARING FUZZY CLASSIFICATIONS

The two output membership functions created in both models are categorized as low and high. The actual value from the test data was evaluated using the parameters of these membership functions to produce a fuzzy set represented by two points (high and low). This fuzzy set represents the degree of belongingness (μ) to each of the two categories (low and high). The predicted output from the Mamdani model is a fuzzy set represented by the given points. Based on the relative contributions from each output membership function, the predicted fuzzy set of given points was reduced to a fuzzy set of three points. The relative contributions from each output membership function were estimated by integrating the predicted fuzzy set over the range of the membership function. Equations (3) were used to develop the predicted fuzzy classification:

$$\mu_P(x) = \frac{\int f(x).\mu_i(x)dx}{\int \mu_i(x)dx} \quad \ldots\ldots\ldots\ldots(3)$$

For each test case, an actual fuzzy classification and a predicted fuzzy classification were obtained. The modified Hamming distance measure (3) was used to determine the similarity between the two fuzzy sets. Apart from a comparison to actual values, the ambiguity associated with each predicted value was quantified using an entropy measure (1) as defined in the theory section.

## 3.4 DEFUZZIFYING THE PREDICTED OUTPUT

The centroid method was used to defuzzify the output of the Mamdani models. The crisp predictions were compared to the actual values from the test data and entropy value was calculated. This is a common form of comparison utilized for most modeling strategies. However, defuzzifying the output results in a loss of information regarding the ambiguity of the prediction. In the absence of actual values, the confidence in the prediction can be determined based on the degree of ambiguity.

## 3.5 INTUITIONISTIC FUZZY SETS

Intuitionistic fuzzy set was introduced first time by Atanassov, which is a generalization of an ordinary Zadeh fuzzy set. Let X be a fixed set. An intuitionistic fuzzy set A in X is an object having the form $A = \{(x, \mu_A(x), v_A(x))|\ x \in X\}$ where the functions $\mu_A(x), v_A(x) : X \rightarrow [0,1]$ are the degree of membership and the degree of non-membership of the element $x \in X$ to $A$ , respectively; moreover, $0 \leq \mu_A(x) + v_A(x) \leq 1$ must hold.

Obviously, each fuzzy set may be represented by the following intuitionistic fuzzy set

$$A = \{(x, \mu_A(x), 1 - \mu_A(x)|\ x \in X\}$$

## 3.6 DISTANCE BETWEEN INTUITIONISTIC FUZZY SET

In Szmidt and Kacprzyk [7], [8], it is shown why in the calculation of distances between the intuitionistic fuzzy sets one should use all three terms describing them. Let A and B be two intuitionistic fuzzy set in $= \{x_1, x_2, \ldots, x_n\}$ . Then the distance between A and B while using the three term representation (Szmidt and Kacprzyk) may be as follows.

The Hamming distance:

$$d_{IFS}(A,B) = \frac{1}{2}\sum_{i=1}^{n}(|\mu_A(x_i) - \mu_B(x_i)| + |v_A(x_i) - v_B(x_i)|$$

The Euclidean distance:

$$e_{IFS}(A,B) = \sqrt{\frac{1}{2}\sum_{i=1}^{n}((\mu_A(x_i) - \mu_B(x_i))^2 + (v_A(x_i) - v_B(x_i))^2)}$$

The normalized Hamming distance:

$$l'_{IFS}(A,B) = \frac{1}{2n}\sum_{i=1}^{n}(|\mu_A(x_i) - \mu_B(x_i)| + |v_A(x_i) - v_B(x_i)|)$$

The normalized Euclidean distance:

$$q'_{IFS}(A,B) = \sqrt{\frac{1}{2n}\sum_{i=1}^{n}((\mu_A(x_i) - \mu_B(x_i))^2 + (v_A(x_i) - v_B(x_i))^2}$$

## 3.7 THE HAUSDROFF DISTANCE

Given two intervals $U = [u_1, u_2]$ and $V = [v_1, v_2]$ of Intuitionistic fuzzy set, the hausdroff metric is defined [9], $d_H(U,V) = \max\{|u_1 - v_1|, |u_2 - v_2|\}$. The hausdroff metric applied to two intuitionistic fuzzy sets, $A(x) = [\mu_A(x), 1 - v_A(x)]$ and $B(x) = [\mu_B(x), 1 - v_B(x)]$, given the following:

$$d_H(A(x), B(x)) = \max\{|\mu_A(x_i) - \mu_B(x_i)|, |v_A(x_i) - v_B(x_i)|\}$$

The following two term representation hausdroff distances between intuitionistic fuzzy sets have been proposed [9]:

The Hamming distance:

$$d_H(A,B) = \sum_{i=1}^{n} \max\ \{|\mu_A(x_i) - \mu_B(x_i)|, |v_A(x_i) - v_B(x_i)|\}$$

The normalized Hamming distance:

$$l_H(A,B) = \frac{1}{n}\sum_{i=1}^{n} \max\ \{|\mu_A(x_i) - \mu_B(x_i)|, |v_A(x_i) - v_B(x_i)|\}$$

The Euclidean distance:

$$e_H(A,B) = \sqrt{\sum_{i=1}^{n} max((\mu_A(x_i) - \mu_B(x_i))^2, (v_A(x_i) - v_B(x_i))^2)}$$

The normalized Euclidean distance:

$$q_H(A,B) = \sqrt{\frac{1}{n}\sum_{i=1}^{n} max((\mu_A(x_i) - \mu_B(x_i))^2, (v_A(x_i) - v_B(x_i))^2)}$$

The Extended haudroff - normalised Hamming distance:

$$\boldsymbol{l_{EH}(s(p_i), d_k) = \frac{1}{5}\sum_{j=1}^{5} max\{|\mu_j(p_i) - \mu_j(d_k)|\}}$$

In comparing an actual fuzzy set to the predicted fuzzy set, a small Hamming distance is ideal. In our study, the model-testing phase involved comparison of predicted and actual fuzzy classifications (low and high). From the results in Table 1, the proposed distance measure is better than the Hamming distance at distinguishing between different levels of classification. In cases e and f, the Hamming distance(HD) gave the same value for different predicted fuzzy classifications[10]. The extended Hausdroff distance gave different values that effectively distinguish between these cases.

## 4. EXAMPLE

Data were collected at the 12-month follow-up phase of a study of job strain and cardiovascular risk, details of which have been published previously [10]. Participants in the original sample were 162 junior and high school teachers, selected on the basis of scores on a work stress measure (37) as having high (28 men and 52 women) or low (32 men and 50 women) job strain scores. Eighty-five (52.5%) were classroom teachers, and 77 (47.5%) had additional administrative roles. One hundred thirty-seven teachers took part in the 12-month phase (84.6%), which consisted of ambulatory blood pressure monitoring and a psychiatric interview (to be reported elsewhere) in addition to cortisol measurements. Of the 25 who did not participate at 12 months, 10 had left teaching or retired, 7 were seriously ill or pregnant, 1 experienced equipment failure, and 7 did not respond to our invitation. Comparisons between the 137 participants and 25 who dropped out of the study revealed no significant differences in gender, job strain scores, age, grade of employment, or scores on negative affect or anger expression. An additional 15 of the 137 individuals refused to sample saliva during the working day, mainly because they envisaged that data collection might be embarrassing or inconvenient at school. Statistical comparisons of these individuals with the remainder again identified no differences on demographic or psychological variables.

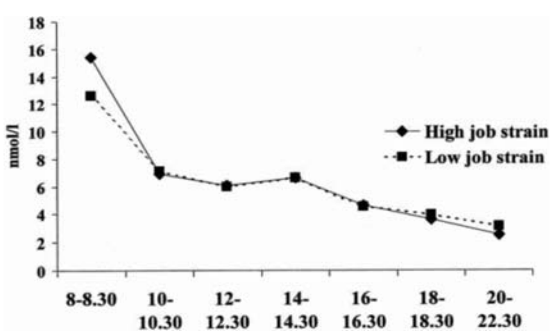



*Figure 2. Mean concentration of saliva free cortisol in high and low job strain groups across the day and evening.*

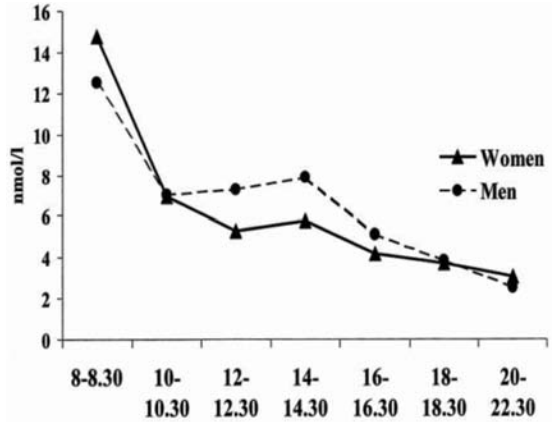



*Figure 3. Mean concentration of saliva free cortisol in men and women across the day and evening.*

*Fuzzy function of the given figure 2 and 3 is defined as*

$$f(x) = \begin{cases} -5x + 1.5, x \in [0,0.2] \\ 0.5, x \in [0.2,0.4] \\ -x + 0.9, x \in [0.4,0.7] \end{cases}$$

$$A_1(x) = \begin{cases} 6.67x, x \in [0,0.15] \\ 1, x \in [0.15,0.25] \\ -6.67x + 2.67, x \in [0.25,0.4] \\ 0, otherwise \end{cases}$$

$$A_2(x) = \begin{cases} 5x - 1, x \in [0.2,0.4] \\ 1, x \in [0.4,0.5] \\ -5x + 3.5, x \in [0.5,0.7] \\ 0, otherwise \end{cases}$$

Corresponding Fuzzy diagram are given in figure 4.

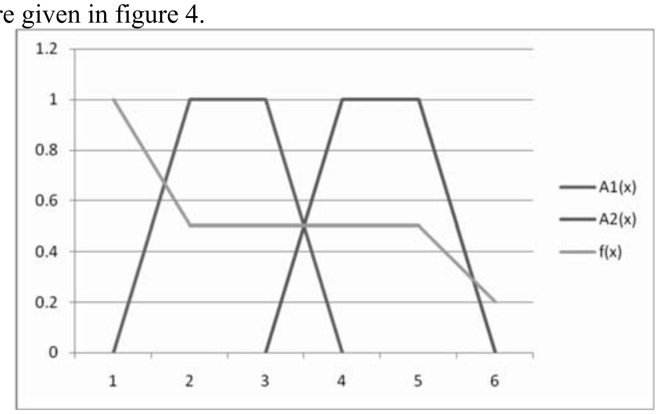



*Figure 4. Fuzzy Mean concentration of saliva free cortisol in high and low job strain groups across the day and evening and Fuzzy Mean concentration of saliva free cortisol in men and women across the day and evening.*

Saliva sampling was conducted on a working day at schools. Participants were asked to take eight saliva samples at 2-hour intervals, and a 30-minute time window was allowed for each sample. Participants were asked to not consume any caffeine, citrus drinks, or food for at least 60 minutes before the saliva sample was taken. The schedule sampling sequence was therefore 8:00 to 8:30, 10:00 to 10:30, 12:00 to

12:30, 14:00 to 14:30, 16:00 to 16:30, 18:00 to 18:30, 20:00 to 20:30, and 22:00 to 22:30 hours. The first sample of the day was always obtained at schools after explanation of the procedure by the investigators. Saliva samples were collected in Salivettes, which were stored at −30°C until analysis. After defrosting, samples were spun at 3000 rpm for 5 minutes, and 100 µl of supernatant was used for duplicate analysis involving a time-resolved immunoassay with fluorescence detection.

| Case | | Actual fuzzy classification | | Predicted fuzzy classification | | HD | Predicted Distance |
|---|---|---|---|---|---|---|---|
| | Time | High $\mu_A(x_i)$ | Low $\mu_A(x_i)$ | High $\mu_B(x_i)$ | Low $\mu_B(x_i)$ | | |
| a | 8 – 8.30 | (1, 0) | (0.83, 0.17) | (0.9, 0.1) | (0.8, 0.2) | 0.13 | 0.43 |
| b | 10 – 10.30 | (0.45, 0.55) | (0.45, 0.55) | (0.8, 0.2) | (0.8, 0.2) | 0.4 | 0.34 |
| c | 12 – 12.30 | (0.39, 0.61) | (0.39, 0.61) | (0.5, 0.5) | (0.5, 0.5) | 0.16 | 0.13 |
| d | 14 – 14.30 | (0.44, 0.54) | (0.44, 0.54) | (0.4, 0.6) | (0.4, 0.6) | 0.16 | 0.11 |
| e | 16 – 16.30 | (0.31, 0.69) | (0.31, 0.69) | (0.5, 0.5) | (0.5, 0.5) | 0.4 | 0.18 |
| f | 18 – 18.30 | (0.24, 0.76) | (0.27, 0.73) | (0.4, 0.6) | (0.5, 0.5) | 0.4 | 0.16 |
| g | 20 – 22.30 | (0.17, 0.83) | (0.21, 0.79) | (0.4, 0.6) | (0.5, 0.5) | 0.51 | 0.18 |
| Entropy Value | | (0.57, 0.43) | (0.64, 0.36) | (0.64, 0.36) | (0.8, 0.2) | | |

| Various Distance Between Intuitionistic fuzzy set | Women | Men |
|---|---|---|
| Hamming Distance | 0.54 | 0.55 |
| Euclidean Distance | 0.39 | 0.49 |
| Normalized Hamming Distance | 0.15 | 0.16 |
| Normalized Euclidean Distance | 0.15 | 0.19 |
| **Hausdorff Hamming Distance** | **0.53** | **0.54** |
| **Hausdorff Euclidean Distance** | **0.39** | **0.49** |
| **Hausdorff Normalized Hamming Distance** | **0.15** | **0.16** |
| **Hausdorff Normalized Euclidean Distance** | **0.15** | **0.19** |

*Table 1: Comparison of the various distances of Fuzzy Mean concentration of saliva free cortisol in high and low job strain groups across the day and evening*

| Case | | Actual fuzzy classification | | Predicted fuzzy classification | | HD | Predicted Distance |
|---|---|---|---|---|---|---|---|
| | Time | Women $\mu_A(x_i)$ | Men $\mu_A(x_i)$ | Women $\mu_B(x_i)$ | Men $\mu_B(x_i)$ | | |
| a | 8 – 8.30 | (1, 0) | (0.83, 0.17) | (0.9, 0.1) | (0.8, 0.2) | 0.13 | 0.43 |
| b | 10 – 10.30 | (0.6, 0.4) | (0.6, 0.4) | (0.8, 0.2) | (0.8, 0.2) | 0.7 | 0.35 |
| c | 12 – 12.30 | (0.34, 0.66) | (0.5, 0.5) | (0.5, 0.5) | (0.5, 0.5) | 0.22 | 0.15 |
| d | 14 – 14.30 | (0.37, 0.63) | (0.53, 0.47) | (0.4, 0.6) | (0.4, 0.6) | 0.08 | 0.11 |
| e | 16 – 16.30 | (0.27, 0.73) | (0.33, 0.67) | (0.5, 0.5) | (0.5, 0.5) | 0.38 | 0.17 |
| f | 18 – 18.30 | (0.25, 0.75) | (0.25, 0.75) | (0.4, 0.6) | (0.5, 0.5) | 0.39 | 0.16 |
| g | 20 – 22.30 | (0.2, 0.8) | (0.19, 0.81) | (0.4, 0.6) | (0.5, 0.5) | 0.52 | 0.17 |
| Entropy Value | | (0.52, 0.48) | (0.66, 0.36) | (0.71, 0.29) | (0.8, 0.2) | | |

| Various Distance Between Intuitionistic fuzzy set | Women | Men |
|---|---|---|
| Hamming Distance | 0.59 | 0.62 |
| Euclidean Distance | 0.51 | 0.56 |
| Normalized Hamming Distance | 0.17 | 0.18 |
| Normalized Euclidean Distance | 0.19 | 0.21 |
| **Hausdorff Hamming Distance** | **0.58** | **0.61** |
| **Hausdorff Euclidean Distance** | **0.51** | **0.56** |
| **Hausdorff Normalized Hamming Distance** | **0.18** | **0.17** |
| **Hausdorff Normalized Euclidean Distance** | **0.19** | **0.21** |

*Table 2: Comparison of the Hamming various distances of Fuzzy Mean concentration of saliva free cortisol in men and women across the day and evening.*

## Extended Hausdroff Distance

| | |
|---|---|
| High and Low | 0.152 and 0.154 |
| Women and Men | 0.16 and 0.168 |

## CONCLUSION

There were significant differences between groups in job strain and in its components job demands, job control, and skill utilization. The high job strain group reported greater demands, lower control, and less skill utilization than the low job strain group as inputs. Negative affect was significantly higher among high job strain individuals, and anger-in scores were also greater. There were no differences in anger-out ratings between groups. Using multiple knowledge sources, membership functions and rules were developed to provide generalized models not optimized for a specific data set. Apart from correlation estimates of actual and defuzzified predictions, an alternative analysis was performed involving comparison of actual and predicted fuzzy classifications. Various distances measure were used to compare actual and fuzzy classifications. The extended hausdroff distance often used to compare distances between fuzzy sets.

## REFERENCES

1. Zadeh L. A. (1965) Fuzzy sets, Information and Control, 8 pp 338-353.
2. Attanasov K(1999) Intuitionistic fuzzy sets: Theory and Applications. Physica-Verlag, Heidelberg and New York.
3. Ross T. J, Fuzzy Logic with Engineering Applications, Mc- Graw-Hill Inc., New York, 1995.
4. Hung W, A note on entropy of intuitionistic fuzzy sets, Int. J. Uncert. Fuzz. Knowledge-Based Syst. 11 (2003) 627–633.
5. Szmidt E., Kacpryzk J., Distances between intuitionistic fuzzy sets, Fuzzy Sets Syst. 114 (2000) 505–518.
6. Pham D.T, Castellani. M, Action aggregation and defuzzification in Mamdani-type fuzzy systems, in: Proceedings of the Institute of Mechanical Engineers Part C, J. Mech. Eng. Sci. 216 (2002) 747– 759.
7. Eulalia Szmidt and Janusz Kacprzyk, Intuitionistic fuzzy sets – two and three term representation in the context of a Hausdroff distance, Acta Universitatis Matthiae Bell11, series Mathematics 19 (2011) 53-62.
8. Szmidt.E and Kacprzyk J (2000) Distance between intuitionistic fuzzy sets. Fuzzy sets and systems, 114(3), 505-518.
9. Grzegorzewski, (2004) Distances between intuitionistic fuzzy sets and / or interval- valued fuzzy sets based on the hausdroff metric. Fuzzy Sets and Systems, 48:319-328,2004.
10. P. Senthil Kumar, B. Mohamed Harif, (2014) Rule-based Mamdani-type Fuzzy Modeling of perceived stress, and cortisol responses to awakening, IJERA, vol.4, 29-35.
11. S. Lakshmi & B. Geetha Rani (2012). "Stochastic model for increasing generalized failure rate for the physiological & Psychological effects of compassion & anger. " Aryabhatta J. of Maths & Info. Vol. 4, (1) pp 151-156.

Aryabhatta Journal of Mathematics & Informatics Vol. 6, No. 2, July-Dec, 2014 ISSN : 0975-7139
Journal Impact Factor (2013) : 0.489

# A RELIABILITY MODEL ON A CEMENT GRINDING SYSTEM WITH FAILURE IN ITS NINE COMPONENTS

**Ritu Gupta* & Dr. Gulshan Taneja****
* Research Scholar, Department of Mathematics, M.D. University, Rohtak, Haryana
** Professor, Department of Mathematics, M.D. University, Rohtak, Haryana
Email : r26jan@gmail.com, drgtaneja@gmail.com

**ABSTRACT :**
***The present study investigates the reliability and profit analysis of a cement grinding system with failure in the nine important components namely; Belt Conveyor, Bucket Elevator, Separator, Roller Press, Diverting Gate, Process Fan, Cyclone, Ball Mill and Fly Ash System. Only one type of failure has been considered in each of these components except that in the Diverting Gate. In Diverting Gate, two types of failure- minor and major has been taken into consideration. Also in case of Fly Ash System, occurrence of failure does not always results to the failure of the complete system. Data on failure times and cost of repairs have been collected from Shree Cement Ltd., Khushkhera, Rajasthan, India. The system has been analysed by using semi – Markov processes and regenerative point technique. Mean Time to System Failure (MTSF) and various other measures of system effectiveness have been obtained. Profit is also evaluated and graphical study is made to draw various important conclusions.***
**Keywords:** ***Reliability, Cement Grinding System, Measures of System Effectiveness, Profit Analysis.***

## 1. INTRODUCTION

Researchers' both from academia and industries have developed a number of reliability models and analysed various measures of system effectiveness. Mokaddis et al. [1997] discussed cost analysis of two dissimilar unit cold standby redundant system subject to inspection and two types of repair. Parashar and Taneja [2007] discussed the reliability and profit evaluation of a PLC hot standby system based on master-slave concept and two types of repair facilities. Goyal et al. [2009] developed a model for a 2-unit cold standby system working in a sugar mill with operating and rest periods. Mathew et al. [2009] discussed profit evaluation of a single unit CC plant with scheduled maintenance. Padmavathi el al. [2012] discussed reliability analysis of an evaporator of a desalination plant with online repair and emergency shutdown. Sharma and Gupta [2012] discussed cost benefit analysis of three unit redundant system model with correlated failures and repairs. Hashim et al. [2013] discussed reliability analysis of phased mission system considering the concept of sensitivity, uncertainty and common cause failure using the GO-FLOW methodology. Jain et al. [2013] analysed a two unit automatic power factor controller system with inspection and four types of failure. Taneja G and Singh Dalip [2013] discussed reliability analysis of a power generating system through gas and steam turbines with scheduled inspection.

The review of the academic literature revels that the models have been developed for a number of situations arising in industries but still many situations have been left unattended. One of the situations that has left unattended is the reliability modelling on cement grinding system with failure in its nine important components. Bridging this gap, the present study, therefore, develops a reliability model on cement grinding system.

Cement is an important constituent of concrete which is an essential material for construction in modern infrastructure. The grinding of cement clinker is an intermediate step in cement manufacturing process. On the basis of data/information gathered from Shree Cement Limited, India, the study investigates the reliability and cost-benefit analysis of the cement grinding process. The cement grinding system comprises mainly the following nine components namely:

| | | |
|---|---|---|
| (1) Belt Conveyor | (2) Bucket Elevator | (3) Separator |
| (4)Roller Press | (5) Diverting Gate | (6) Process Fan |
| (7) Cyclone | (8) Ball Mill | (9) Fly Ash System |

Diverting gate is the component for which two types of failure – minor and major have been observed and hence considered. Minor failure leads to partial failure due to which system does not become inoperable i.e. it still works whereas a major failure leads to complete failure of the system. On occurrence of the failure in fly ash system, it does not go to failed state immediately and may remain operable for some stipulated time period during which the efforts may be made to remove or repair the faults. However, if the faults are not removed within the stipulated time period, the system becomes inoperable i.e. goes to failed state. Various measures of the system effectiveness and reliability characteristics such as mean time to system failure (MTSF), availability, expected number of replacements or repairs of the nine components, expected number of visits by the repairman and profit function are evaluated in steady state using semi-Markov processes and regenerative point technique. Various important conclusions have been drawn from the graphs plotted for the model

## NOTATIONS

| | | |
|---|---|---|
| O | : | cement grinding system is operative |
| $\lambda_i$ | : | constant failure rate of $i^{th}$ component of the system; i = 1,2,...............,9 |
| $G_i(t)$, $g_i(t)$ | : | cdf and pdf of repair time of $i^{th}$ component; i = 1,2,3,4,6,7,8,9 |
| $p_1$, $q_1$ | : | probability of minor and major leakage in diverting gate |
| $G_{51}(t)$, $g_{51}(t)$ | : | cdf and pdf of repair time for minor leakage in diverting gate. |
| $G_{52}(t)$, $g_{52}(t)$ | : | cdf and pdf of repair time for major leakage in diverting gate. |
| $p_2$ | : | probability that failure in fly ash system is repaired before the fly ash in the bin is consumed completely |
| $q_2$ | : | probability that fly ash in the bin is consumed completely but the component is not repaired |
| I(t), i(t) | : | pdf and cdf of allowable time during which dry fly ash is there in the bin. |
| $F_{ri}$ | : | completely failed $i^{th}$ component under repair; i = 1,2,.................,9 |
| $pf_{r5}$ | : | partially failed $5^{th}$ component under repair |
| $O_{ir}$ | : | online repair is going on after the failure of fly ash system but within the stipulated time |
| $q_{ij}(t)$, $Q_{ij}(t)$ | : | probability density function (p.d.f.), cumulative distribution function (c.d.f.) of first passage time from a regenerative state i to a regenerative state j without visiting any other regenerative state in (0,t] |
| $A_i(t)$ | : | probability that the system is in up state at the instant t given that the system entered regenerative state i at t=0 |

$ER_i^j(t)$ : expected number of replacements/repairs in $j^{th}$ component at instant t given that the system started from the regenerative state i at t=0; j= I,II,........,X

$V_i(t)$ : expected number of visits of the repairman in (0,t] given that the system entered regenerative state i at t=0

**Transition Probabilities and Mean Sojourn Times:**

A transition diagram showing the various states of the system is shown in Fig.1. The epochs of entry into states 0 to 11 are regeneration points and hence these states are regenerative states. States 0, 5 and 10 are up states. States 1 to 4, 6 to 9 and 11 are failed states. The non zero elements $p_{ij} = \lim_{s\to 0} q_{ij}*(s)$ are given below:

$$p_{0j} = \frac{\lambda j}{\sum_{i=1}^{9}\lambda_i} \qquad (\text{ j= }1,2,3,4)$$

$$p_{05} = \frac{p_1\lambda_5}{\sum_{i=1}^{9}\lambda_i}$$

$$p_{06} = \frac{q_1\lambda_5}{\sum_{i=1}^{9}\lambda_i}$$

$$p_{0j} = \frac{\lambda_{j-1}}{\sum_{i=1}^{9}\lambda_i} \qquad (\text{ j=}7,8,9,10)$$

$p_{i,0}=1$ (i=1,2,.......9)

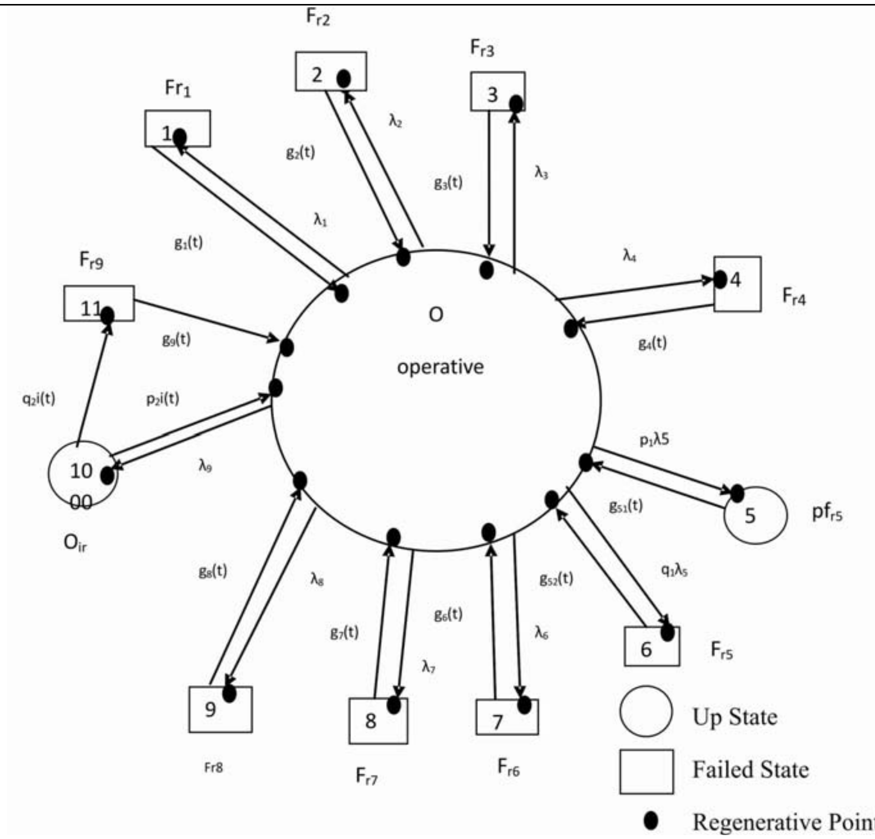
$p_{10,0}=p_2$

$p_{10,11}=q_2$

$p_{11,0}=1$

By these transition probabilities, it can be verified that $\sum_{j=1}^{10} p_{0,j} = 1, p_{10,0} + p_{10,11} = 1$

and $p_{11,0} = 1, p_{i,0} = 1(i = 1,2,.....,9)$




The mean sojourn time ($\mu_i$) in state i is given by:

$$\mu_0 = \int_0^\infty e^{-\left(\sum_{i=1}^{9}\lambda_i\right)t}\,dt = \frac{1}{\sum_{i=1}^{9}\lambda_i}$$

$$\mu_k = -g_k{}^{*\prime}(0), (k = 1,2,3,4)$$
$$\mu_5 = -g_{51}{}^{*\prime}(0), \mu_6 = -g_{52}{}^{*\prime}(0)$$
$$\mu_7 = -g_6{}^{*\prime}(0), \mu_8 = -g_7{}^{*\prime}(0), \mu_9 = -g_8{}^{*\prime}(0),$$
$$\mu_{10} = -i^{*\prime}(0), \mu_{11} = -g_9{}^{*\prime}(0)$$

The unconditional mean time taken by the system to transit for any regenerative state j when the time is counted from epoch of entrance into state i is given as:

$$m_{ij} = \int_0^\infty t q_{ij}(t)\,dt$$

Thus, $$\sum_{j=1}^{10} m_{0,j} = \mu_0, m_{i,0} = \mu_i (i = 1,2,....,9), m_{10,0} + m_{10,11} = \mu_{10}, m_{11,0} = \mu_{11}$$

**MEASURES OF SYSTEM EFFECTIVENESS**

Various measures of system effectiveness obtained in steady state using the arguments of the theory of regenerative process are:

(1) The Mean Time to System Failure (MTSF) = N/D

(2) The Availability of the System ($A_0$) = $N_1/D_1$

(3) Expected Number of Replacements/Repairs of parts in Belt Conveyor:
$ER_0^{I} = N_2 / D_1$

(4) Expected Number of Replacements/Repairs of parts in Bucket Elevator:
$ER_0^{II} = N_3 / D_1$

(5) Expected Number of Replacements/Repairs of parts in Separator:
$ER_0^{III} = N_4 / D_1$

(6) Expected Number of Replacements/Repairs of parts in Roller Press:
$ER_0^{IV} = N_5 / D_1$

(7) Expected Number of Replacements/Repairs of parts in Diverting Gate on Minor Failure:
$ER_0^{V} = N_6 / D_1$

(8) Expected Number of Replacements/Repairs of parts in Diverting Gate on Major Failure:
$ER_0^{VI} = N_7 / D_1$

(9) Expected Number of Replacements/Repairs of parts in Process Fan:
$ER_0^{VII} = N_8 / D_1$

(10) Expected Number of Replacements/Repairs of parts in Cyclone:
$ER_0^{VIII} = N_9 / D_1$

(11) Expected Number of Replacements/Repairs of parts in Ball Mill:
$ER_0^{IX} = N_{10} / D_1$

(12) Expected Number of Replacements/Repairs of parts in Fly Ash System:
$ER_0^{X} = N_{11} / D_1$

(13) Expected Number of Visits by the Repairman $(V_0) = N_{12}/D_1$

where

$$N = \mu_0 + p_{05}\mu_5 + p_{0,10}\mu_{10}$$

$$N_1 = \mu_0 + p_{05}\mu_5 + p_{0,10}\mu_{10}$$

$N_2 = p_{01}$, $N_3 = p_{02}$, $N_4 = p_{03}$, $N_5 = p_{04}$, $N_6 = p_{05}$, $N_7 = p_{06}$, $N_8 = p_{07}$, $N_9 = p_{08}$, $N_{10} = p_{09}$,

$N_{11} = p_{0,10}$, $N_{12} = \sum_{i=1}^{10} p_{0i} = 1$

$$D = 1 - p_{05} - p_{0,10}\, p_{10,0}$$

$$D_1 = \mu_0 + \sum_{i=1}^{10} p_{0i}\, \mu_i + \mu_{11}\, p_{0,10}\, p_{10,11}$$

**PROFIT ANALYSIS**

Expected profit incurred to the system is given as:

$P = C_0A_0 - C_1ER_0^{I} - C_2ER_0^{II} - C_3ER_0^{III} - C_4ER_0^{IV} - C_5ER_0^{V} - C_6ER_0^{VI} - C_7ER_0^{VII} - C_8ER_0^{VIII} - C_9ER_0^{IX} - C_{10}ER_0^{X} - C_{11}V_0$

where

$C_0$ = revenue per unit up time of the system

$C_1$ = cost per replacement/repair of parts in Belt Conveyor

$C_2$ = cost per replacement/repair of parts in Bucket Elevator

$C_3$ = cost per replacement/repair of parts in Separator

$C_4$ = cost per replacement/repair of parts in Roller Press
$C_5$ = cost per replacement/repair of parts in Diverting Gate on minor failure
$C_6$ = cost per replacement/repair of parts in Diverting Gate on major failure
$C_7$ = cost per replacement/repair of parts in Process Fan
$C_8$ = cost per replacement/repair of parts in Cyclone
$C_9$ = cost per replacement/repair of parts in Ball Mill
$C_{10}$ = cost per replacement/repair of parts in Fly Ash System
$C_{11}$ = cost per visit of the repairman

**RESULTS AND DISSCUSSION**

The following particular case is considered for graphical study:

$g_i(t) = \alpha_i e^{-\alpha_i t}$ , i =1,2,..........,9 , i≠5

$g_{51}(t) = \alpha_{51} e^{-\alpha_{51} t}$ , $g_{52}(t) = \alpha_{52} e^{-\alpha_{52} t}$ , $i(t) = \beta e^{-\beta t}$

The following values have been estimated from the gathered data/information:

$\lambda_1$=0.0004235, $\lambda_2$=0.0005802 , $\lambda_3$=0.0003948 , $\lambda_4$=0.0008738 , $\lambda_5$=0.0008158 ,$\lambda_6$=0.0002789, $\lambda_7$=0.0004236 , $\lambda_8$=0.0003778 , $\lambda_9$=0.0002783 , $\alpha_1$=0.1892333 , $\alpha_2$=0.0871105,$\alpha_3$=0.0666667, $\alpha_4$=0.0993984 , $\alpha_{51}$=0.2692308 , $\alpha_{52}$=0.1165049, $\alpha_6$=0.097643 , $\alpha_7$=0.0862069, $\alpha_8$=0.0453258 $\alpha_9$=0.1164902 , β=3, $p_1$=0.5392 , $q_1$=0.4608 , $p_2$=0.1 , $q_2$=0.9 , $C_o$=1540 , $C_1$=55495.69 , $C_2$=105689.43 , $C_3$=15161.76 , $C_4$=1442167.1 , $C_5$=339.29 , $C_6$=1075, $C_7$=11020.69 , $C_8$=25280 , $C_9$=211525 , $C_{10}$=17536.36 ,$C_{11}$=20000

Using the above estimated values, the following measures of system effectiveness are obtained:

Table 1

| Measure | Value |
|---|---|
| MTSF | 251.7539484 |
| $A_0$ | 0.9569234 |
| $ER_0^{I}$ | 0.0004046 |
| $ER_0^{II}$ | 0.0005543 |
| $ER_0^{III}$ | 0.0003771 |
| $ER_0^{IV}$ | 0.0008347 |
| $ER_0^{V}$ | 0.0004202 |
| $ER_0^{VI}$ | 0.0003591 |
| $ER_0^{VII}$ | 0.0002664 |
| $ER_0^{VIII}$ | 0.0004047 |
| $ER_0^{IX}$ | 0.0003609 |
| $ER_0^{X}$ | 0.0002659 |
| $V_0$ | 0.0042478 |
| P | 3.4579974 |

Various graphs have also been plotted using the above particular case. All of these graphs cannot be shown here but some of the graphs are shown in Figs 2 to 5 as a sample.

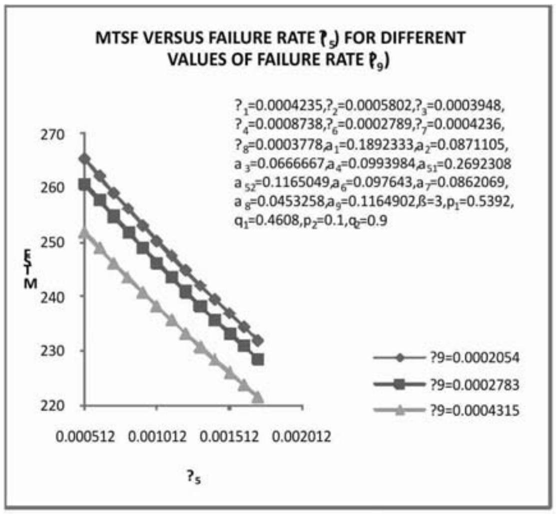



Fig. 2

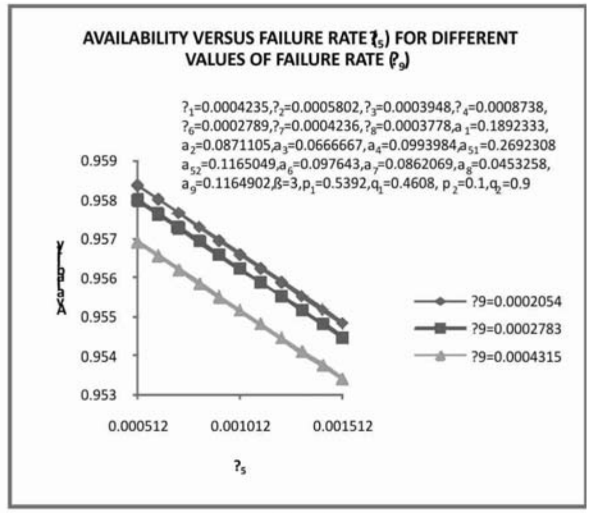



Fig. 3

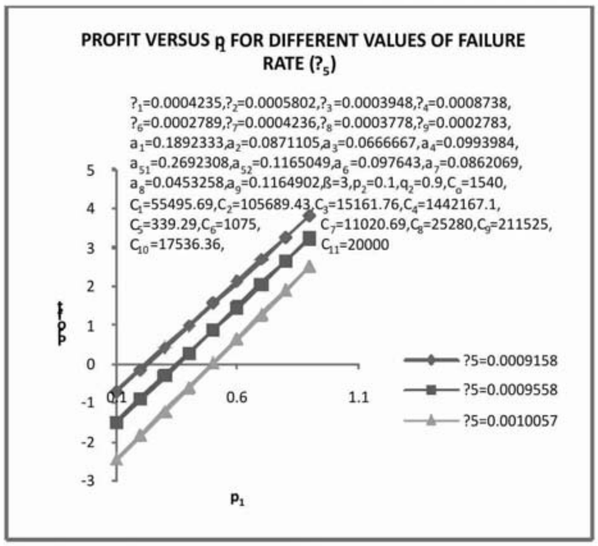



Fig. 4

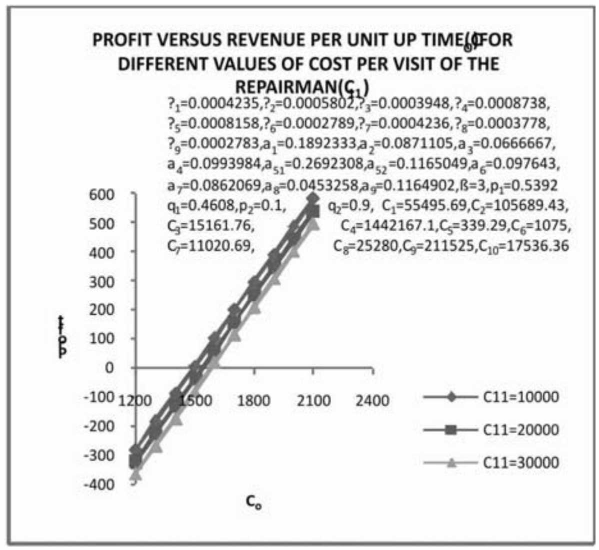



Fig. 5

**CONCLUSION**

Following conclusions are drawn on the basis of the graphs, irrespective of the fact whether they are being shown here or not:

- The MTSF and Availability gets decreased as the failure rate ($\lambda_5$) increases and also gets lowered for higher values of failure rate ($\lambda_9$).

Other interpretations are given in Table 2. The values of those parameters which have not been mentioned in each case in the table are the same as mentioned in the beginning of "RESULTS AND DISSCUSSION".

| S.No. | Graph | Other fixed parameters | Profit | | For | Profit ≥ 0 If |
|---|---|---|---|---|---|---|
| | | | Increases | Decreases | | |
| 1 | Profit versus $\lambda_5$ | $\beta=3, p_1=0.5392, C_o=1540, C_1=55495.69, C_{11}=20000$ | - | With increase in $\lambda_5$ and $\lambda_9$ | $\lambda_9 = 0.0002054$ | $\lambda_5 \leq 0.0011749$ |
| | | | | | $\lambda_9 = 0.0002783$ | $\lambda_5 \leq 0.0010216$ |
| | | | | | $\lambda_9 = 0.0003154$ | $\lambda_5 \leq 0.0009435$ |
| 2 | Profit versus $p_1$ | $\lambda_9=0.0002783, \beta=3, C_o=1540, C_1=55495.69, C_{11}=20000$ | With increase in $p_1$ | With increase in $\lambda_5$ | $\lambda_5=0.0009158$ | $p_1 \geq 0.1840283$ |
| | | | | | $\lambda_5=0.0009558$ | $p_1 \geq 0.3276139$ |
| | | | | | $\lambda_5=0.0010057$ | $p_1 \geq 0.4907336$ |
| 3 | Profit versus $C_0$ | $\lambda_5=0.0008158, \lambda_9=0.0002783, \beta=3, p_1=0.5392, C_1=55495.69$ | With increase in $C_0$ | With increase in $C_{11}$ | $C_{11}=10000$ | $C_0 \geq 1491.9959828$ |
| | | | | | $C_{11}=20000$ | $C_0 \geq 1536.3863382$ |
| | | | | | $C_{11}=30000$ | $C_0 \geq 1580.7766936$ |
| 4 | Profit versus $C_0$ | $\lambda_5=0.0008158, \lambda_9=0.0002783, \beta=3, p_1=0.5392, C_{11}=20000$ | With increase in $C_0$ | With increase in $C_1$ | $C_1=26354.42$ | $C_0 \geq 1524.0662824$ |
| | | | | | $C_1=55495.69$ | $C_0 \geq 1536.3863384$ |
| | | | | | $C_1=84636.96$ | $C_0 \geq 1548.7063944$ |
| 5 | Profit versus $\lambda_9$ | $\lambda_5=0.0008158, p_1=0.5392, C_o=1540, C_1=55495.69, C_{11}=20000$ | - | With increase in $\lambda_9$ and $\beta$ | $\beta=1$ | $\lambda_9 \leq 0.0003868$ |
| | | | | | $\beta=3$ | $\lambda_9 \leq 0.0003761$ |
| | | | | | $\beta=5$ | $\lambda_9 \leq 0.0003740$ |

## REFERENCES


1. Parashar B and Taneja G (2007) “Reliability and Profit Evaluation of a PLC Hot Standby System Based on a Master Slave Concept and Two Types of Repair Facilities”, IEEE Transactions on Reliability, 56(3), 534-539.
2. Goyal A, Taneja G and Singh D V (2009) “Analysis of a 2-Unit Cold Standby System Working in a Sugar Mill with Operating and Rest Periods”, Caledonian J. Engg., 5(1), 1-5.
3. Mathew A G, Rizwan S M, Majumdar M C, Ramachandran K P & Taneja G (2009)“ Profit Evaluation of a Single Unit CC Plant with Scheduled Maintenance”, Caledonian journal of Engineering; 5: 1-5
4. Padmavathi N, Rizwan S M, Pal Anita and Taneja G (2012)“Reliability Analysis of an Evaporator of a Desalination Plant with Online Repair and Emergency Shutdown”, Aryabhatta Journal of Mathematics & Informatics, 4(1), 1-12.
5. Sharma V K and Gupta S P (2012)“Cost Benefit Analysis of Three Unit Redundant System Model with Correlated Failures and Repairs”, Journal of Pure and Applied Science & Technology,Vol.2(1),pp1-10.
6. Hashim M, Hidekazu M T, Ming Yang (2013) “Reliability Analysis of Phased Mission System by Considering the Concept of Sensitivity Analysis, Uncertainty Analysis and Common Cause Failure Analysis using the GO-FLOW Methodology”, Research Journal of Applied Sciences, Engineering and Technology Vol. 5 No.12, 3465-3475.
7. Jain Roosel, Taneja G and Bhatia P K (2013) “Analysis of a Two Unit Automatic Power Factor Controller System with Inspection and Four Types of Failure” , Advanced Modelling and Optinization (AMO)”, 15, 487-497.
8. Taneja G and Singh Dalip (2013) “Reliability Analysis of a Power Generating System Through Gas and Steam Turbines with Scheduled Inspection” Aryabhatta Journal of Mathematics & Informatics,5(2), 373-380.

Aryabhatta Journal of Mathematics & Informatics Vol. 6, No. 2, July-Dec, 201 ISSN : 0975-7139
Journal Impact Factor (2013) : 0.489

# A GENERALIZED DOUBLE ENDED STOCHASTIC QUEUE SYSTEM WITH EXCESS CUSTOMER DEMAND IN REAL WORLD SITUATIONS

**Reeta Bhardwaj*, T. P. Singh** and Vijay Kumar*****

* & ***Assistant professor, Department of Mathematics, Amity University, Punchgaon, Manesar, Gurgaon (India)

**Professor, Department of Mathematics, Yamuna Institute of Engg. & Technology, Gadholi, Yamuna Nagar (India)

Email: - bhardwajreeta84@gmail.com*,tpsingh78@yahoo.com**, vijaykumar.goldy@gmail.com***

**ABSTRACT :**

***A double ended queue model is a system where customer demanding and resource supply arrive to a work station in poisson process. When there is a pair of customer and resource in a system, they immediately transact the required system and leave the system. Thus, there cannot be non-zero numbers of customers and resources simultaneously in the system.***

***The double ended queue system can be applied to model various social and economic situations in real world scenario. We analyzed the various queue parameters like mean queue length, waiting time and the cost functions etc. due to imbalance of customer demand and resources. The analysis has been made analytically. The queuing behavior in different situations has been examined and a comparative study has been made with the work already done through simulation. The main objective of the paper is to explore the situation when the stochastic system becomes unstable due to excess customers. The effect of customer's reduction factor and resource expansion factor has been examined graphically. The limitations of the model has also been given and extensions are suggested.***

**Key Words:** ***Double ended, Poisson process, customer reduction factor, resource expansion factor.***

## 1. INTRODUCTION

Stochastic model provides the basic frame work for analyzing the natural as well as socio-economic phenomenon in real world context. The analysis of double ended queue system has attracted the attention of several investigators. Kendall [1951] was the first who discussed the double ended queue problem in which the passengers and taxis arrive at a taxi stand in Poisson's stream with constant mean rate $\lambda$ and $\mu$. No limit was placed on the numbers of passengers or taxis that can form a queue at the stand. Kashyap [1966] considered the double ended queue with limiting waiting space for both passengers and taxis with Poisson arrival. Perry and Staje [1999] explored an inventory system for perishable commodities with finite shelf size and finite waiting room, for demand units. Conolly and Parthasarathy[2002] studied the effect of impatient behavior of customers in the context of double ended queue. Mendoza, Sedaghat and Yon [2009] developed an optimization model for the double ended queue due to imbalance of supply. Kim et al [2010] developed a simulation design for extended double ended queue model and studied sensitive analysis on performance measures by changing parameters and impact factors such as batch size, processing time, probability of paring failure etc.Arti and Singh T.P. [2014] through his case study showed that how queuing theory satisfies the stochastic model when applied in real time scenario.

We recall that a good paper dealing approximation with the problem of balancing queue system with demand and supply is due to, Mendoza, Sedaghatand Yon [2009, 2014] who studied a double ended queue model for demand/supply system where demand and supply queues both have finite maximum possible lengths. These authors generated a number of scenarios using Monte Carlo's technique and the results obtained in these scenarios are used to find regression equations that express the policy factor.

Along with the research line traced by above mentioned researchers, we present the double ended stochastic queue model in general form with excess customers may be considered as excess demand queue in the sense of Mendoza et al [2014]. An effort has been explored to find an optimal policy in these situations. The analysis has been made mathematically rather than simulation. The model parameters affecting the optimal value of policy factors have been analyzed mathematically & graphically. The effect of changing the reduction factor on mean queue length with excess customer demand queue and excess resources supply queue has been presented in tabular form and the comparative study with discouraged customers or excess resources has been made. A graphical analysis has been made and is compared with the simulation study done by Mendoza et al [2014].We present numerical results for the model at the resource level and customer level. We address four questions (i). What is the general behavior of these matrices? (ii) What are the sensitivities on changing the parametric value? (iii) How do the system level matrices compared with the results given by simulation study? (iv) How do the customer level matrices compared with those given by simulation? Limitations of the model and further scope of the research has also been highlighted in this paper.

Following, the introductory part and the literature survey, the remainder of the paper proceeds as follows. In section 2, we present the social and economic situations where the model can be applied. In section 3, the model has been framed with relevant notations.Section 4 derived the steady state equations for the model and its stability conditions and provides formulae for system expected performance. Various queue characteristics for both customers and resources have been derived along with reduction factor in section 5. Some of these results are known while some are new. In section 6, we derive matrix on the performance of the system for excess customer demand queue and makes an analytic study of cost parameter due to excess customers. The results for model have been compared with the simulation study already made by Mandoza etal [2014] in Section 7. We discuss our results in section 8 and present further scope in research direction.

## 2. APPLICATION OF THE MODEL IN SOCIAL AND ECONOMIC SITUATION

Practically, the model can be used for a variety of social or economic situations where the system is demanding and providing services in bothways. In an orderly taxi rank at an airport or taxi stand, on one side a queue is formed by the arrival of stream of passengers who wait for taxi while on the other side a queue of taxis wait for the passengers. At freight ports, on one side a queue of cargo containers to be shipped, while the other queue consist of waiting ships in their berths. In business organizations balancing the demand of goods and supply of services are very common. A similar situation exists at a stock exchange, on one side there is a queue of stock waiting for sale, while the other queue consists of potential buyers of these stocks. It has been observed in many crossing networks, both market orders (an order to buy and sell stocks at the prevailing market price) and limit orders (an order to buy or sell a stock at a specific prize or better) are expected, which means that at any point in time there may be an excess of orders that wish to buy or sell a stock.

We also find that in research funding cases which are obtained through a competitive process, the excess research scholars can be supposed in form of demanding unit for fellowships or funds. The potential research proposals are evaluated in R&D department and only some of the most promising proposals receive funding. The researchers request to a granting agency may be UGC/CSIR for financially support their proposals. We can assume proposals request for sponsorship or fellowship or funding, follow a Poisson process.

We find a very practical situation of double ended queue system in ware housing. Automatic storage and retrieval systems (AS/RS) are widely used in ware houses. AS/RS using a queue model with two linked queues, one for the items waiting for storage and other of finite capacity for the request for an item to be removed from storage based on the size of storage racked can be supposed as an example of a double ended queue. Queue 1 is considered to be arrival queue, where items wait to be placed into storage, queue 2 is considered to be the queue in which the request for retrieval is arrived. Queue 2 is depended on the numbers of racks.

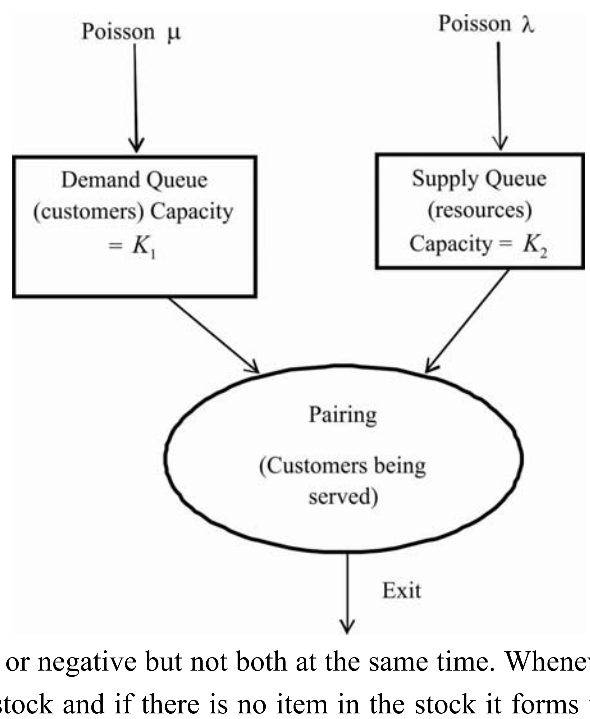
## 3. THE MODEL

We present a general double ended stochastic queue model in which the customers and commodity (resources) queue have maximum possible length $K_1$ and $K_2$ respectively with the state of system one dimensional index. Customers arriving in a group may be taken as one unit assuming that a group does not consist of more customers than the capacity of resources. Excess resource results in a positive index while excess customers result in negative index i.e,

$n > 0$ Indicates numbers of resources are waiting.

$n = 0$ Indicates neither resources nor customers are waiting.

$n < 0$ Indicates – n customers are waiting. The numerical value gives the customers waiting.




The queue can be either positive or negative but not both at the same time. Whenever, there is a need of one unit, it will be satisfied by resource in stock and if there is no item in the stock it forms the queue and will wait until the resource is available and vice versa. We find various queue characteristics when there is excess of customers and also find the effect of reduction factor causes due to customer demand queue on expected queue length. The cost analysis due to imbalance of excess customer demands queue and resources have been discussed mathematically in detail.

**Notations**

$\lambda$- Average rate of resources
$\mu$-Average rate of customers
$\alpha$ – Customers reduction factor
$\alpha\mu$ – New customer reduction factor in which $0 < \alpha < 1$
$\rho$ – Utilization factor $= \dfrac{\lambda}{\alpha\mu}$ (due to excess customers)
$\beta$ – Resource expansion factor
$\lambda\beta$ – New resource expansion factor in which $\beta > 1$
$\rho$ – Utilization factor $= \dfrac{\lambda\beta}{\mu}$ (due to excess resources)
$C_1$ – Cost per unit time of one unit of excess customers in the queue
$C_2$ – Cost per unit time of one unit of excess resources in the queue
$C_\alpha$ – Cost incurred per time unit in reducing the customer rate by one unit
$K_1$ – Mean length of customers queue.
$K_2$ – Mean length of resources queue.

## 4. STEADY STATE EQUATIONS OF THE MODEL DUE TO EXCESS CUSTOMERS

The transient analysis of queuing system is not always easy to be performed and it leads very often to non-manageable mathematical formulae. Hence it is feasible to study the system under consideration in steady state.

Let $P_n$ be the steady state probability that the system is in state n here, $n = -k_1, -k_1+1, ................0,1,2....., k_2-1, k_2$. The differential difference balance equation for steady state with customer reduction factor α due to excess customers can be summarized as follows:

$$\lambda P_{-k_1} = \alpha\mu P_{-k_1+1} \tag{1}$$

$$(\lambda + \alpha\mu)P_n = \lambda P_{n-1} + \alpha\mu P_{n+1} \tag{2}$$

$$\lambda P_{k_2-1} = \alpha P_{k_2} \tag{3}$$

**Solution Methodology**

From equation **(**1)

$$P_{-k_{1+1}} = \frac{\lambda}{\alpha\mu} P_{-k_1}$$

$$P_{-k_{1+1}} = \rho P_{-k_1} \qquad \text{where } \rho = \frac{\lambda}{\alpha\mu}$$

$$P_{-k_1+2} = \rho P_{-k_1+1} = \rho^2 P_{-k_1}$$

$P_{-k_1+3} = \rho^3 P_{-k_1}$ and so on…………………..

$$P_{-k_1+n} = \rho^n P_{-k_1} \tag{4}$$

In general, $P_n = \rho^{k_1+n} P_{-k_1}$

On applying the normalized condition

$$\sum_{n=-k_1}^{k_2} P_n = 1 \tag{5}$$

$$P_{-k_1} + P_{-k_1+1} + .............+ P_0 + P_1.......+ P_{k_2} = 1$$

$$P_{-k_1}\left[1+\rho+\rho^2+.............\rho^{k_1+k_2}\right] = 1$$

Which on simplification

$$P_{-k_1} = \frac{(1-\rho)}{1-\rho^{k_1+k_2+1}} \tag{6}$$

From (4) we get,

$$P_n = \begin{cases} \dfrac{(1-\rho)\rho^{k_1+n}}{1-\rho^{k_1+k_2+1}}, & if\ \rho \neq 1 \\ \dfrac{1}{k_1+k_2+1}, & if\ \rho = 1 \end{cases} \tag{7}$$

## 5. QUEUE CHARACTERISTICS/PERFORMANCE MEASURES:

Obtaining performance measures is essential in order to utilize our queue model in redesign of the system. Further these performance measures allow us to determine how system is behaving on applying the reduction factor with excess customer demand queue.

Comparison on these performance measures for various value of the reduction factor will provide insight into what changes would be appropriate for the given conditions. The model can also be applied to determine the behavior of system and ultimately helps in deciding best optimal policy for the system.

5.1 Probability that waiting space is empty

$$P_0 = \begin{cases} \dfrac{(1-\rho)\rho^{k_1}}{1-\rho^{k_1+k_2+1}} & if\ \rho \neq 1 \\ \dfrac{1}{k_1+k_2+1} & if\ \rho = 1 \end{cases}$$

5.2 Probability that there is queue of resources

$$P_r = \sum_{1}^{k_2} P_n = \begin{cases} \dfrac{\left(1-\rho^{k_2}\right)\rho^{k_1+1}}{1-\rho^{k_1+k_2+1}} & if\ \rho \neq 1 \\ \dfrac{k_2}{k_1+k_2+1} & if\ \rho = 1 \end{cases}$$

5.3 Probability that there is queue of customers

$$P_c = \sum_{-k_1}^{-1} P_n = \begin{cases} \dfrac{\left(1-\rho^{k_1}\right)}{1-\rho^{k_1+k_2+1}} & if\ \rho \neq 1 \\ \dfrac{k_1}{k_1+k_2+1} & if\ \rho = 1 \end{cases}$$

5.4 The mean queue length of resources

$$L_r = \sum_{1}^{k_2} nP_n = \begin{cases} \dfrac{\rho^{k_1+1}\left[1-(k_2+1)\rho^{k_2} + k_2\rho^{k_2+1}\right]}{(1-\rho)\left(1-\rho^{k_1+k_2+1}\right)} & if\ \rho \neq 1 \\ \dfrac{k_2(k_2+1)}{2(k_1+k_2+1)} & if\ \rho = 1 \end{cases}$$

5.5 The mean queue length of customers

$$L_c = \sum_{-k_1}^{-1} -nP_n = \begin{cases} \dfrac{k_1-(k_1+1)\rho+\rho^{k_1+1}}{(1-\rho)\left(1-\rho^{k_1+k_2+1}\right)} & if\ \rho \neq 1 \\ \dfrac{k_1(k_1+1)}{2(k_1+k_2+1)} & if\ \rho = 1 \end{cases}$$

5.6 Probability that resources are lost due to limiting waiting space $= P_{k_2}$

5.7 Probability that customer sare lost due to limiting waiting space $= P_{-k_1}$

5.8 Mean number of resources lost per unit time $= \lambda P_{k_2}$

5.9 Mean number of customers lost per unit time $= \mu P_{-k_1}$

5.10 Mean waiting time for resources $W_r = \dfrac{L_r}{\lambda}$

5.11 Mean waiting time for customers $W_c = \dfrac{L_c}{\lambda}$

## 6. ANALYTICAL STUDY OF QUEUE PARAMETERS

When customers demand become higher than the resource parameter we have only two possibilities (i) either discourage the excess customers demand (ii) or increase the resource. In either the cases the mean queue length and other queue parameter will be affected and a cost will be associated in balancing and optimum decision depending upon the total cost selecting any one of these options.

**Case 1: Balancing the system by reducing excess demands**

**(A):Mean queue length due to excess customers.**

Let $L_c$ be the mean queue length due to excess customer. $\alpha$ is considered as customer reduction factor so that the new customer rate in terms of demand rate become $\alpha\mu$ $(0 < \alpha < 1)$.

From parameter 5.5, we find the following values of mean queue length due to excess customers, on applying a customer reduction factor $\alpha$ $(0 < \alpha < 1)$. The different values are calculated mathematically and are presented in the following table for two different sets.

**Table 1: Between mean queue length of customers $L_c$ and reduction factor $\alpha$** Let

$\lambda = 3,\ \mu = 5$

$k_1 = 10,\ k_2 = 15$

| $\alpha$ | $L_c$ |
|---|---|
| 0.1 | 0 |
| 0.2 | 0 |
| 0.3 | 0 |
| 0.4 | 0.004 |
| 0.5 | 0.18 |
| 0.6 | 2.11 |
| 0.7 | 5.38 |
| 0.8 | 7.17 |
| 0.9 | 8.03 |
| 1.0 | 8.50 |

$\lambda = 3,\ \mu = 6$

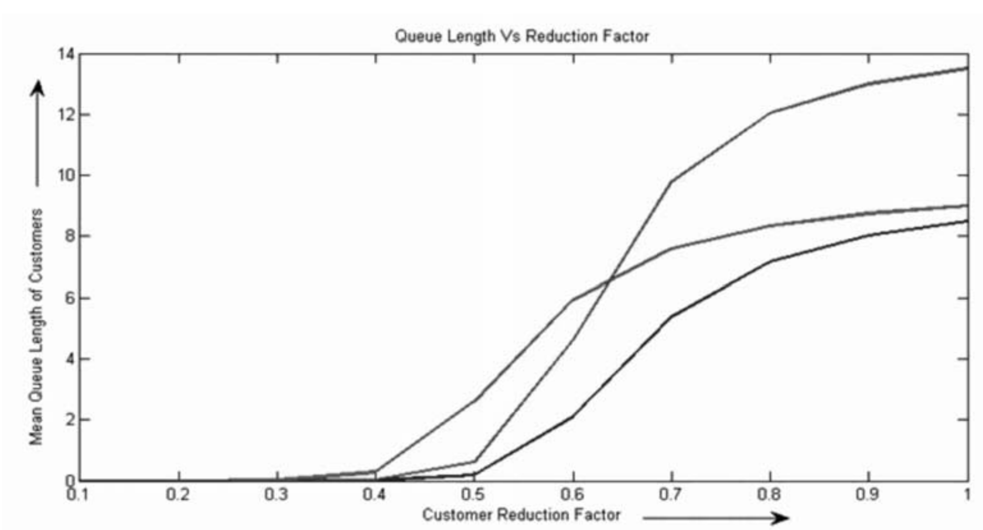
$k_1 = 10,\ k_2 = 10$

| $\alpha$ | $L_c$ |
|---|---|
| 0.1 | 0 |
| 0.2 | 0 |
| 0.3 | 0.008 |
| 0.4 | 0.29 |
| 0.5 | 2.61 |
| 0.6 | 5.93 |
| 0.7 | 7.59 |
| 0.8 | 8.34 |
| 0.9 | 8.75 |
| 1.0 | 9.00 |

$\lambda = 3,\ \mu = 5$

$k_1 = 15,\ k_2 = 10$

| $\alpha$ | $L_c$ |
|---|---|
| 0.1 | 0 |
| 0.2 | 0 |
| 0.3 | 0.0009 |
| 0.4 | 0.034 |
| 0.5 | 0.62 |
| 0.6 | 4.61 |
| 0.7 | 9.77 |
| 0.8 | 12.04 |
| 0.9 | 13.00 |
| 1.0 | 13.5 |



## GRAPHICAL ANALYSIS

The mean queue length in the system, for customer queue and resource queue has been presented graphically. The graph shows that as, the customer reduction factor is greater than 1, the queue length grows higher and almost becomes constant. In this case, the asymptotes of the curve shifted to left indicating that the system gets unstable faster, since the resource rate is lesser. When the reduction factor is less than 1, the mean queue length is comparatively lesser but grows highly. At a reduction factor about 0.6, the queue length is minimum. It is because of high utilization of the service pattern depending upon the excess customer arrival rate and the resource rate. Low utilization would indicate that there is a room for increasing the use of storage facility. Hence, a smaller reduction factor would be more efficient. High utilization would indicate the system is being used efficiently. However, there is a little scope for increasing utilization. It creates instability in the system therefore reduction factor can't be taken greater than 1.

### (B):Cost parameters due to excess customer

In this study, we focus our attention mainly on practical and social situations where customers or demand factor is higher than the resources or supply factors i.e., $\mu > \lambda$

Let $C_1$ be the cost per time unit of one unit of excess customer or demand for service in the queue. $C_2$ be the cost per time unit of one unit of excess resources or supply in the queue. $\alpha$ is considered as customer reduction factor so that the new customer rate in terms of demand rate become $\alpha\mu$ $(0<\alpha<1)$ and $C_\alpha$ be the cost incurred per time unit in reducing the customer rate by one unit. Let $P_n$ be the steady state probability that the system is in state $n$ where $n=-k_1,-k_1+1,\dots\dots\dots\dots\dots 0,1,2\dots\dots k_2$.

The expected total cost when a customer reduction factor is taken into consideration is given by

$$C(\alpha)=C_1\sum_{n=-k_1}^{-1}-nP_n+C_2\sum_{n=1}^{k_2}nP_n+C_\alpha\mu(1-\alpha) \tag{8}$$

First summation indicates expected under resource cost, second summation represents over resource cost and third term is expected cost of reducing the customers (either by increasing the fare price or redesigning the queue system) rate from $\mu$ to $\alpha\mu$ in order to balance the system.

Substitute the value of $P_n$ from (4) in (8), we get

$$C(\alpha)=\begin{cases}\dfrac{\left[c_1k_1(k_1+1)+c_2k_2(k_2+1)\right]}{2(k_1+k_2+1)}+C_\alpha\mu(1-\alpha) & when \quad \rho=1\\[2ex] \dfrac{-c_1\left(-k_1+\rho+k_1\rho-\rho^{k_1+1}\right)+c_2\left[\rho^{k_1+1}-(1+k_2)\rho^{k_1+k_2+1}+k_2\rho^{k_1+k_2+2}\right]}{(1-\rho)\left(1-\rho^{k_1+k_2+1}\right)} & \\[2ex] +C_\alpha\mu(1-\alpha) & when \quad \rho\neq1\end{cases} \tag{9}$$

$$=\frac{f_1(\rho)}{f_2(\rho)}+C_\alpha\mu(1-\alpha) \tag{10}$$

Where, $f_1(\rho)=-c_1\left(-k_1+\rho+k_1\rho-\rho^{k_1+1}\right)+c_2\left[\rho^{k_1+1}-(1+k_2)\rho^{k_1+k_2+1}+k_2\rho^{k_1+k_2+2}\right]$

$f_2(\rho)=(1-\rho)\left(1-\rho^{k_1+k_2+1}\right)$

## OPTIMAL ANALYSIS

We find the following values that minimize the expected total cost on applying a customer reduction factor $\alpha$ ($0 < \alpha < 1$). The different values are calculated mathematically and are presented in the following table for two different sets.

**Table 2: Between expected cost $C(\alpha)$ and reduction factor $\alpha$** Let

$\lambda = 1,\ \mu = 2$

$k_1 = 15,\ k_2 = 25$

$c_1 = 1,\ c_2 = 1.1,\ c_\alpha = 1.25$

| $\alpha$ | $C(\alpha)$ |
|---|---|
| 0.1 | 29.47 |
| 0.2 | 28.76 |
| 0.3 | 27.60 |
| 0.4 | 24.63 |
| 0.5 | 12.89 |
| 0.6 | 11.66 |
| 0.7 | 13.28 |
| 0.8 | 13.83 |
| 0.9 | 14.00 |
| 1.0 | 14.00 |

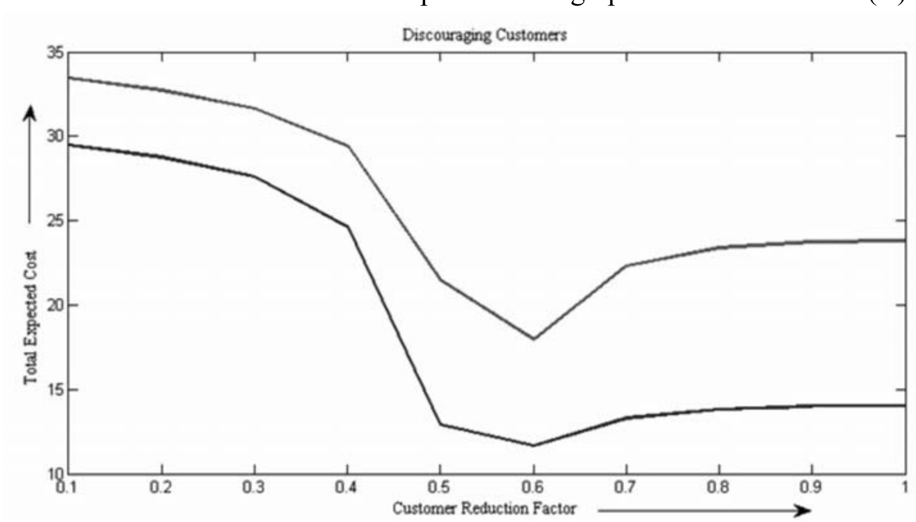
$\lambda = 1.2,\ \mu = 2.2$

$k_1 = 25,\ k_2 = 25$

$c_1 = 1,\ c_2 = 1.25,\ c_\alpha = 1.25$

| $\alpha$ | $C(\alpha)$ |
|---|---|
| 0.1 | 33.44 |
| 0.2 | 32.72 |
| 0.3 | 31.64 |
| 0.4 | 29.46 |
| 0.5 | 21.48 |
| 0.6 | 17.95 |
| 0.7 | 22.31 |
| 0.8 | 23.40 |
| 0.9 | 23.73 |
| 1.0 | 23.80 |

The above tabular values has been presented in graphs between $\alpha$ and $C(\alpha)$




## GRAPHICAL ANALYSIS

The graph shows the typical shapes of the relation-ship between minimum expected total cost $C(\alpha)$ and customer reduction factor $\alpha$. We find in most cases that $C(\alpha)$ has two inflection points when $\alpha$ lies between 0 and 1. When $\alpha$ is 1 or more the value of $C(1)$ is higher than the minimum. When $\alpha \geq 1$ either the customer demand should be discouraged otherwise many customers, after waiting for service leave the system. In order to balance the system we should apply the customer reduction factor between 0 and 1. With the above said data, 0.6 gives approximate the least cost model for both cases i.e., case of expected mean queue length of customers as well as in expected total cost, as this value attains the high utilization of the system.

The formula can be used numerically to find the policy factor that would minimize the total expected cost in order to balance the queuing system. By comparing the minimum total expected cost, with the help of graph either by discouraging customers or by increasing resources is the best policy.

**Case ii: Balancing the system by increasing resources**

**(A):Mean queue length on increasing the resources**

In this the rate of customer demand remains unchanged and we introduce the resource expansion factor $\beta(\beta > 1)$ so that the utilization factor $\rho = \dfrac{\lambda\beta}{\mu}$.

$L_r$ be the mean queue length on increasing the resources.

Steady state differential equations can be expressed as

$$\lambda\beta P_{-k_1} = \mu P_{-k_1+1} \tag{11}$$

$$(\lambda\beta + \mu)P_n = \lambda\beta P_{n-1} + \mu P_{n+1} \tag{12}$$

$$\lambda\beta P_{k_2-1} = \mu P_{k_2} \tag{13}$$

On solving, we get the similar results as already given in section 4 and queue characteristic will also be the same as given in section 5. From parameter 5.4, we find the following values of mean queue length on increasing the resources, on applying a resource expansion factor $\beta$ ($\beta > 1$). The different values are calculated mathematically and are presented in the following table for two different sets.

**Table 3: Between mean queue length of resources $L_r$ and expansion factor $\beta$ Let**

$\lambda = 2,\ \mu = 3$

$k_1 = 15,\ k_2 = 25$

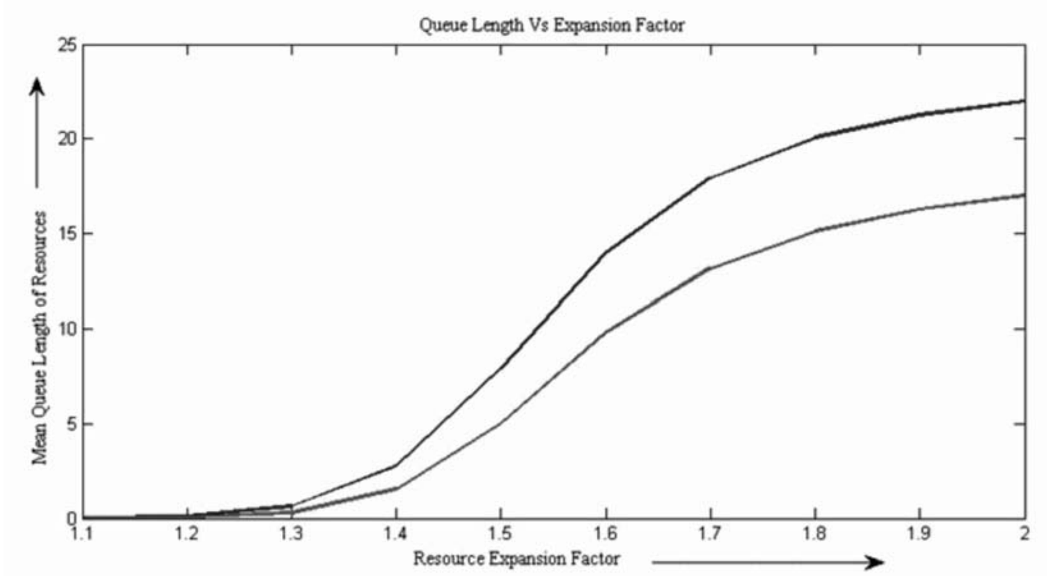
| $\beta$ | $L_r$ |
|---|---|
| 1.1 | 0.026 |
| 1.2 | 0.13 |
| 1.3 | 0.66 |
| 1.4 | 2.77 |
| 1.5 | 7.92 |
| 1.6 | 13.97 |
| 1.7 | 17.93 |
| 1.8 | 20.06 |
| 1.9 | 21.26 |
| 2.0 | 22.00 |

$\lambda = 2,\ \mu = 3$

$k_1 = 20,\ k_2 = 20$

| $\beta$ | $L_r$ |
|---|---|
| 1.1 | 0.005 |
| 1.2 | 0.043 |
| 1.3 | 0.29 |
| 1.4 | 1.54 |
| 1.5 | 5.00 |
| 1.6 | 9.82 |
| 1.7 | 13.19 |
| 1.8 | 15.13 |
| 1.9 | 16.28 |
| 2.0 | 17.00 |

Mean queue length of resources and resource expansion factor has been presented graphically.




In case, if resource expansion factor is increased, the curve increases exponentially and the system become unstable faster.

**(B): Cost parameters on increasing the resources**

$C_\beta$ be the cost incurred per time unit in increasing the resource rate by one unit. Here the expected cost becomes:

$$C(\beta) = C_1 \sum_{n=-k_1}^{-1} -nP_n + C_2 \sum_{n=1}^{k_2} nP_n + C_\beta \lambda(\beta - 1) \tag{14}$$

First summation indicates expected under resource cost, second summation represents over resource cost and third term is expected cost of increasing the resource rate from $\beta$ to $\lambda\beta$

After putting $P_n$ from equation (7) in equation (14) the expected total cost to the system, when resource expansion factor is applied, is given by

$$C(\alpha) = \begin{cases} \dfrac{\left[c_1 k_1 (k_1+1) + c_2 k_2 (k_2+1)\right]}{2(k_1+k_2+1)} + C_\beta \lambda(\beta-1) & when \quad \rho = 1 \\ \dfrac{-c_1\left(-k_1+\rho+k_1\rho-\rho^{k_1+1}\right) + c_2\left[\rho^{k_1+1} - (1+k_2)\rho^{k_1+k_2+1} + k_2\rho^{k_1+k_2+2}\right]}{(1-\rho)\left(1-\rho^{k_1+k_2+1}\right)} & \\ +C_\beta \lambda(\beta-1) & when \quad \rho \neq 1 \end{cases}$$

$$= \frac{f_1(\rho)}{f_2(\rho)} + C_\beta \lambda(\beta-1) \tag{15}$$

Where, $f_1(\rho) = -c_1\left(-k_1+\rho+k_1\rho-\rho^{k_1+1}\right) + c_2\left[\rho^{k_1+1} - (1+k_2)\rho^{k_1+k_2+1} + k_2\rho^{k_1+k_2+2}\right]$

$f_2(\rho) = (1-\rho)\left(1-\rho^{k_1+k_2+1}\right)$

We find the following values that minimize the expected total cost on applying a resource expansion factor $\beta$ ( $\beta > 1$ ). The different values are calculated mathematically and are presented in the following table for two different sets.

**Table 4: Between expected cost $C(\beta)$ and expansion factor $\beta$** Let

$\lambda = 1,\ \mu = 2$

$k_1 = 10,\ k_2 = 25$

$c_1 = 1,\ c_2 = 1.1,\ c_\beta = 1.25$

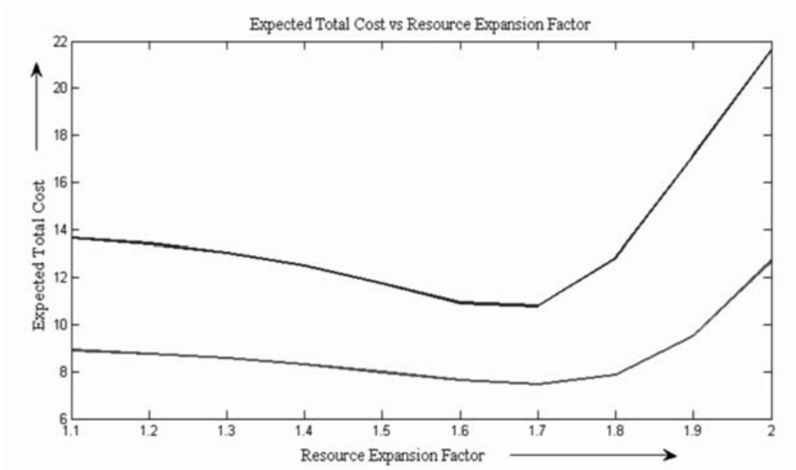
| β | C(β) |
|---|---|
| 1.1 | 8.90 |
| 1.2 | 8.76 |
| 1.3 | 8.57 |
| 1.4 | 8.30 |
| 1.5 | 7.97 |
| 1.6 | 7.64 |
| 1.7 | 7.46 |
| 1.8 | 7.87 |
| 1.9 | 9.53 |
| 2.0 | 12.70 |

$\lambda = 1.2,\ \mu = 2.2$

$k_1 = 15,\ k_2 = 25$

$c_1 = 1,\ c_2 = 1.25,\ c_\beta = 1.25$

| β | C(β) |
|---|---|
| 1.1 | 13.65 |
| 1.2 | 13.41 |
| 1.3 | 13.04 |
| 1.4 | 12.49 |
| 1.5 | 11.74 |
| 1.6 | 10.92 |
| 1.7 | 10.79 |
| 1.8 | 12.81 |
| 1.9 | 17.15 |
| 2.0 | 21.61 |




**GRAPHICAL ANALYSIS**

The minimum value of $C(\beta)$ occurs for $\beta > 1$ and has been presented graphically. We have shown the parametric value for some fix value and it has found that in most of the cases the minimum cost $C(\beta)$ occurs at $\beta > 1$ and $C(1)$ is higher than the minimum.

In a nut shell, the formula can be used to find numerically the policy factor that would be minimized the total expected cost to balance the unbalancing queue system either by increasing the resources or decreasing the customer demand. The best policy is one which leads to the smallest expected unitary cost

## 7. CUSTOMER LEVEL COMPARISON TO A SIMULATED MODEL

We now consider how the cost parameters in our model derived mathematically can be compared with the performance of an individual customer in simulated system recently given by Mandoza etal [2014]. Through its simulation study and randomly generated numbers studied in three different experiments, 929 scenarios in one experiment, 1440 in second and 337 in third experiment and find out that approximatelyin 93 % instances where $C(\alpha)$ has two inflection point and minimum occurs at $\alpha$ between 0 and 1. In third scenario, he illustrated that $C(\alpha)$ has one inflection point and minimum occurs at a value of $\alpha$ closed to one. When the demand is discouraged while in case of increasing the supply the min occurs at a value $\beta$ greater than one and in scenario 337 the cost has a minimum at a value $\beta$ slightly greater than one. In our analytical study we point out that agreement between the mathematical result and simulation result is approximately confirmed by the comparison of cost analysis in both cases that is on discouraging the customers and increasing the resources in general.

## 8. LIMITATION OF THE MODEL AND FURTHER EXTENSION

In our model, it is assumed that the inter arrival time follows an exponential distribution with batch size 1, processing time $t = 0$ and there is no pairing failure. This indicates that $n$ exponential arrived customer demand require only one resource and pairing occurs instantaneously with no possible matching failure but in real world situation there are various instances where these assumptions are not fit. The pairing and batch size can be a random variable $(>1)$. The inter arrival time may follow general or Poisson distribution rather than the exponential distribution.

We end this paper with suggestions for

i) In this paper we have assumed that the patience time customers queue and resources are exponentially distributed. It would be interested to study the situation when the distributions are general and to establish relation for $x(t): t > 0$ under similar parameter regime.

ii) In this work we assume that the arrival pattern of customer and resources are independent of the state of the system. It would be interested to consider an extension where the arrivals process are counting process whose intensity parameters depend on the state of double ended queue.

iii) Even a simple change in one of these conditions, can make the mathematical formulation very complicated. The analytical approach presents a tremendous computational burden. In this case, simulation or experimentation on the system is far better to apply.

## CONCLUDING REMARKS

To summarize our results, we find that the measures we derive above are good in the sense that they provide insight into the system behavior while comparing favorably with the simulation model results. We show that due to excess of customer demand, a high service level is achieved. A balance service level is achieved either on increasing the resources with resource expansion factor $\beta$ ( $\beta > 1$ ) (about 1.7 from the said data) or by discouraging the customers with reduction factor $\alpha$ ( $\alpha < 1$ ) (about 0.6)

Secondly, we have found that the measures are more sensitive to changes in customer demand rate and less sensitive to change in resource rate. For both the system level matrices and customer level matrices there is a close correspondence between the mathematical model and simulated system. We conclude, our model is quite attractive because of its simplicity, tractability and its accuracy in approximating the performance in the simulated system.

## REFERENCES

1. Kendal, D. G. (1951). Some problems in the theory of queues. *Journal of the Royal Statistical Society*, Series B, 13(2), 151 - 185.
2. Kashyap, B. R. K. (1966). The double-ended queue with bulk service and limited waiting space. *Operations Research, 14*, 822-834.

3. Perry, D., Stadje, W. (1999). Perishable inventory systems with impatient demands, *Mathematical Method of Operation Research*, 50, 77-90.
4. Conolly B. W. and Parthasarthy, P. R. (2002), Double ended queue with impatience. *Computer Opertation Research*, 29, 2053-2072
5. Mendoza, G., Sedaghat, M., Yoon, K.P. (2009). Queuing models to balance system with excess supply. *International Business& Economics Research Journal*, 91-103.
6. Kim, W. K., Yoon, K. P., Mendoza, G., Sedaghat, M. (2010). Simulation model for extended double-ended queuing. *Computers and Industrial Engineering, 59*(2), 209-219.
7. Arti Tyagi, M. S. Saroa and Singh T. P. (2014). Application of stochastic queue model in a restaurant a case study. *Aryabhatta Journal of Mathematics And Informatics*, 6(1), 115 - 118
8. Mendoza, G., Sedaghat, M., Yoon, K. P., Melnyk, O.(2014). Balancing queueing systems with excess demand, *International Journal of Management & Information Systems, 18(3),* 173 -184.

Aryabhatta Journal of Mathematics & Informatics Vol. 6, No. 2, July-Dec, 2014 ISSN : 0975-7139
Journal Impact Factor (2013) : 0.489

# A COMPARATIVE ANALYSIS OF NEURAL NETWORK AND DECISION TREES IN FORECASTING.

**Arumugam P.* & Christy, V.****
*Department of Statistics, Manonamaniam Sundaranar University, Tirunelveli
sixfacemsu@gmail.com and Christy.eben@gmail.com

**ABSTRACT :**
***In a wide range of analysis, decision trees and neural networks are used for predictive analysis. There are researchers who compared the performance of decision trees and Neural networks, however very few with Classification and Regression Tree (CART) and Neural networks (NNs). Here we performed a three way comparison of CART, Random Forest (RF) and NNs models using a continuous and categorical dependent variable for prediction. A web usage sales data set is used to run these models. Measurement of different predictive accuracy methods are used to compare the performance of the models. Experimental results of test data of the model is used here to predict the accuracy.***
**Keywords:** ***Decision Trees, Neural Networks, Classification and Regression Tree (CART), Random Forest, Data Mining.***

## INTRODUCTION

Statistical methods like regression analysis, multivariate analysis and pattern recognition models have been applied to a wide range of decisions in many disciplines. These models are used to make decisions with the traditional statistical methodology. Neural networks share the advantages with the many other data mining tools. It has many advantage over classical models used to analyze data. Regression analysis, is that they can fit data where the relation between independent and dependent variables is non-linear and where the specific form of the nonlinear relationship is unknown. Also, decision trees, a method of splitting data into homogeneous clusters with similar expected values for the dependent variable, is often less effective when the predictor variables are continuous than when they are nominal (or categorical). Neural networks work well with both nominal and continuous variables. They do not require that the relationships between predictor and dependent variables be linear whether or not the variables are transformed. The neural network method is more robust and has better predictive accuracy than classical methods. Classification and Regression Tree (CART) models use tree-building algorithms, which are a set of if-then (split) conditions that permit prediction or classification of cases. A CART model that predicts the value of continuous variables from a set of continuous or categorical predictor variables is referred as in this model. For the prediction of the value of categorical variable from a set of continuous and/or categorical predictor variables, a set of data on classification-type CART model is used. The most advantageous of decision tree based models is CART, this tree based models are scalable to large problems and can handle smaller data set than NNs models [1].

In data mining decision trees methods are widely accepted though it also has some weak points like data fragmentation. Sometimes decision trees have relatively poor accuracy compared to other knowledge models like neural networks. In order to overcome this problem, a large number of decision trees are generated randomly for the same data set, and used simultaneously for prediction. Random Forest is one of the method. The idea of building multiple trees arose early on with the development of the multiple inductive learning (MIL) algorithm Williams [5]. In building a single decision tree, it was noted that often there was very little difference in choosing between alternative variables. For example, two or more variables might not be distinguishable in terms of their ability to partition the data into more homogeneous datasets. The MIL algorithm builds all equally good models and then combines them into one model, resulting in a better overall model. It is a statistical method for classification and was first introduced by Leo Breiman in 2013. It is a decision –tree based supervised learning algorithm. The algorithm is an ensemble classification which is unsurpassable inaccuracy among current data mining algorithms. It also can handle a very large number of attributes and can learn fast. It has been widely used in many classification problems and has produced a high classification accuracy. Random forest models are generally very competitive with nonlinear classifiers such as artificial neural nets and support vectormachines. It is known to be robust for irrelevant features with very good performance. Decision Trees and Neural Networks are widely used for classification in data mining or machine learning. Each model has its own characteristics and finding appropriate model with smallest error rate for the given data sets is crucial for the success of data mining.

Our research objective was to compare the predictive ability of CART, RF and NNs methods using sales web usage data. Comparison of predictive abilities of statistical and NNs models are plentiful in the literature. So, random forest algorithm is also applied to data sets to compare with CART and Neural networks.

It is also widely recognized that the effectiveness of any model is largely dependent on the characteristics of data used to fit the model. In our research, a three-way comparison involving CART, RF and NNs models are performed. The prediction errors of these three models are compared where the independent variables are continuous and predictive variables is categorical.

## RELATED WORKS:

We have a lot of difference in learning time between neural networks and decision trees. Generally, it takes far longer time to train neural networks than decision trees. But the two knowledge models are used widely, because of their own good points. There are two kinds of networks based on how the networks are interconnected – feed-forward neural networks and recurrent neural networks. RBF networks are one of the most popular feed- forward networks. The training time of RBF networks is relatively shorter than other neural network algorithms. A good point of RBF networks is their good prediction accuracy with small-sized data sets, which is also true for other neural networks. When we have very large data sets for training, we may use decision tree algorithms to save training time. There have been a lot of efforts to build better decision trees with respect to accuracy.[1][2] Early decision tree algorithmis a fast and dirty type algorithm that was developed in early 90's, and is often referred in literaturebecause of its wide availability. Random forest uses many decision trees simultaneously for prediction so that it can avoid the negative effect of irrelevant features. A good point of decision tree algorithms is their scalability so that they are also good for very large data sets. The bagging decision trees or random forest is the best predictors in experiment. But some omitted the possible performance comparison with neural networks. Numerous applications of NNs models in Marketing and data mining are available in the literature. It has been observed that NNs provides better prediction than discriminant analysis and logistic regression in brand choicedecision.

So we want to see some comparative analysis to generate better prediction accuracy for the data sets empirically.

## ORGANIZATION OF DATA:

In this study we have used a set of three data sets with account details with different type of variables and the web data is based on two different CRM(Customer Relationship Management) sources. Because of the change in technology we have two marketing data with different sets of variables and profile data set. The three data set contained total of 35 variables and almost 1000 records. Among 35 available variables, initially we choose 22 variables considered to be most intuitively related to leads outcome, and ran a correlation analysis. Based on the results of correlation analysis of the three data set, the 18 variables considered to be significant contributor towards the prediction of the dependent variable of the data set. (Sales leads Yes/No), are selected.

## RESEARCH MODELS AND PREDICTIVE ACCURACY PROCEDURES

## CLASSIFICATION AND REGRESSION TREE (CART) MODEL

Regression-type CART model isa non-parametric procedure for predicting continuous dependent variable with categorical and/or continuous predictor variables where the data is partitioned into nodeson the basis of conditional binary responses to questions involving the predictor variable, Sales. 'CART models use abinary tree to recursively partition the predictor space into subsets in which the distribution of sales **y** is successively more homogenous'. For example, CART procedure derives conditional distribution of sales **y** given **x** independent variables.[1] The partitioning procedure searches through all values of predictor variables to find the independent variablethat provides best partition into child nodes. The best partition is the one that minimizes the weighted variance. The distribution $f(\mathbf{y} \mid \O i)$ of **lynx** represents the situation that x lies in the region corresponding to the ith terminal node. Although numerous articles have compared NNs models with linear,non-linear and hybrid Regression models, very few havecompared the predictive ability of CART model with NNsand/or regression models.Razi and Athappilly (2005) compared NNswith CART model. Since CART is a non-parametricprocedure for predicting continuous dependent variable with categorical predictor variables, we find this model anatural fit for prediction with the variable set chosen for this study. In R programming rpart is the package which is used to run CART tree.

A random subset of 70% of the observations is chosen and will be used to identify a training dataset. The random number seed is set, using set. seed(), so that we will always obtain the same random sample for illustrative purposes and repeatability. Choosing different random sample seeds is also useful, providing empirically an indication of how stable the models are

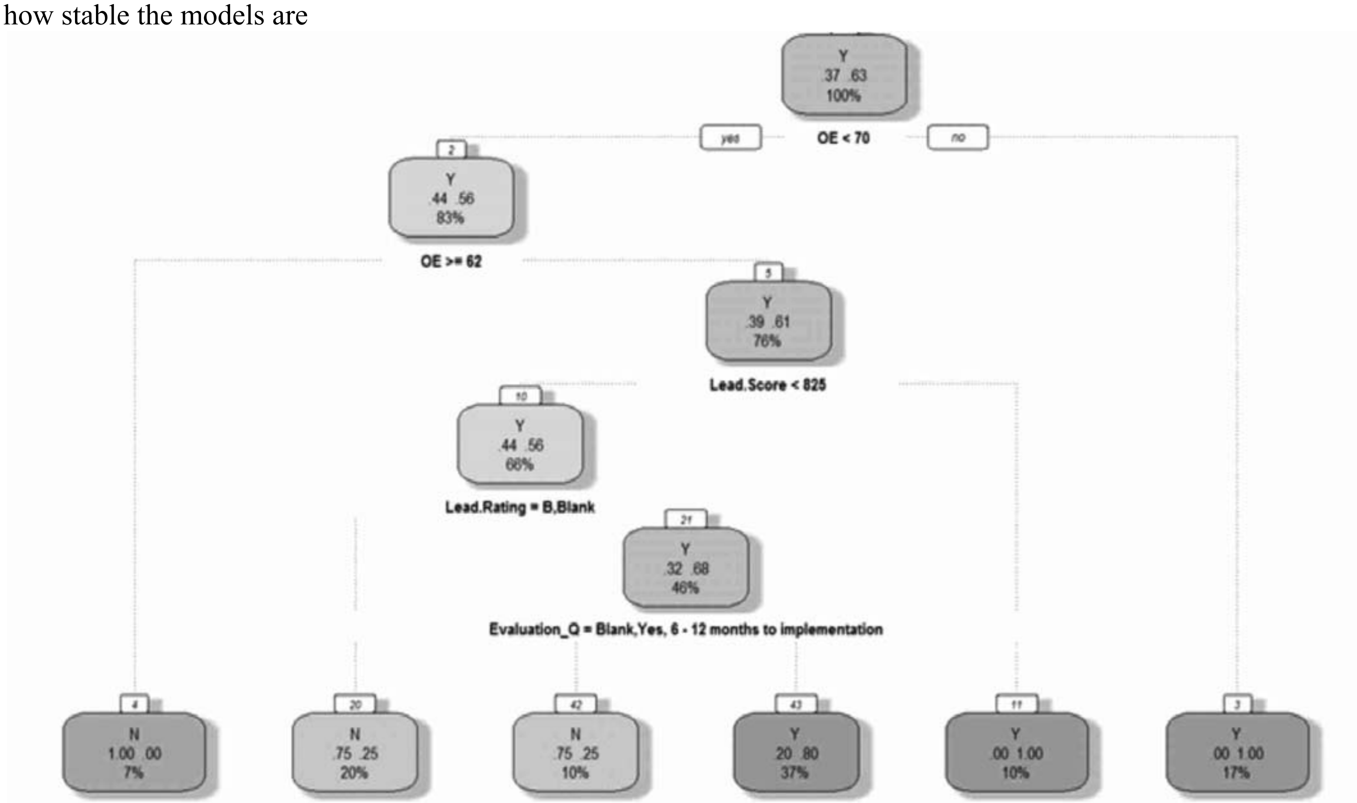



*Fig 1. CART Tree for Marketing Data.*

The above figure represents the CART decision tree rules for the used marketing data set. We have specified the minimum splits as 20 and it is nothing but the minimum number of observation exist in a node resulting from a split before a split will be performed. The complexity parameter (CP) set is 0.010. This is used to control the size of the decision tree and to select the optimal tree size. The tree construction does not continue unless it would decrease the overall lack of fit by a factor of CP. Setting CP to zero is the maximum depth of the tree.

**Neural Networks:**

We choose to use NNs method because it handlesnonlinearity associated with the data well. NNs methodsimitate the structure of biological neural network. Processingelements (PEs) are the neurons in a Neural Network.

Each neuron receives one or more inputs, processes those inputs, and generates a single or more output. Main components of information processing in the Neural Networks are: Inputs,Weights, Summation Function (weighted average of all input data going into a processing element (PE), Transformation function and Outputs (Fig. 2).

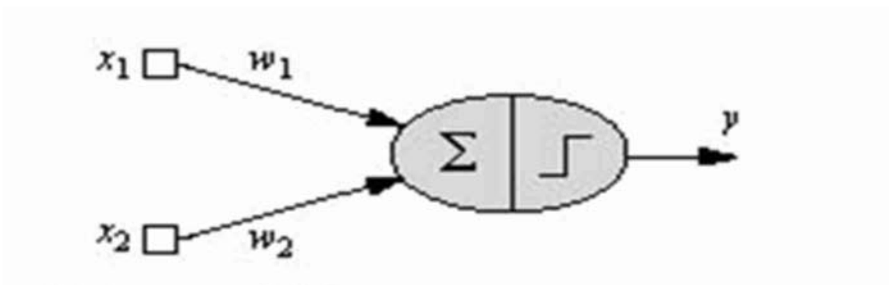



***Fig 2. Neural Network.***

**RANDOM FOREST:**

A random forest is a classifier consisting of a collection of tree structured classifiers $\{h(\mathbf{x},\Theta_k), k=1, ...\}$ where the$\{\Theta_k\}$ are independent identically distributed random vectors and each tree casts a unit vote for the most popular class at input **x** .Given an ensemble of classifiers $h_1(\mathbf{x}),h_2(\mathbf{x}), ... ,h_K(\mathbf{x})$, and with the training set drawn at random from the distribution of the random vector Y,**X**, define the margin function as

$$mg(\mathbf{X},Y) = av_k I(h_k(\mathbf{X}) = Y) \max_{j \neq Y} av_k I(h_k(\mathbf{X}) = j) .$$

Where $I(\cdot)$ is the indicator function. The margin measures the extent to which the average number of votes at $\mathbf{X},Y$ for the right class exceeds the average vote for any other class.

In random forests, $h_k(\mathbf{X}) = h(\mathbf{X}, \Theta_k)$ . For a large number of trees, it follows from the Strong Law of Large Numbers and the tree structures used. The random forest algorithm tends to produce quite accurate models because the ensemble reduces the instability that we can observe when we build single decision trees.

The random forest algorithms will often build from 100 to 500 trees. In deploying the model, the decisions made by each of the trees are combined by treating all trees as equals. The final decision of the ensemble will be the decision of the majority of the constituent trees. The randomness used by a random forest algorithm is in the selection of both observations and variables. It is this randomness that delivers considerable robustness to noise, outliers, and over fitting, when compared with a single-tree classifier. The performance of the random forest model is indicated using the OOB(Out Of Bag) error estimate.

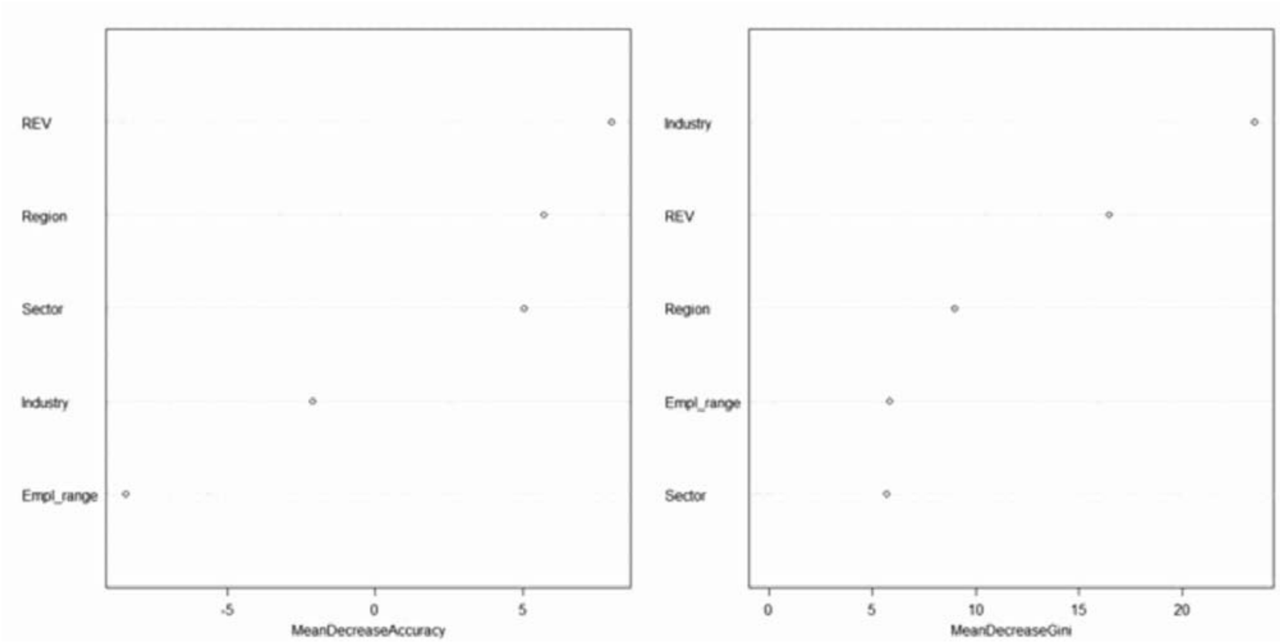



***Fig 3. Relative Importance of variable for Profile data using random forest.***

Random Forest has the difficulties in understanding the discovered knowledge. With the help of variable imporatnce we can get the idea of the knowledge discovery of building 500 decision trees. The Variable Importance is a piece of information about the accuracy measures.

In the above figure 3, the mean decrease measure is the importance measure which is a scaled average of the prediction accuracy of each variable. The calculation is based on a process of randomly permuting the values of a variable across the observations and measuring the impact on the predictive accuracy of the resulting tree. The larger the impact then the more important the variable is. Thus this measure reports the mean decrease in the accuracy of the model. The actual magnitude of the measure is not as relevant as the relative positioning of variables by the measure. The mean decrease Gini is the measure of importance is the total decrease in a decision tree node's impurity (the splitting criterion) when splitting on a variable. The splitting criterion used is the Gini index. This is measured for a variable over all trees giving a measure of the mean decrease in the Gini index of diversity relating to the variable.

**EVALUATION METHODS:**

**CONFUSION MATRIX:**

It is a specific table layout that allows visualization of the performance of an algorithm, typically a supervised learning one. Each column of the matrix represents the instances in a predicted class, while each row represents the instances in an actual class. The name stems from the fact that it makes it easy to see if the system is confusing two classes. Let us define an experiment from Y positive instances and N negative instances. The four outcomes can be formulated in a 2×2 contingency table or confusion matrix, as follows:

| Prediction | Actual Value Y | Actual Value N |
|---|---|---|
| Y | TRUE Positive | FALSE Positive |
| N | FALSE Negative | True Negative |

| Confusion Matrix Error Rate in % | | | |
|---|---|---|---|
| **Data File** | **CART** | **RF** | **NN** |
| **Old Market data** | **30%** | **8%** | **19%** |
| **New Market data** | **22%** | **3%** | **0%** |
| **Profile data** | **39%** | **20%** | **24%** |

From the above table we can see the comparison result of the three models. Here random forest model showed the lower error rate.

## RECEIVER OPERATING CHARACTERISTIC (ROC) ANALYSIS:

The sensitivity and specificity of a diagnostic test depends on more than just the "quality" of the test--they also depend on the definition of what constitutes an abnormal test. The position of the cutpoint will determine the number of true positive, true negatives, false positives and false negatives. We may wish to use different cutpoints for different clinical situations if we wish to minimize one of the erroneous types of test results. ROC Curve is a plot of the true positive rate against the false positive rate for the different possible cutpoints of test data for the model. The area under the curve (AUC) is a measure of model accuracy.It is related to the gini coefficient($G_1$) by the formula $G_1 = 2AUC - 1$, where:

$$G_1 = 1 - \sum_{k=1}^{n} (X_k - X_{k-1})(Y_k + Y_{k-1})$$

In order to establish statistical significance, we carry out statistical tests to comparatively evaluate prediction accuracy between CART, Random Forest and NNs methods based on sales data set which contains two Marketing and one profile dataset.

| ROC Area in % | | | |
|---|---|---|---|
| **Data File** | **CART** | **RF** | **NN** |
| **Old Market data** | **73%** | **95%** | **88%** |
| **New Market data** | **82%** | **100%** | **100%** |
| **Profile data** | **61%** | **85%** | **86%** |

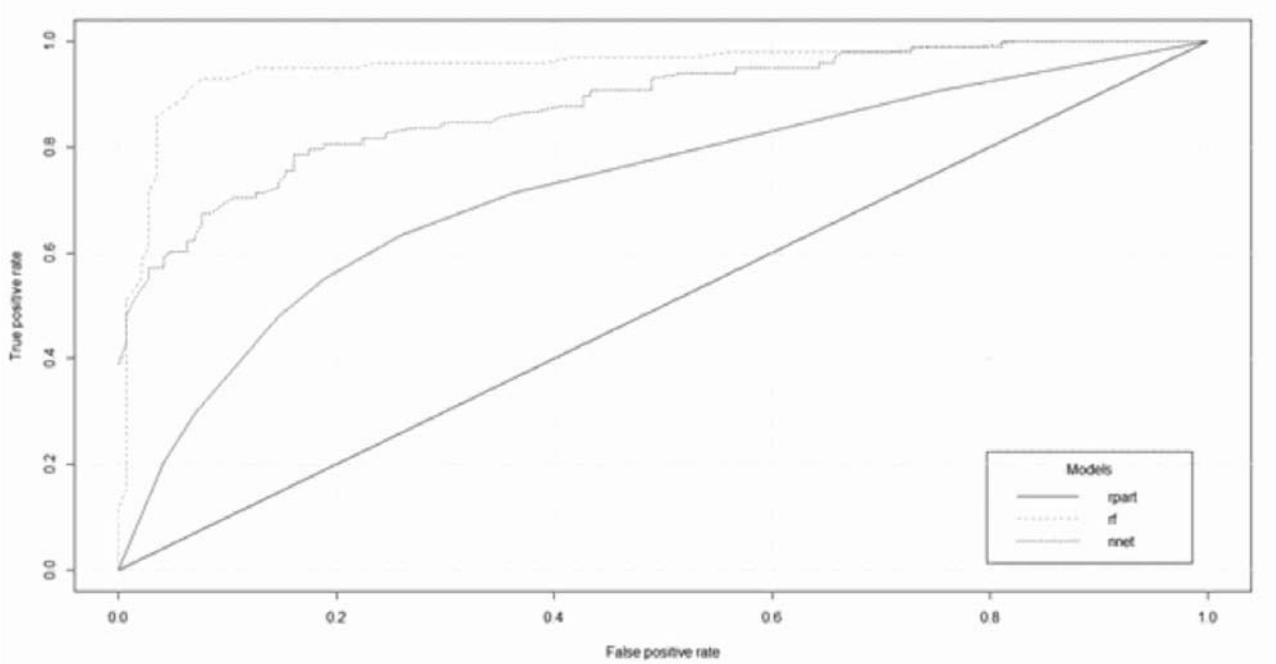



*Fig 4. ROC area for CART, RF and NN.*

## RESULTS AND ANALYSIS:

The data here we use contains the result of Neural Networks , Random Forest and CART using the Sales lead data. Random Forest gives us good result due to their bagging procedure.

A confusion matrix is appropriate whenpredicting a categorical target. The confusion matrix displays the predicted versus the actual results in a table. From the above two tables we can see that the Random Forest showed a very good result.The ROC is probably the most popular, but each can provide some different insights based essentially on the same performance data.The ROC has a form and interpretation similar to the risk chart, though it plots different measures on the axes. ROCR is used by Rattle togenerate these charts. We can differentiate each chart by the measures used on the two axes.An ROC chart plots the true positive rate against the false positive rate. The sensitivity/specificity chart plots the true positive rate against thetrue negative rate. The lift chart plots the relative increase in predictiveperformance against the rate of positive predictions. The precision/recallchart plots the proportion of true positives out of the positive predictionsagainst the true positive rate.

**Table 3. Confusion Matrix of Test data using Random Forest Method.**

| Marketing +Profile Using Random Forest | | | |
|---|---|---|---|
| | Predicted | | ER = 11% |
| Actual | N | Y | Total |
| N | 137 | 6 | 143 |
| Y | 20 | 78 | 98 |
| Total | 157 | 84 | 241 |

Based on the comparison of three methods the ROC area of Random forest method gives good result. We can see the test result of the modeling based on random forest in Table 3.Here the false negative and false positive rate is low compared to the other methods.

## CONCLUSION:

We can see that the Random forests is an effective tool in prediction compared to CART and Neural Networks. Because of the Law of LargeNumbers they do not over fit. Injecting the right kind of randomness makesthem accurate classifiers and regressors. Furthermore, the framework in termsof strength of the individual predictors and their correlations gives insight intothe ability of the random forest to predict. Using out-of-bag estimation makes concrete otherwise theoretical values of strength and correlation.

## REFERENCES:

1. Muhammad A. Razi, Kuriakose Athappilly (2005). A comparative predictive analysis of neural networks (NNs), nonlinear regression and classification and regression tree (CART) models. Expert Systems with Applications, vol 29 (2005) 65–74.
2. Jaesoo Kim, Heejune Ahn (2009). A New Perspective for Neural Networks: Application to a Marketing Management Problem.Journal of Information Science and Engineering. Vol 25, 1605-1616.
3. Hyontai Sug (2010). A comparison of RBF networks and random forest in forecasting ozone day. International Journal of Mathematics and Computers in simulation. Issue 3. Vol 4
4. .Frank M. Thiesing ,Oliver Vornberger.(1997). Sales Forecating Using Neural Networks. Proceedings ICNN'97, Houston,Texas,9-12 June 1997 Vol 4 pp2125 – 2128, IEEE 1997.
5. Graham Williams, Data Mining with Rattle and R, Springer Books 2011.
6. Breiman and Cutler's random forests for classification and regression (2013). R Package 'random Forest' version 4.6-7.
7. Yanchang Zhao (2012). R and Data Mining: Examples and Case Studies. Published by Elsevier in December 2012.

Aryabhatta Journal of Mathematics & Informatics Vol. 6, No. 2, July-Dec, 2014 ISSN : 0975-7139
Journal Impact Factor (2013) : 0.489

# SCHEDULE RISK ANALYSIS AND MANAGEMENT IN SOFTWARE PROJECTS USING SIMULATION

**Nitika Bansal*, Dr. Rajesh Garg**, Ravikant Jaiswal*****
*Research Scholar, (CSE Deptt), Ganpati Institute of Technology and Management, Bilaspur,
**Lect. (ECE Deptt), Seth Jai Prakash Polytechnic for Engineering, Damla,
***Lect. (CSE Deptt.) Ganpati Institute of Technology and Management, Bilaspur
*rajesh_damla@yahoo.co.in, ** nitikabpr@gmail.com, *** girlcutester@gmail.com

**ABSTRACT :**
***Schedule is the key factor for success of any software project. To maintain the scheduling of software project is very difficult task because it is very hard to estimate the project completion time. For analyzing the schedule risk in software project. Monte-Carlo Simulation is mainly used to calculate the activity times of a project network many numbers of times e.g. 1000 times, each time choosing the activity time randomly using probability distribution. The Monte Carlo simulation technique is also used in research work to provide greater flexibility in estimating risk index and project completion time. The main objective of work is to analyze the schedule risk in software project by calculating the risk index of each activity in the network of software project.***
**Keywords :** ***Schedule Risk, Monte-Carlo Simulation, SASR, Risk Index***

## 1. INTRODUCTION

Today in every aspect of life, Software systems are used and have made an impact on human life, but the development of these software systems is not an easy task. No. of software projects are cancelled due to many unknown problems i.e. risk which is a combination of the probability of a negative event and its impacts on the software project. *Risks* are those events or conditions that *may* occur and whose occurrence if it does take place gives negative outcome of the project. Risk arises because of lack of software risk management. The lack of risk management results in schedule slippage, budget overrun etc. The essential factors of the project success are the quality, the time and the budget [1].So, software risk management is essential to diminish the possibilities of risk in software projects.

### A. Software risk management

Risk analysis and management is a series of steps that help a software team to understand and manage uncertainty. “Software risk management can be defined as a way to manage risks”. In other words, it concerns all activities that are performed to reduce the uncertainties associated with certain tasks. In the context of projects, risk management reduces the impacts of undesirable events on a project. Risk management in any project requires undertaking decision-making activities [2]. Risk management in software projects has number of uses. It helps to save projects from failing due to non-completion of projects within the specified schedule, budget constraints etc.

### B. Role of simulation in Risk Management

Simulation is powerful tool used for risk management in any software project [6]. Simulation is a process of designing a model of a real system and conducting experiments with this model for the purpose of understanding the behavior of the system and/or evaluating various strategies for the operation of the system. Risks in a software project take many forms such as cost, schedule, quality etc. The key to manage risks in any software project is to identify them on the early stages of development of the project and develop a suitable risk mitigation plan.

## 2. DESIGNING OF SIMULATOR FOR ANALYZING SCHEDULE RISK (SASR)

Simulator is a software tool resulting from application of modeling and simulation to real life projects. It involves modeling of project or system under the study and represents the system in the form of network model.

The project network of the process of designing of system card on which the simulator is applied for analyzing the schedule risk is given in figure 1.

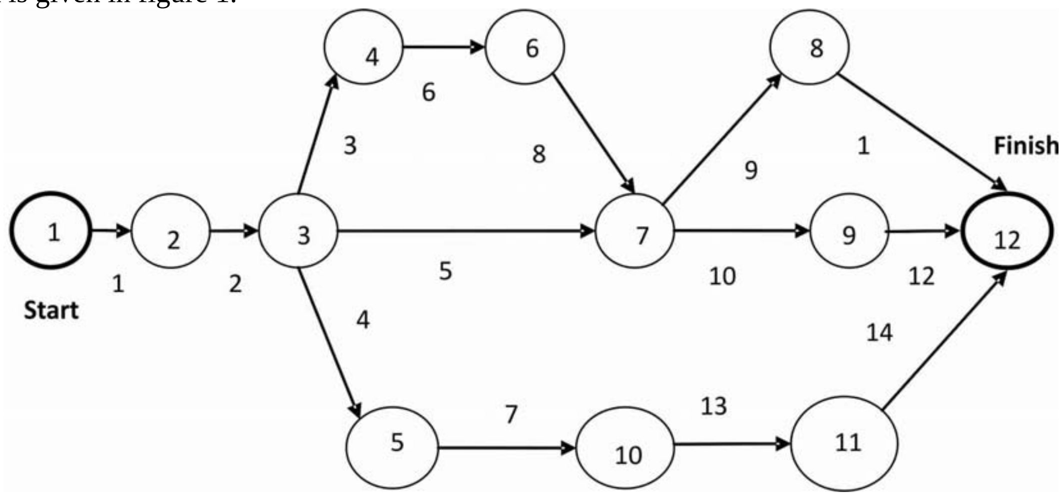



**Figure 1. N\w representation of System Card design process**

With respect to each activity of the project three parameters optimistic time X[n], pessimistic time Y[n] and most likely time Z[n] is generated given in the Table 1.

The Table 1 represents the start of an activity S[n], Finish of an activity F[n], Optimistic time, Pessimistic time and most likely time estimates with activity number.

**Table 1. Estimated time duration of each activity**

| Activity no. | S[n] | F[n] | X[n] | Z[n] | Y[n] | Mean($\mu_n$) | Sigma($\sigma_n$) |
|---|---|---|---|---|---|---|---|
| 1 | 1 | 2 | 1 | 2 | 3 | 3.0 | 0.3 |
| 2 | 2 | 3 | 2 | 2 | 5 | 4.2 | 0.2 |
| 3 | 3 | 4 | 3 | 4 | 6 | 4.0 | 0.3 |
| 4 | 3 | 5 | 2 | 5 | 6 | 5.0 | 0.3 |
| 5 | 3 | 7 | 2 | 5 | 7 | 5.3 | 0.3 |
| 6 | 4 | 6 | 1 | 3 | 5 | 3.2 | 0.2 |
| 7 | 5 | 10 | 2 | 3 | 4 | 5.0 | 0.3 |
| 8 | 6 | 7 | 2 | 3 | 5 | 4.0 | 0.7 |
| 9 | 7 | 8 | 3 | 4 | 5 | 4.0 | 0.3 |
| 10 | 7 | 9 | 3 | 3 | 4 | 4.8 | 0.5 |
| 11 | 8 | 12 | 4 | 5 | 7 | 5.2 | 0.5 |
| 12 | 9 | 12 | 2 | 2 | 4 | 3.8 | 0.5 |
| 13 | 10 | 11 | 1 | 2 | 3 | 4.0 | 0.3 |
| 14 | 11 | 12 | 2 | 2 | 3 | 2.2 | 0.5 |

Once the activities time duration ranges have been established, the schedule risk analysis simulator can determine the risk during the project schedule.

Table 1 also shows the three time estimates for each activity and their corresponding mean time and standard deviations. The data given in this table is input to the simulator to compute risk index and project completion time.

The whole procedure of designing a simulator for analyzing schedule risk is described in the algorithm_1. In this algorithm N represent the total no. of activities, M represent the total number of nodes, n represent the activity number, X[n] represent the optimistic time for activity no. n , Z[n] represent the most likely time of activity number n, and Y[n] represent the pessimistic time of an activity.

**Algorithm_1: SASR**

***Step 1:*** *Read the input data for the project network which involves no. of activities (N), no. of nodes (M),start of an activity(S[n]),finish of an activity(F[n]),simulation run (how many time to run, SRUN) and a counter variable (count).*

***Step 2:*** *Assign three time estimates to each activity based on probability distribution i.e. X[n], Z [n] and Y[n].*

***Step 4:*** *Calculate the mean and sigma for each activity using following formulas:*

$$Mean = \mu_{n=}(X[n] + Z[n] + Y[n])/6$$

$$Standard\ deviation\ (Sigma) = \sigma_n = (Y[n]-X[n])/6$$

***Step 5:*** *Initialize the run counter*

***Step 6:*** *Apply Box Muller transformation to compute the activity duration using following two equations:*

$$T_n = \sigma_n * S + \mu_n$$

$$S = (-2\log_e r1)^{1/2} \cos(2\pi r2)$$

***Step 7:*** *Calculate project completion times and risky activities of the project network using following steps:*

1. *Perform the forward pass computation on the network.*
2. *Perform the backward pass computation on the network.*

***Step 8****: Increment the run counter by 1.*

***Step 9****: Run it for SRUN times.*

***Step 10****: Compute and print all the risk indexes, activity durations and project completion times.*

## 3. RESULTS OF SIMULATOR

After running the project on the simulator for 1000 no. of times, the results of simulator are given in the form of tables and graphs. The main aim of this simulator is to calculate the risk index of each and every activity of the software project. The activity duration are said to be normally distributed given in table 2.

**Table 2. Generated activity durations**

| Activity no. | Generated activity durations for 1000 runs | | | | | |
|---|---|---|---|---|---|---|
| | 1st | 2nd | 3rd | 4th | ------------------- | 1000th |
| 1 | 4.2 | 4.4 | 4.9 | 4.4 | ------------------- | 4.5 |
| 2 | 4.3 | 3.7 | 4.2 | 4.2 | ------------------- | 3.7 |
| 3 | 4.8 | 5.3 | 5.2 | 5.6 | ------------------- | 5.0 |
| 4 | 6.1 | 6.0 | 5.8 | 6.0 | ------------------- | 5.7 |
| 5 | 3.2 | 3.1 | 3.3 | 3.2 | ------------------- | 3.0 |
| 6 | 5.5 | 5.1 | 4.8 | 4.3 | ------------------- | 4.8 |
| 7 | 4.6 | 2.5 | 3.4 | 3.0 | ------------------- | 2.8 |
| Activity no. | Generated activity durations for 1000 runs | | | | | |
| | 1st | 2nd | 3rd | 4th | ------------------- | 1000th |
| 8 | 4.5 | 3.8 | 3.3 | 4.3 | ------------------- | 3.9 |
| 9 | 3.5 | 2.8 | 3.2 | 2.9 | ------------------- | 2.9 |
| 10 | 5.7 | 5.5 | 5.6 | 5.6 | ------------------- | 5.8 |
| 11 | 5.1 | 4.1 | 5.5 | 4.8 | ------------------- | 5.7 |
| 12 | 3.3 | 4.5 | 3.8 | 4.2 | ------------------- | 3.8 |
| 13 | 4.0 | 4.2 | 4.6 | 4.0 | ------------------- | 4.3 |
| 14 | 1.1 | 1.8 | 2.8 | 2.5 | ------------------- | 2.1 |

After calculating the activity durations, project completion times are calculated. Table 3 shows the project completion times (PCT) calculated for 1000 simulation runs. Using this frequency of project completion time is determined which is presented in the graph 5.2

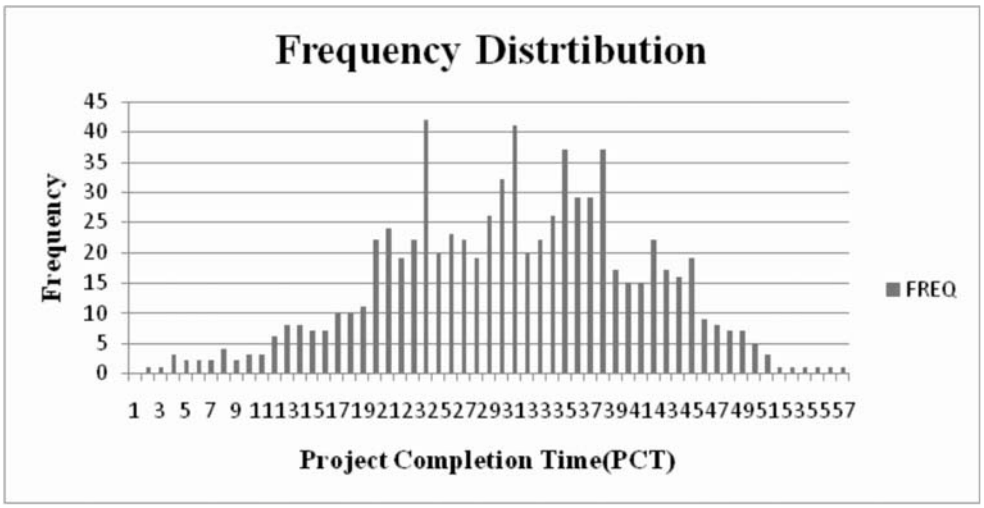
**Table 3. Project completion time (PCT)**

| Generated project completion time (in weeks) for 1000 runs | | | | | | |
|---|---|---|---|---|---|---|
| Running order | PCT | Running order | PCT | | Running order | PCT |
| 1 | 26.0 | 11 | 25.0 | …………….. | 991 | 23.3 |
| 2 | 23.4 | 12 | 23.3 | …………….. | 992 | 26.2 |
| 3 | 24.6 | 13 | 26.0 | …………….. | 993 | 25.0 |
| 4 | 24.0 | 14 | 25.1 | …………….. | 994 | 24.1 |
| 5 | 26.2 | 15 | 23.3 | …………….. | 995 | 25.1 |
| 6 | 24.6 | 16 | 25.5 | …………….. | 996 | 25.1 |
| 7 | 26.0 | 17 | 26.5 | …………….. | 997 | 25.0 |
| 8 | 25.3 | 18 | 25.3 | …………….. | 998 | 23.6 |
| 9 | 24.0 | 19 | 24.6 | …………….. | 999 | 26.2 |
| 9 | 24.0 | 19 | 24.6 | …………….. | 999 | 26.2 |
| 10 | 25.4 | 20 | 25.1 | …………….. | 1000 | 23.5 |

Graph 1 show the frequency distribution of project completion time among the 1000 simulations runs within the specified intervals as given in table 3.

Here the duration of 24 weeks occurs for the maximum times during simulation runs that mean most of the times the Process of designing the card will be completed in 24 weeks.

Table 4 shows the activities with their respective risk index; in this table the bolded activities are much more risky as compare to others.



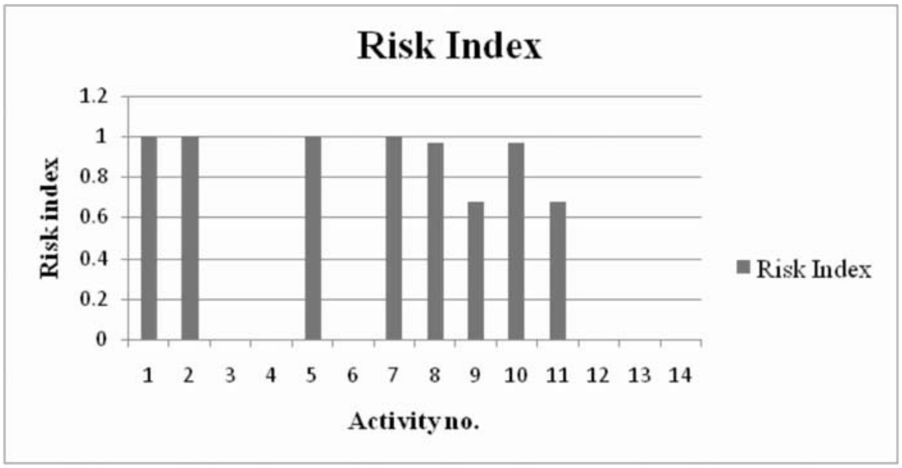

**Graph 1 Probability distribution of project completion time**

- Activity no. 1, 2, 5 and 7 has the risk index of 1 that means these activities are risky 100%.
- Activity no. 8 and 10 has risk index of 0.97 that means these activities are 97% risky in this process of designing System card.

**Table 4. Risk Index**

| Activity no. | Risk Index |
|---|---|
| **1** | **1** |
| **2** | **1** |
| 3 | 0 |
| 4 | 0 |
| **5** | **1** |
| 6 | 0 |
| **7** | **1** |
| **8** | **0.97** |
| 9 | 0.68 |
| **10** | **0.97** |
| 11 | 0.68 |
| 12 | 0 |
| 13 | 0 |
| 14 | 0 |

Graph 2 shows the plotted form of table 5 that is it shows the critical activities with their respective risk (critical) indexes.




**Graph 2. Activities v\s Risk index**

From the graph one can easily identify the critical or risky activities to deal with. The activities with risk index lies in the range of 0.8 to 1.0 are considered to be more risky as compare to others. Hence the activities 1, 2, 5, 7, 8 and 10 needs to be taken care very carefully to reduce the chances of risk occurrence in project.

## 4. CONCLUSION

The research concludes that now a day's software systems are used in every aspect of human life, but the development of these software systems is not an easy task. Number of software projects are cancelled due to many unknown problems which arises because of lack of software risk management. There are risks in every projects can be of any type, but avoiding it does not make any sense.

Simulation helps in identifying the areas of higher risks where more concentration is required to get desired results. It is also concluded that higher the Risk (criticality) index of an activity, higher is the risk involved. Computing Risk index of activities help in making decisions during project development and planning schedules for projects.

## 5. FUTURE SCOPE

Further scope of simulator includes analyzing the other risk factors like cost, performance etc. The no. of improvement elements can be added for risk management in future work, various software risk analysis techniques can be combined with the tools like simulation. Future plan for simulator also include the implementation of other related real life projects on the simulator. One more thing can be added to simulator is to set the range for risk index e.g. 0.5 to 1 and if activities' risk index lies in this range then the activity is not acceptable that means it is risky to continue with.

## REFERENCES

1. Jakub Miler & Janusz Górski (2001), "*Implementing risk management in software projects*", 3rd national conference on software engineering, Otwack, Poland.
2. Subhas C. Misra, Vinod Kumar and Uma kumar (2006), "*Different Techniques for Risk Management in Software Engineering: A Review*", ASAC Banff, Alberta.

3. Roger S. Pressman (2006), "*Software Engineering- A practitioner approach*", fifth edition, Mc Graw Hill.
4. Linda Westfall (2001), *"Software Risk Management"*, the Westfall Team PMB 383, 3000 Custer Road, Suite 270 Plano, TX 75075.
5. Abdullah Al Murad Chowdhury and Shamsul Arefeen (2011), "*Software Risk Management: Importance and Practices*", IJCIT, Volume 02, Issue 01.
6. Deo, Narsingh (2003), "*System Simulation with Digital Computer*", Prentice Hall of India, New Delhi.
7. Sarita G., Varali Kaushal etal. (2010) "Statistical analysis. Influence on marketing and finance strategies". Aryabhatta J. of Maths & Info. Vol. 2 (1) pp 95-102
8. Anu Maria (1997), *"Introduction to Modeling and Simulation*", Proceedings of the winter simulation conference.
9. Shenila Makhani, Abdul Hafeez Khan and Safeeullah Soomro (2010)*," Project Management Risk Sensitivity Analysis*", Journal of Information & Communication Technology, Vol. 4, No. 1, 38-48.
10. Haneen Hijazi, Thair Khdour and Abdulsalam Alarabeyyat (2012), "*A Review of Risk Management in Different Software Development Methodologies*", International Journal of Computer Applications (0975 – 8887), Volume 45– No.7.
11. Rajesh Garg & Vikram Singh [2010] "Simulator for performance evaluation of process scheduling policies for embedded Real time operating system". Aryabhatta J. of Maths & Info. Vol. 2 (2)

Aryabhatta Journal of Mathematics & Informatics Vol. 6, No. 2, July-Dec, 2014 ISSN : 0975-7139
Journal Impact Factor (2013) : 0.489

# ORDERING POLICY FOR ITEMS WITH VARIABLE DETERIORATION UNDER TRADE CREDIT AND TIME DISCOUNTING

**Dr. Deep Shikha*, Dr. Hari Kishan** and Megha Rani*****
*& **Department of Mathematics, D.N. College, Meerut, India
***Department of Mathematics, RKGIT, Ghaziabad, India.

**ABSTRACT :**
***In this paper, an inventory model has been developed for deteriorating items under the assumptions of trade credit and time discounting. The deterioration rate has been considered as linear function of time, i.e.*** $\theta t$ ***.The demand rate has been considered as linear function of time. Shortages are not allowed.***
**Key-words:** ***Ordering policy, deterioration, trade credit and discount.***

## 1. INTRODUCTION

In the classical EOQ model, it is assumed that the retailer must be paid for the items at the time of delivery. However, in real life situation, the supplier may offer the retailer a delay period, which is called the trade credit period, for the payment of purchasing cost to stimulate his products. During the trade credit period, the retailer can sell the products and can earn the interest on the revenue thus obtained. It is beneficial for the retailer to delay the settlement of the replenishment account up to the last moment of the permissible delay allowed by the supplier.

Several researchers discussed the inventory problems under the permissible delay in payment condition. **Goyal** (1985) discussed a single item inventory model under permissible delay in payment. **Agrawal & Jaggi** (1995) discussed the inventory model with an exponential deterioration rate under permissible delay in payment. **Chang et. al.** (2002) extended this work for variable deterioration rate. **Liao, et al.** (2000) discussed the topic with inflation. **Jamal, et. al**. (1997) and **Chang & Dye** (2001) extended this work with shortages. **Huang & Shinn** (1997) studied an inventory system for retailer's pricing and lot sizing policy for exponentially deteriorating products under the condition of permissible delay in payment. **Jamal, et. al.** (2000) and **Sarkar, et. al.** (2000) obtained the optimal time of payment under permissible delay in payment. **Teng** (2002) considered the selling price not equal to the purchasing price to modify the model under permissible delay in payment. **Shinn & Huang** (2003) obtained the optimal price and order size simultaneously under the condition of order-size dependent delay in payments. They assumed the length of the credit period as a function of retailer's order size and the demand rate to be the function of selling price. **Chung & Huang** (2003) extended this work within the EPQ framework and obtained the retailer's optimal ordering policy. **Huang** (2003) extended this work under two level of trade credit. **Huang** (2005) modified the **Goyal's** model. **Megha Rani, Hari Kishan and Shiv Raj Singh** (2011) discussed the inventory model of deteriorating products under supplier's partial trade credit policy.

While determining the optimal ordering policy, the effect of inflation and time value of money cannot be ignored. **Buzacott** (1975) developed an EOQ model with inflation subject to different types of pricing policies. **Madhu Jain,** etal. (2010) and **Hari Kishan** etal. (2012) developed multi items inventory model of deteriorating products with declined demand and premises delay payment. Futher **Sarkar** etal (2000), **Teng** etal (2002) and Hari Kishan etal (2014) also worked in this direction. **Huang** (2005) developed an ordering policy for deteriorating items under trade credit and time discounting.

In this paper, an inventory model has been developed for deteriorating items under the assumptions of trade credit and time discounting. The deterioration rate has been considered as linear function of time, i.e. $\theta t$ . The work has been extended for the case of variable deterioration and variable demand rate.

## 2. ASSUMPTIONS AND NOTATIONS:

**Assumptions:** The following assumptions are considered in this paper:

(i) The demand rate is linear function of time which is given by *at+b*.

(ii) Time horizon is finite given by *H*.

(iii) Shortages are not allowed.

(iv) The deteriorating rate is deterministic and linearly varying with time.

(v) During the time the account is not settled, the generated sales revenue is deposited in an interest bearing account. When $T \geq M$ , the account is settled at *T*=*M* and we start paying for the interest charges on items in stock. When $T \leq M$ , the account is settled at *T*=*M* and we need not to pay any interest charge.

**Notations:** The following notations have been used in this chapter:

(i) The demand rate=*at+b* where *a* and *b* are positive constants.

(ii) $A$ = the ordering cost per order.

(iii) *c*= the unit purchasing price.

(iv) *h*=unit holding cost per unit time excluding interest charges.

(v) *M*= the trade credit period.

(vi) *H*= length of planning horizon.

(vii) *T*=the replenishment cycle time in years.

(viii) *n*= number of replenishment during the planning horizon.

(ix) $\theta t$ =the deterioration rate of the on hand inventory.

(x) $I_e$ = interest earned per Re per year.

(xi) $I_c$ = interest charged per Re per year.

(xii) $I(t)$ = stock level at any time *t*.

(xiii) *Q*= maximum stock level.

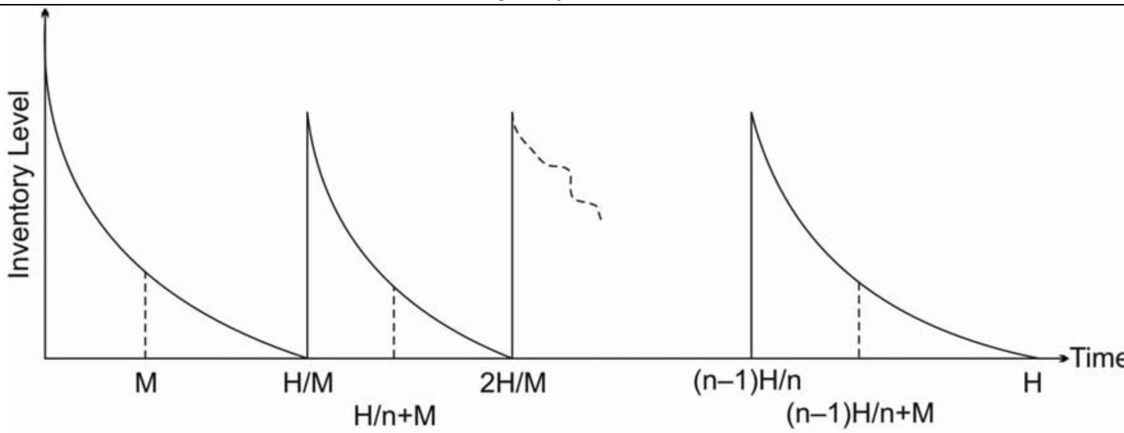
(xiv) *TVC*(*T*)= the annual total relevant cost, which is a function of *T*.

(xv) $T^*$ = the optimal cycle time of *TVC*(*T*).

(xvi) $Q^*$ = the optimal order quantity.

## 3. MATHEMATICAL MODEL:

The total time horizon *H* has been divided in *n* equal parts of length *T* so that $T = \frac{H}{n}$. Therefore the reorder times over the planning horizon H are given by $T_j = jT$ , $(j = 0,1,2...,n-1)$. This model is given by fig. 1.




**(Fig. 1)**

Let $I(t)$ be the inventory level during the first replenishment cycle. This inventory level is depleted due to demand and deterioration. The governing differential equation of the stock status during the period $0 \le t \le T$ is given by

$$\frac{dI}{dt} = -(at+b) - \theta t q \, , \; 0 \le t \le T. \qquad \ldots(1)$$

The boundary conditions are

$$I(0) = Q \, , \qquad \ldots(2)$$

and $q(T) = 0$ $\qquad \ldots(3)$

## 4. ANALYSIS

Solving equation (1) and using boundary condition (3), we get

$$I(t) = \left[\frac{a}{2}\left(T^2 - t^2\right) + b(T-t) + \frac{a\theta}{8}\left(T^4 - t^4\right) + \frac{b\theta}{6}\left(T^3 - t^3\right)\right] e^{-\frac{\theta T^2}{2}}. \qquad \ldots(4)$$

Consequently, the initial inventory after replenishment becomes

$$I(0) = Q = \frac{a}{2}T^2 + bT + \frac{a\theta}{8}T^4 + \frac{b\theta}{6}T^3. \qquad \ldots(5)$$

Since there are $n$ replenishments in the entire time horizon $H$, the present value of the total replenishment cost is given by

$$C_r = A\sum_{j=0}^{n-1} e^{-jRT}$$

$$= A\frac{\left(1-e^{-RH}\right)}{\left(1-e^{-\frac{RH}{n}}\right)}. \qquad \ldots(6)$$

and the present value of the total purchasing cost is given by

$$C_p = c\sum_{j=0}^{n-1} I(0)e^{-jRT}$$

$$= c\left[\frac{a}{2}T^2 + bT + \frac{a\theta}{8}T^4 \frac{b\theta}{6}T^3\right]\frac{\left(1-e^{-RH}\right)}{\left(1-e^{-\frac{RH}{n}}\right)}. \qquad \ldots(7)$$

The present value of the holding cost during the first replenishment cycle is given by

$$h_1 = h\int_0^T I(t)e^{-RT}e^{-\frac{\theta t^2}{2}}dt$$

$$= h\int_0^T\left[\frac{a}{2}\left(T^2-t^2\right)+b(T-t)+\frac{a\theta}{8}\left(T^4-t^4\right)+\frac{b\theta}{6}\left(T^3-t^3\right)\right]e^{-RT}\left(1-\frac{\theta t^2}{2}+\ldots\right)dt$$

$$= h\left[\frac{a}{2}\left(\frac{2Te^{-RT}}{R^2}+\frac{2e^{-RT}}{R^3}+\frac{T^2}{R}-\frac{2}{R^3}\right)+b\left(\frac{e^{-RT}}{R^2}+\frac{T}{R}-\frac{1}{R^2}\right)\right.$$

$$-\frac{a\theta}{8}\left(\frac{8T^2e^{-RT}}{R^3}+\frac{16Te^{-RT}}{R^4}+\frac{16e^{-RT}}{R^5}-\frac{T^4}{R}+\frac{4T^2}{R^3}-\frac{24}{R^5}\right)$$

$$-\frac{b\theta}{6}\left(\frac{6Te^{-RT}}{R^3}+\frac{12e^{-RT}}{R^4}-\frac{T^3}{R}+\frac{6T}{R^3}-\frac{12}{R^4}\right)$$

$$-\frac{a\theta^2}{16}\left(\frac{4T^5e^{-RT}}{R^2}+\frac{28T^4e^{-RT}}{R^3}+\frac{120T^3e^{-RT}}{R^4}+\frac{360T^2e^{-RT}}{R^5}+\frac{720Te^{-RT}}{R^6}+\frac{720e^{-RT}}{R^7}+\frac{2T^4}{R^3}-\frac{720}{R^7}\right)$$

$$\left.-\frac{b\theta^2}{12}\left(\frac{3T^4e^{-RT}}{R^2}+\frac{18T^3e^{-RT}}{R^3}+\frac{60T^2e^{-RT}}{R^4}+\frac{120Te^{-RT}}{R^5}+\frac{120e^{-RT}}{R^5}+\frac{2T^3}{R^3}-\frac{120}{R^6}\right)\right]. \qquad \ldots(8)$$

The present value of the total holding cost over the time horizon H is given by

$$C_h = h_1\sum_{j=0}^{n-1}e^{-jRT}$$

$$= h\left[\frac{a}{2}\left(\frac{2Te^{-RT}}{R^2}+\frac{2e^{-RT}}{R^3}+\frac{T^2}{R}-\frac{2}{R^3}\right)+b\left(\frac{e^{-RT}}{R^2}+\frac{T}{R}-\frac{1}{R^2}\right)\right.$$

$$-\frac{a\theta}{8}\left(\frac{8T^2e^{-RT}}{R^3}+\frac{16Te^{-RT}}{R^4}+\frac{16e^{-RT}}{R^5}-\frac{T^4}{R}+\frac{4T^2}{R^3}-\frac{24}{R^5}\right)$$

$$-\frac{b\theta}{6}\left(\frac{6Te^{-RT}}{R^3}+\frac{12e^{-RT}}{R^4}-\frac{T^3}{R}+\frac{6T}{R^3}-\frac{12}{R^4}\right)$$

$$-\frac{a\theta^2}{16}\left(\frac{4T^5e^{-RT}}{R^2}+\frac{28T^4e^{-RT}}{R^3}+\frac{120T^3e^{-RT}}{R^4}+\frac{360T^2e^{-RT}}{R^5}+\frac{720Te^{-RT}}{R^6}+\frac{720e^{-RT}}{R^7}+\frac{2T^4}{R^3}-\frac{720}{R^7}\right)$$

$$-\frac{b\theta^2}{12}\left(\frac{3T^4e^{-RT}}{R^2}+\frac{18T^3e^{-RT}}{R^3}+\frac{60T^2e^{-RT}}{R^4}+\frac{120Te^{-RT}}{R^5}+\frac{120e^{-RT}}{R^5}+\frac{2T^3}{R^3}-\frac{120}{R^6}\right)\Bigg]\times\frac{\left(1-e^{-RH}\right)}{\left(1-e^{-\frac{RH}{n}}\right)}. \qquad \ldots(9)$$

Since the inventory model considers the effect of delay in payments there are two distinct types of cases in the inventory system.

**Case I:** $M \le T = \dfrac{H}{n}$:

In this case, the interest payable is given by

$$I_{p1}^{1} = cI_c\int_M^T I(t)e^{-Rt}dt$$

$$= cI_c\Bigg[\frac{a}{2}\left(\frac{2Te^{-RT}}{R^2}+\frac{2e^{-RT}}{R^3}+\frac{\left(T^2-M^2\right)e^{-RM}}{R}-\frac{2Me^{-RM}}{R^2}-\frac{2e^{-RM}}{R^3}\right) \qquad +b\left(\frac{e^{-RT}}{R^2}+\frac{(T-M)e^{-RM}}{R}-\frac{e^{-RM}}{R^2}\right)$$

$$+\frac{a\theta}{8}\left(\frac{4T^3e^{-RT}}{R^2}+\frac{12T^2e^{-RT}}{R^3}+\frac{24Te^{-RT}}{R^4}+\frac{24e^{-RT}}{R^5}\right.$$

$$\left.+\frac{\left(T^4-M^4\right)e^{-RM}}{R}-\frac{4M^3e^{-RM}}{R^2}-\frac{12M^2e^{-RM}}{R^3}-\frac{24Me^{-RM}}{R^4}-\frac{24e^{-RM}}{R^5}\right)$$

$$+\frac{b\theta}{6}\left(\frac{3T^2e^{-RT}}{R^2}+\frac{6Te^{-RT}}{R^3}+\frac{6e^{-RT}}{R^4}\right.$$

$$\left.+\frac{\left(T^3-M^3\right)e^{-RM}}{R}-\frac{3M^2e^{-RM}}{R^2}-\frac{6Me^{-RM}}{R^3}-\frac{6e^{-RM}}{R^4}\right)\Bigg]. \qquad \ldots(10)$$

The present value of the total interest payable over the time horizon $H$ is given by

$$I_{p1}^{H} = \sum_{j=0}^{n-1} I_{p1}^{1}e^{-jRT}$$

$$= cI_c\Bigg[\frac{a}{2}\left(\frac{2Te^{-RT}}{R^2}+\frac{2e^{-RT}}{R^3}+\frac{\left(T^2-M^2\right)e^{-RM}}{R}-\frac{2Me^{-RM}}{R^2}-\frac{2e^{-RM}}{R^3}\right) \qquad +b\left(\frac{e^{-RT}}{R^2}+\frac{(T-M)e^{-RM}}{R}-\frac{e^{-RM}}{R^2}\right)$$

$$+\frac{a\theta}{8}\left(\frac{4T^3e^{-RT}}{R^2}+\frac{12T^2e^{-RT}}{R^3}+\frac{24Te^{-RT}}{R^4}+\frac{24e^{-RT}}{R^5}\right.$$

$$\left.+\frac{\left(T^4-M^4\right)e^{-RM}}{R}-\frac{4M^3e^{-RM}}{R^2}-\frac{12M^2e^{-RM}}{R^3}-\frac{24Me^{-RM}}{R^4}-\frac{24e^{-RM}}{R^5}\right)$$

$$+\frac{b\theta}{6}\left(\frac{3T^2e^{-RT}}{R^2}+\frac{6Te^{-RT}}{R^3}+\frac{6e^{-RT}}{R^4}\right.$$

$$\left.+\frac{\left(T^3-M^3\right)e^{-RM}}{R}-\frac{3M^2e^{-RM}}{R^2}-\frac{6Me^{-RM}}{R^3}-\frac{6e^{-RM}}{R^4}\right)\Bigg]\frac{\left(1-e^{-RH}\right)}{\left(1-e^{-\frac{RH}{n}}\right)}. \qquad \ldots(11)$$

The present value of the total interest earned during the first replenishment cycle is given by

$$I_{e1}^{1} = cI_e \int_0^T (at+b)te^{-Rt}dt$$
$$= cI_e\left[ a\left( -\frac{T^2e^{-RT}}{R} - \frac{2Te^{-RT}}{R^2} - \frac{2e^{-RT}}{R^3} + \frac{2}{R^3} \right)\right.$$
$$\left. + b\left( \frac{Te^{-RT}}{R} - \frac{e^{-RT}}{R^2} + \frac{2}{R^2} \right)\right]. \qquad \ldots(12$$

Hence the present value of the total interest earned over the time horizon $H$ is given by

$$I_{e1}^{H} = \sum_{j=0}^{n-1} I_{e1}^{1} e^{-jRT}$$
$$= cI_e\left[ a\left( -\frac{T^2e^{-RT}}{R} - \frac{2Te^{-RT}}{R^2} - \frac{2e^{-RT}}{R^3} + \frac{2}{R^3} \right)\right.$$
$$\left. + b\left( \frac{Te^{-RT}}{R} - \frac{e^{-RT}}{R^2} + \frac{2}{R^2} \right)\right]\left( \frac{1-e^{-RH}}{1-e^{-RH/n}} \right). \qquad \ldots(13)$$

Therefore the total present value of the costs over the time horizon $H$ is given by

$$TVC_1(n) = C_R + C_p + C_h + I_{p1}^{1} - I_{e1}^{H}. \qquad \ldots(14)$$

**Case II:** $M > T = \dfrac{H}{n}$:

In this case, the present value of the total interest earned during the first replenishment cycle is given by

$$I_{e2}^{1} = cI_e\left[ \int_0^T (at+b)te^{-Rt}dt + (M-T)e^{-RT}\int_0^T (at+b)dt \right]$$
$$= cI_e\left[ -a\left( \frac{T^2e^{-RT}}{R} + \frac{2Te^{-RT}}{R^2} + \frac{2e^{-RT}}{R^3} - \frac{2}{R^3} \right) - b\left( \frac{Te^{-RT}}{R} + \frac{e^{-RT}}{R^2} - \frac{1}{R^2} \right)\right.$$
$$\left. + (M-T)e^{-RT}\left( \frac{aT^2}{2} + bT \right)\right]. \qquad \ldots(15)$$

Hence the present value of the total interest earned over the time horizon $H$ is given by

$$I_{e2}^{H} = \sum_{j=0}^{n-1} I_{e1}^{1} e^{-jRT}$$
$$= cI_e\left[ -a\left( \frac{T^2e^{-RT}}{R} + \frac{2Te^{-RT}}{R^2} + \frac{2e^{-RT}}{R^3} - \frac{2}{R^3} \right) - b\left( \frac{Te^{-RT}}{R} + \frac{e^{-RT}}{R^2} - \frac{1}{R^2} \right)\right.$$
$$\left. + (M-T)e^{-RT}\left( \frac{aT^2}{2} + bT \right)\right]\left( \frac{1-e^{-RH}}{1-e^{-RH/n}} \right). \qquad \ldots(16)$$

Therefore the total present value of the costs over the time horizon $H$ is given by

$$TVC_2(n) = C_R + C_p + C_h - I_{e2}^{H}. \qquad \ldots(17)$$

At $M{=}T = \dfrac{H}{n}$, we have $TVC_1(n){=}TVC_2(n)$. Thus we have

$$TVC(n) = \begin{cases} TVC_1(n), T = \dfrac{H}{n} \geq M \\ TVC_2(n), T = \dfrac{H}{n} \leq M \end{cases}$$

## 5. ALGORITHM

The following algorithm is used to derive the optimal values of *n, T, Q* and $TVC(n)$:

**Step 1.** Start by choosing a discrete value of *n* equal or greater than 1.

**Step 2.** If $T = \dfrac{H}{n} \geq M$ for different integral values of n then $TVC_1(n)$ is derived from expression (14). If $T = \dfrac{H}{n} \leq M$ for different integral values of *n* then $TVC_2(n)$ is derived from expression (17).

**Step 3.** Step 1 and 2 are repeated for all possible values of *n* with $T = \dfrac{H}{n} \geq M$ until the minimum value of $TVC_1(n)$ is found from expression (7.14). Let $n = n_1^*$ be such value of *n*. For all possible values of n with $T = \dfrac{H}{n} \leq M$ until the minimum value of $TVC_2(n)$ is found from expression (7.17). Let $n = n_2^*$ be such value of *n*. The values $n_1^*, n_2^*, TVC_1(n*)$ and $TVC_2(n*)$ constitute the optimal solution.

**Step 4.** The optimal number of replenishment *n** is selected such that

$$TVC(n*) = \begin{cases} TVC_1(n), T = \dfrac{H}{n*} \geq M \\ TVC_2(n), T = \dfrac{H}{n*} \leq M \end{cases}$$

The optimal value of ordered quantity *Q** is derived by substituting *n** in the expression (5) and optimal cycle time T* is given by $T = \dfrac{H}{n*}$.

## 6. NUMERICAL EXAMPLE:

The demand rate *a*=10, *b*=1000, replenishment cost *A*=$ 60 per order, the holding cost *h*=$ 2 per unit per year, per unit item cost *c*=$ 8 per unit, deterioration parameter $\theta = 0.15$, net discount rate of inflation *R*=$ 0.1 per $ per year, the interest earned per $ per year by supplier $I_e = 0.15/\$$, the interest charged per $ per year by supplier $I_e = 0.12/\$$, planning horizon *H*=5 years, $\mu = 50$ days(=0.139 years) and *M*=60 days(=0.167 years) by assuming 360 days per year.

By the application of computational procedure, the results are shown in the following table:

| | No. of replenishment n | Cycle time T | Order quantity Q | Total present value of costs |
|---|---|---|---|---|
| CaseI: $M \leq T$ | 26 | 0.192 | 192.016 | 42251.47 |
| | 27 | 0.185 | 185.029 | 42249.53 |
| | 28 | 0.179 | 178.940 | 42250.38 |
| CaseII: $M \geq T$ | 37 | 0.135 | 135.009 | 42145.31 |
| | 38 | 0.132 | 132.013* | 42144.46* |
| | 39 | 0.128 | 128.018 | 42147.12 |

*quantities are the optimal quantities.

## 7. CONCLUSIONS:

In this paper, an inventory model has been developed for deteriorating items under the assumptions of trade credit and time discounting. The deterioration rate has been considered as linear function of time, i.e. $\theta t$. The demand rate has been taken linear function of time. This model can further be extended for other forms of demand rate and deterioration rate. Shortage case can also be included in future study.

## REFERENCES:


1. **Aggarwal, S.P. & Jaggi, C.K. (1995):** Ordering Policies of Deteriorating Items under Permissible Delay in Payment. ***Jour. Oper. Res. Soc.*, 46(5), 658-662.**
2. **Chang, H.J. & Dye, C.Y. (1999):** An EOQ Model for Deteriorating Items with Time Varying Demand and Partial Backlogging. ***Jour. Oper. Res. Soc.* 50, 1176-1182.**
3. **Chang, C.T., Quyang , L.Y. and Teng, J.T. (2003):** An EOQ Model for Deteriorating Items under Supplier Credits Linked to Ordering Quantity. ***Appl. Math. Mod.* 27, 983-996.**
4. **Chung, K.J. (1998):** A Theorem on the Determination of Economic Order Quantity under Conditions of Permissible Delay in Payments. ***Comp. & Oper. Res*., 25, 49-52.**
5. **Goyal, S.K. (1985):** Economic Order Quantity under Conditions of Permissible Delay in Payments. ***Jour. Oper. Res. Soc.*, 36, 335-338.**
6. **Huang, Y.F. (2003):** Optimal Retailers Ordering Policies in the EOQ Model under Trade Credit Financing. ***Jour. Oper. Res. Soc.*, 54, 1011-1015**
7. **Huang, Y.F. (2005):** Retailer's Inventory Policy under Supplier's Partial Trade Credit Policy. ***Jour. Oper. Res. Soc. Japan,* 48(3), 173-182.**
8. **Hari Kishan, etal (2012),** Inventory model of Deteriorating products with life time under declining demand and permissible delay in payment. Aryabhatta J. of Maths & Info. Vol. 4 (2) pp 307-3014.
9. **Hari Kishan, etal (2014),** "Multi Item**,** Inventory model of Deteriorating products with life time." Aryabhatta J. of Maths & Info. Vol. 6 (1) pp 87-96.
10. **Jamal, A.A.M., Sarkar, B.R. and Wang, S. (1997):** An Ordering Policy for Deteriorating Items with Allowable Shortage and Permissible Delay in Payments. ***Jour. Oper. Res. Soc.* 48, 826-833.**
11. **Liao, H.C., Tsai, C.H. and Su, C.T. (2000):** An Inventory Model with Deteriorating Items under Inflation When a Delay in Payments is Permissible. ***Inter. Jour. Prod. Eco.* 63, 207-214.**
12. **Megha Rani, Hari Kishan and Shiv Raj Singh (2011):** Inventory Model of Deteriorating Products under Supplier's Partial Trade Credit Policy. ***Inter, Tran. Math. Sci. & Comp.* 4(1), 2011, 143-156.**
13. **Madhu Jain etal (2010),** "Inventory Model with deterioration Inflation & permissible delay in payment. Aryabhatta J. of Maths & Info. Vol. 2 (2) pp 165-174.
14. **Sarkar, B.R., Jamal, A.M.M. and Wang, S. (2000):** Supply Chain Models for Perishable Products under Inflation and Permissible Delay in Payment. ***Comp. Oper. Res*. 27, 59-75.**
15. **Shinn, S.W. & Huang, H. (2003):** Optimal Pricing and Ordering Policies for Retailers under Permissible Delay in Payments. ***Comp. & Oper. Res.*, 30, 35-50.**
16. **Teng, J.T., Chang, H.J., Dye, C.Y. and Hung, C.H. (2002):** An Optimal Replenishment Policy for Deteriorating Items with Time-varying Demand and Partial Backlogging. ***Op. Res. Let.*, 30, 387-393.**

# SIMULATOR DESIGNED FOR REAL TIME EMBEDDED SYSTEM SOFTWARE BASED ON PRIORITY SCHEDULING ALGORITHM

**Lucky*, Dr. Rajesh Garg**,Munish Suri****
*Research Scholar (ECE Deptt.), Yamuna Institute of Engineering & Technology, Gadholi, Yamuna Nagar
**Lect.(ECE Deptt.), Damla Polytechnic for Engineering, Yamuna Nagar
***Lect(ECE Deptt.), Yamuna Institute of Engineering & Technology, Gadholi, Yamuna Nagar
*lucky.bakshi27@gmail.com, **rajesh_damla@yahoo.co.in, ***dr.munishsuri@gmail.com

**ABSTRACT :**
***Today numerous appliances like digital camera, laser printers, mobile phones and microwave ovens etc are powered by something beneath the sheath that makes them to do what they do. These tiny microcontrollers, working on basic assembly languages and high level languages are the heart of the appliances called Embedded Systems. An RTOS is responsible for managing the resources and hosting the applications that run on the embedded system but is designed specially to run with very precise timing and a high degree of reliability. The objective of this study is to design a simulator for real time embedded system software in stochastic environment. A comparative study is then made in terms of waiting and turnaround times for a number of processes based on two cases of priority scheduling policies used to design the simulator. The policy with minimum of these two parameters is considered the better one.***

**Keywords :** ***Embedded System, RTOS, Priority Scheduling, Waiting time, Turnaround time.***

## 1. INTRODUCTION

An embedded system is a combination of hardware, software and other mechanical parts designed to perform a specific function. The RTOS embedded in a system is operating system software used for real time programming and scheduling. Real-time system’s performance is specified in terms of ability to make decisions in a timely manner. These important calculations have deadline for completion and a missed deadline causes an accident. The damage caused by this miss will depend on the application. Thus it becomes necessary to have the knowledge of the embedded system prior to its hardware designing.

Simulation is used to observe the dynamic behavior of model of a real or imaginary system. By simulating a complex system, we are able to understand its behavior at a low cost. In embedded systems, simulation has potential use in the following: Exploring various designs alternatives, functional simulations, high-level performance and power estimations. Various researchers have done a lot of work in the field of real time embedded system software through designing different types of simulators. Liu and Leyland [1973] for the first time study priority driven algorithm. Yashuwanth et.al [2010] developed a modified round robin algorithm. Scheduling decision for embedded software plays an important role in system performance. The designer should keep in mind how to select the right scheduling algorithm in order to save him from error prone and time consuming tasks at the final stage of system design. Garg and Vikram Singh [2010] developed a simulator for performance evaluation of CPU scheduling policies for RTOS in crisp set under stochastic environment. Mehdi and Neshat et.al [2012] proposed an algorithm which optimizes the average waiting and response time for system process and compared the performance of this algorithm with the classical scheduling algorithm under non-fuzzy environment. Singh T.P et.al [2013] developed a heuristic algorithm for general priority scheduling in fuzzy environment and obtained optimal solution to the general machine scheduling problem. Recently, Silky, Sachin and Singh T.P [2014] made a comparative study of CPU scheduling algorithms for RTOS in which CPU burst time has been considered fuzzy in nature. In our study we have designed a simulator for real time embedded system software and the comparative study has been made in terms of waiting and turnaround times for various processes.

## 2. DESIGNING OF SIMULATOR FOR REAL TIME EMBEDDED SYSTEM

The simulator for real time embedded system software has been developed in C++ language under windows operating system on an INTEL compatible machine to execute preemptive and non-preemptive priority scheduling algorithm. The total number of processes is entered by the user and the scheduling algorithm is implemented by calling the respective function. The final results are then displayed to the user.

**Algorithm 1: Simulation Design Main**

**Step-1**: *Input the total no. processes from user.*
*Read Number of Processes in variable PROCESS*

**Step-2**: *Simulate preemptive scheduling.*
*Call Preemptive_Sch (PROCESS,CAT[],Burst[],Priority[])*

**Step-3:** *Simulate non-preemptive scheduling.*
*Call Non_Preemptive_Sch(PROCESS,CAT[],Burst[],Priority[])*

**Step-4**: *Display the output.*

**Algorithm 2: Preemptive_Sch (PROCESS, CAT [], Burst [], Priority [])**

**Step-1**: *[Initialization]*
*Total Waiting Time = 0*
*Avg Waiting Time = 0*
*Total Turn around Time = 0*
*Avg Turn around Time = 0*
*Count = 0*
*BtSum= 0*
*Time = 0*

**Step-2**: *[Read the process workload on the system from disk file(s) in integer variable PROCESS]*

**Step-3**: *Repeat for count =0 to Process-1*
*BtSum = BtSum + Burst (count)*
*[end of step (3) loop]*

**Step-4**: *[Initialize the variable inCPU to the current value of count variable that hold the total number of processes] inCPU = count*

**Step-5**: *Repeat for time = 0 to BtSum*

**Step-6**: *Repeat for i = 0 to PROCESS-1*
*IF CAT(i) <= time AND Priority(i)< Priority(inCPU) THEN*
*Set inCPU = i*
*[end of IF]*
*[end of step (6) loop]*

**Step-7**:*[Decrement TimeRemaining array by 1 for inCPU counter, TimeRemaining is the array that holds the service time left for the processes]*
*TimeRemaining(inCPU)= TimeRemaining(inCPU)-1*

**Step-8**: *IF TimeRemaining(inCPU) = 0 THEN*

*Set FinishTime(inCPU) = time+1*

*inCPU = inCPU-1*

*[end of IF]*

*[end of step(5) loop]*

**Step-9**: *Repeat for i = 0 to PROCESS-1*

*Set WaitingTime(i) = FnishTime() – Burst(i) – CAT(i)*

*TurnaroundTime(i) = WaitingTime(i) + Burst(i)*

*[end of step(9) loop]*

**Step-10**:*[Compute the total waiting time and turnaround times of the all the processes]*

*Repeat for i = 0 to PROCESS-1*

*CumulativeWaitingTime=CumulativeWaitingTime+WaitingTime()*

*CumulativeTurnaroundTime=CumulativeTurnaroundTime+*

*TurnaroundTime(i)*

*[end of step(10) loop]*

**Step-11**: *[Compute the Average waiting time and average turnaround time]*

*AvgWaitingTime = CumulativeWaitingTime/PROCESS*

*AvgTurnaroundTime = CumulativeTurnaroundTime/PROCESS*

**Step-12**: *End.*

**Algorithm 3: Non-Preemptive_Sch (PROCESS, CAT[], Burst[], Priority[])**

**Step-1**: *[Initialization]*

*TotalWaitingTime = 0,*

*AvgWaitingTime = 0,*

*TotalTurnaroundTime = 0,*

*AvgTurnaroundTime = 0,*

*Count = 0,*

*BtSum = 0,*

*Time = 0*

**Step-2**: *[Read the process workload on the system from disk file(s) in integer variable PROCESS]*

**Step-3**: *Repeat for count =0 to Process-1*

*BtSum = BtSum+Burst(count)*

*[end of step(3) loop]*

**Step-4**: *[Initialize the variable inCPU to the current value of count variable that hold the total number of processes]*

*inCPU = count*

**Step-5**: *Repeat for time = 0 to BtSum*

*IF time = 0 THEN*

*Set isIdle = true [isIdle is a Boolean variable]*

*[end of IF]*

*IF inCPU≠-1 AND time=StartTime(inCPU)+Burst(inCPU) THEN*

*Set IsIdle = true;*

*FinishTime(inCPU)= Time;*

*[end of IF]*

*[StartTime and FinishTime are the arrays that hold the starting time and finishing time of all processes that process takes]*

**Step-6**: *Repeat for count =0 to Process-1*

*IF Priority(count)<Priority(inCPU) AND CAT(count)<=Time THEN*

*Set StartTime(count) = time*

*isIdle = false*

*inCPU = count*

*[end of step(6) loop]*

*[end of step(5) loop]*

**Step-7**: *Repeat for i = 0 to PROCESS-1*

*Set WaitingTime(i) = StartTime() – CAT(i)*

*TurnAroundTime(i) = WaitingTime(i) + Burst(i)*

*[end of step(7) loop]*

**Step-8**: *[Compute the total waiting time and turnaround time of the all the processes]*

*Repeat for i = 0 to PROCESS-1*

*CumulativeWaitingTime = CumulativeWaitingTime + WaitingTime(i)*

*CumulativeTurnaroundTime= CumulativeTurnaroundTime + TurnaroundTime(i)*

*[end of step(8) loop]*

**Step-9**: *[Compute the Average waiting time and average turnaround time]*

*AvgWaitingTime = CumulativeWaitingTime/PROCESS*

*AvgTurnaroundTime = CumulativeTurnaroundTime/PROCESS*

**Step-10**: *End.*

## 3. RESULT DISCUSSION

Table 3.1 gives the process information for 100 processes and table 3.2 shows the process information for 500 processes.

**Table 3.1**: Process information of 100 processes

| Process | Burst Time | Arrival Time | Priority |
|---|---|---|---|
| 1 | 6 | 13 | 4 |
| 2 | 12 | 13 | 92 |
| 3 | 2 | 15 | 96 |
| 4 | 1 | 18 | 48 |
| 5 | 5 | 21 | 36 |
| 6 | 6 | 21 | 58 |
| 7 | 32 | 25 | 15 |
| 8 | 3 | 34 | 75 |
| 9 | 1 | 45 | 82 |
| 10 | 2 | 45 | 0 |
| - | - | - | - |
| - | - | - | - |
| - | - | - | - |
| 98 | 1 | 352 | 90 |
| 99 | 4 | 353 | 89 |
| 100 | 4 | 359 | 45 |

**Table3.2**: Process information of 500 processes

| Process | Burst time | Arrival time | Priority |
|---|---|---|---|
| 1 | 1 | 4 | 395 |
| 2 | 3 | 7 | 198 |
| 3 | 13 | 8 | 290 |
| 4 | 5 | 12 | 437 |
| 5 | 3 | 14 | 303 |
| 6 | 7 | 24 | 128 |
| 7 | 18 | 41 | 219 |
| 8 | 7 | 42 | 10 |
| 9 | 2 | 46 | 155 |
| 10 | 7 | 46 | 61 |
| - | - | - | - |
| - | - | - | - |
| - | - | - | - |
| 498 | 5 | 2342 | 390 |
| 499 | 2 | 2363 | 293 |
| 500 | 2 | 2366 | 492 |

Table 3.3 gives the average waiting time and turnaround time for 100 and 500 processes. In preemptive scheduling, for 100 processes, the average waiting time is 163.71 and average turnaround time is 170.25 and in non-preemptive scheduling, the average waiting time is 165.96 and average turnaround time is 172.

In preemptive scheduling, for 500 processes, the average waiting time is 490.504 and average turnaround time is 497.206 and in non-preemptive scheduling, the average waiting time is 494.054 and average turnaround time is 500.756.

**Table 3.3: Waiting time and Turnaround time for 100 and 500 Processes**

| For 100 Process | Preemptive Priority Scheduling | | Non- Preemptive Priority Scheduling | | For 500 Process | Preemptive Priority Scheduling | | Non-Preemptive Priority Scheduling | |
|---|---|---|---|---|---|---|---|---|---|
| **Process No.** | **Waiting Time** | **Turn-around Time** | **Waiting Time** | **Turn-around Time** | **Process No.** | **Waiting Time** | **Turn-around Time** | **Waiting Time** | **Turn-around Time** |
| **1** | 0 | 6 | 0 | 6 | **1** | 0 | 1 | 0 | 1 |
| **2** | 623 | 635 | 7 | 19 | **2** | 0 | 3 | 0 | 3 |
| **3** | 636 | 638 | 636 | 638 | **3** | 2 | 15 | 2 | 15 |
| **4** | 1 | 2 | 1 | 2 | **4** | 21 | 26 | 21 | 26 |
| **5** | 34 | 39 | 59 | 64 | **5** | 16 | 19 | 9 | 12 |
| **6** | 218 | 224 | 237 | 243 | **6** | 0 | 7 | 2 | 9 |
| **7** | 2 | 34 | 7 | 39 | **7** | 68 | 86 | 0 | 18 |
| **8** | 481 | 484 | 492 | 495 | **8** | 0 | 7 | 19 | 26 |
| **9** | 528 | 529 | 539 | 540 | **9** | 26 | 28 | 47 | 49 |
| **10** | 0 | 2 | 19 | 21 | **10** | 16 | 23 | 37 | 44 |
| - | - | - | - | - | - | - | - | - | - |
| - | - | - | - | - | - | - | - | - | - |
| - | - | - | - | - | - | - | - | - | - |
| **98** | 281 | 282 | 292 | 293 | **498** | 379 | 384 | 382 | 387 |
| **99** | 273 | 277 | 284 | 288 | **499** | 8 | 10 | 11 | 13 |
| **100** | 48 | 52 | 51 | 55 | **500** | 948 | 950 | 948 | 950 |
| **Average** | 163.71 | 170.25 | 165.96 | 172.5 | **Average** | 490.504 | 497.206 | 494.054 | 500.756 |

## 4. CONCLUSION

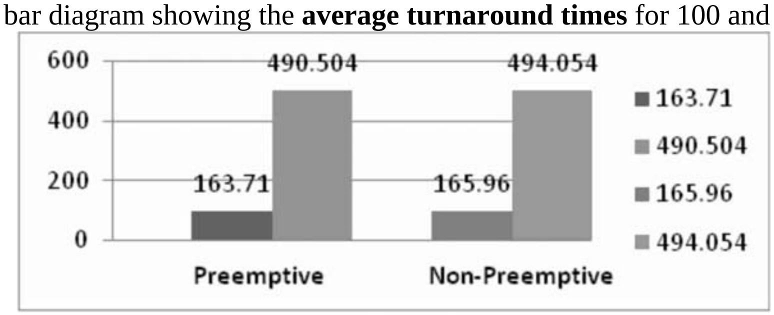
The output of the designed simulator for 100 processes and 500 processes in terms of waiting time and turnaround time are shown in table 4.1

**Table 4.1: Comparative outputs from two scheduling policy simulators**

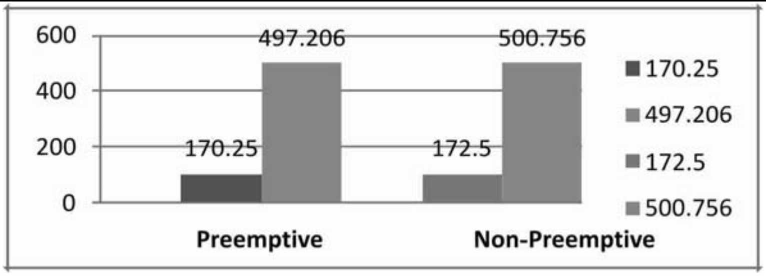
| Processes No | 100 | | 500 | |
|---|---|---|---|---|
| **Parameters** | **Waiting Time** | **Turnaround Time** | **Waiting Time** | **Turnaround Time** |
| **Preemptive** | 163.71 | 170.25 | 490.504 | 497.206 |
| **Non-preemptive** | 165.96 | 172.50 | 494.054 | 500.756 |

Graph 4.1 represents the bar diagram showing the **average waiting times** for 100 and 500 processes.

Graph 4.2 represents the bar diagram showing the **average turnaround times** for 100 and 500 processes.




**Graph 4.1**: Average waiting time




**Graph 4.2**: Average turnaround time

Preemptive scheduled simulator has shown well response as compared to non-preemptive one. But when the processes with short burst time have to preempt and wait for a long whether due to some I/O request or preemptive nature of scheduler in that case the non-preemptive scheduled simulator gives better results than preemptive one. Thus on the whole preemptive scheduled simulator is considered as best among the two but starvation may be possible for the lowest priority processes.

## 5. FUTURE SCOPE

Preemptive priority scheduling can cause problems known as ‘indefinite blocking’ or ‘starvation’ when two processes share same data. Because one process may get interrupted in the middle of updating shared data structures. In this a low priority task can wait forever because there are always some other tasks around that have high priority. One solution to this problem is ‘priority swapping’, in which priorities of tasks get swapped after once they executed using a time slice. Under this scheme a low priority process will eventually get its priority raised high enough to get run. Thus the proposed work can be modified using this concept.

## REFERENCES

1. Terry Regner, Craig lacy; “An introductory study of scheduling algorithms”; Feb 2005.
2. C.L. Liu and J. Leyland,[1973],”Scheduling Algorithm For Multiprogramming in a Hard Real Time Environment”, J.Amar.Compt.Mach, Vol.20 No.1, pp 4MI.
3. Silber Schartz, A. and Galvin, P.B., [2001], “Operating System Concepts”, (6e), Addison-Wesley, New York.
4. Kamal, R, [2006], “Embedded Systems - Architecture, Programming and Design”, (8e), Tata McGraw, New Delhi.
5. Yashuwanth.C ,Dr. R.Ramesh, [2010], ”Design of Real Time Scheduler Simulator and Development of Modified Round Robin Architecture”, IJCSNS International Journal of Computer Science and Network Security,Vol.10 No.3.
6. Sukanya Suranauwarat, [2007], ”A CPU Scheduling Algorithm Simulator”, A proceeding of 3^7th ASEE/IEEE Frontiers in Education Conference.
7. Garg, Rajesh & Singh Vikram, [2010],” Simulator for Performance Evaluation of Process Scheduling Policies for Embedded Real-Time Operating Systems”, Aryabhatta Journal of Mathematics and Informatics Vol.2, No.2.
8. M.V. Panduranaga Rao, K.C. Shet, [2009], **“**A Research in Real Time Scheduling Policy for Embedded System Domain” CLEI Electronic Journal, Vol. 12, No.2.
9. Silky Miglani, Sachin and Singh T.P [2014], “Comparative Study of CPU Scheduling Algorithm for Real-Time Operating System in Uncertainty”, Aryabhatta Journal of Mathematics & Informatics, Vol.6 (1) pp 149-158.
10. Garg Rajesh, Lucky, Anil Raman; “CPU Scheduling - A Review”**,** 2nd International Conference on Futuristic Trends in Engineering & Management 2014 (ICFTEM-2014) (ISSN: 0973-7391) 3rd May, 2014.

Aryabhatta Journal of Mathematics & Informatics Vol. 6, No. 2, July-Dec, 2014 ISSN : 0975-7139
Journal Impact Factor (2013) : 0.489

# THE IMPACT BY RELIABILITY GROWTH IN STRESS INTERVENTION FOR NON-METASTATIC BREAST CANCER THROUGH STOCHASTIC MODEL

**Dr. P. Senthil Kumar*, Ms. N. Umamaheswari****
* Assistant Professor, Department of Mathematics, Rajah Serfoji Government Arts College, Thanjavur- 613 005, Tamilnadu, India.
** Assistant Professor, Department of Science and Humanities, Dhanalakshmi Srinivasan Institute of Technology, Samayapuram- 621 112,
Email: *senthilscas@yahoo.com.**India.Email:maheshreji11@gmail.com.

**ABSTRACT :**
***The paper studies the effects of a cognitive-behavioral stress management (CBSM) intervention in order to examine intervention related changes in serum cortisol among breast cancer patients over a 12- month period in women who were undergoing treatment for non metastatic breast cancer. The most widely used traditional reliability growth tracking model and reliability growth projection model are both included as International Standard and National Standard models. These traditional models address reliability growth based on failure modes surfaced during the test. This paper presents an Extended Model that addresses this practical situation and allows for preemptive corrective actions.***
**Keywords:** *Cortisol; Breast cancer; relaxation; stress management; extended reliability growth model;*
***2010 Mathematics Subject Classification: 60G05, 60K10,62N05,***

## 1. INTRODUCTION

Women diagnosed with breast cancer encounter a number of burdens, including anxiety about treatment and prognosis adjuvant therapy, and disruptions in daily living. Evidence suggests cancer-related stress has a negative impact on health, possibly through neuroendocrine pathways [1], Cortisol, a steroid hormone secreted by the adrenal cortex [2,11], is used to assess hypothalamic-pituitary-adrenal (HPA) axis function and is a reliable measure of physiological stress, Cortisol affects multiple physiological processes, including metabolic and immune responses in [10, 12]. Women living with breast cancer have higher cortisol levels compared with healthy women, and higher cortisol levels are associated with greater disease severity in women with breast cancer [4]. Neuroendocrine regulation of cortisol and other adrenal hormones may contribute to cancer progression and health outcomes through multiple mechanisms.

In today's environment of compressed schedules and limited testing, every opportunity to identify the correct reliability deficiencies in a new design is one of the prime objective. A metric for tracking system reliability before development testing based on preemptive corrective actions for potential problem modes is discussed [7]. In the test-fix-test strategy problem modes are found during testing and corrective actions for these problems are incorporated during the test. For the test-find-test strategy problem modes are found during testing but all corrective actions for these problems are delayed and incorporated after the completion of the test.

## 2. NOTATION

$\lambda$ Scale parameter for model
$\beta$ Shape parameter for model
tTest time
T Total test time

MTBF Mean time between failures

$X_i$ $i^{th}$ successive failure time

N Total number failures

$\lambda_p$ Projected failure intensity

$M_p$ Projected MTBF

$\lambda_{CA}$ Achieved failure intensity

$\lambda_{EM}$ Extended model failure intensity

$M_{EM}$ Extended model MTBF

## 3. BACKGROUND ON THE WIDELY USED TEST-FIX-TEST AND TEST-FIND-TEST MODELS

To lay the groundwork for the Extended Model we first give some background on the two widely used basic models. For reliability growth during test-fix-test development testing states that the instantaneous system MTBF at cumulative test time t is

$$M(t) = \left[\lambda\beta t^{\beta-1}\right]^{-1} \tag{1}$$

Where $0 < \lambda$ and $0 < \beta$ are parameters. The Non-homogeneous Poisson process (NHPP) with intensity in [8] is defined by

$$r(t) = \lambda\beta t^{\beta-1} \tag{2}$$

Thus allowing for statistical procedures based on this process for reliability growth analyses. This model is applicable to text-fix-test data not test-fix-find-test. Estimation procedures, confidence intervals, etc, in[9]. The parameter $\lambda$ is referred to as the scale parameter and β is the shape parameter. For $\beta$ = 1, there is no reliability growth. For $\beta$ < 1, there is positive reliability growth. That is, the system reliability is improving due to corrective actions. For $\beta$ >1, there is negative reliability growth. The basic model the achieved or demonstrated failure intensity at time T, the end of the test is given by r (T). We denote the achieved failure intensity by

$$\lambda_{CA} = r(T) \tag{3}$$

Suppose a development testing program begins at time 0 and is conducted until time T and stopped. Let N be the total number of failures recorded and let $0 < X_1 < X_2 < \ldots\ldots X_N < T$ denote the N successive failure times on a cumulative time scale. We assume that the NHPP assumption applies to this set of data in [3]. Under the basic model the maximum likelihood estimates for $\lambda$ and $\beta$ (Numerator of MLE for $\beta$ adjusted from N to N-1 to obtain unbiased estimate) are

$$\hat{\lambda} = \frac{N}{\hat{T}\beta}, \hat{\beta} = \frac{N-1}{\sum_{i=1}^{N} \log\frac{T}{X_i}} \quad . \tag{4}$$

and $\lambda^* = \dfrac{N}{\hat{T}\beta^*}$ (5)

$$\beta^* = \frac{N-1}{\sum_{j=1}^{n} \log\frac{T}{X_j}} \tag{6}$$

If it is assumed that no corrective actions are incorporated into the system during $\beta$ < 1 the test and then this is equivalent to assuming that for $\lambda_{CA}$ and $\lambda^*_{CA}$ is estimated in [5,6]. The estimated projected failure intensity

$$\lambda_p = \lambda_{AT} + \sum_i^N (1 - E_i) + \hat{E}h(T / AT) \quad (7)$$

$$\lambda^*{}_p = \lambda^*{}_{AG} + \sum_i^N (1 - E^*{}_i) + \hat{E}h(T / AG) \quad (8)$$

The extended model projected failure intensity is

$$\lambda_{EM} = \lambda_{CA} - \lambda_{AT} + \sum_i^N (1 - E_i)\frac{N_i}{T} + \hat{E}h(T / AT) \quad (9)$$

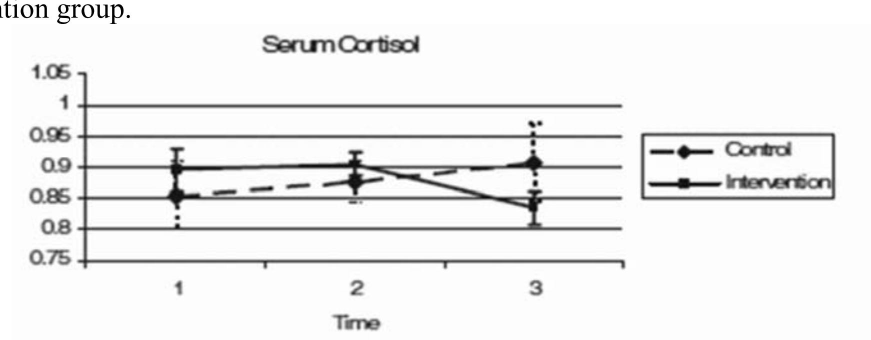
$$\lambda^*{}_{EM} = \lambda^*{}_{CA} - \lambda^*{}_{AT} + \sum_j^N (1 - E_j^*)\frac{N_j}{T} + \hat{E}h(T / AG) \quad (10)$$

The extended model projected MTBT is

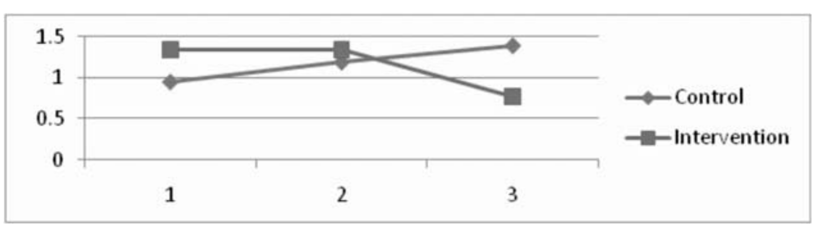
$$M_{EM} = \frac{1}{\lambda_{EM}} \text{ and } M^*{}_{EM} = \frac{1}{\lambda^*{}_{EM}} \quad . \quad (11)$$

## 4. EXAMPLE

The first hypothesis – that women in the intervention group would have decreases in cortisol levels across time compared with women in the control condition was supported. Women who received the intervention demonstrated decreases in cortisol across 12 months. The second hypothesis – that women in the intervention group would report increased ability to relax across time compared with women in the control condition was also supported. Those who received cognitive behavioral stress management (CBSM) increased confidence in their ability to relax across time to a greater degree than control in [4]. Figure (1) shows mean values and 95% confidence intervals for log-transformed serum cortisol values at time points 1,2 and 3 for women assigned to the cortisol and intervention group.




Fig(1)




Fig(2)

## 5. CONCLUSION:

These calculations can be updated continuously throughout the entire reliability growth test and across the test phases, the model can be implemented using data over the test phases. Women who participate in a 10 – week

Cognitive behavioral stress management intervention during treatment for breast cancer show decreases in physiological stress in parallel with increases in perceived relaxation skills. This is the first study demonstrating well-maintained reductions in cortisol after a Cognitive behavioral stress management intervention in cancer patients during and just after treatment in fig (1). At the completion of the reliability growth test, it concludes that from fig (2), the results coincide with the medical findings.

**REFERENCES:**


1. Antoni MH, Latgendorf S, Cole S, Dhahbar F, Sephton S, McDonald P, Stefanek M, Sood A, The influence of biobehavioral factors on tumor biology, pathways and mechanisms, Nat Rev Cancer 2006; 6:240-8.
2. Antoni MH, Psychoneuroendocrinology and psychoneurommunology of cancer. Plausible mechanisms worth pursiomg, Brain behave immun 2003:17:584-591.
3. John S, Usher, Reliability models and misconceptions, Quality Engineering Volume 6 Issue 2, pp 261-271 (October 1993).
4. Kristin M. Phillips, Michael ,Antoni H et al, "Stress management intervention reduces serum cortisol and increases relaxation during treatment for nonmetastatic breast cancer", psychosomatic medicine 70:1044-1049 (2008).
5. Lakshmi S & S Alamelu (2011). "A Mathematical model for arousing effect of CRH between middle aged & young man using a discrete Weibull distribution." Aryabhatta J. of Math. & Info. Vol. 3 (1) pp 135-138.
6. Lakshmi S and Karthik R, Mathematical Model for the secretion ofGnRH in beef cows by using extended Reliability growth model, Bio-Science Research Bulletin. Vol.27 (no.1) 2011:P1 -5.
7. Larry H. Crow, "An Extended Reliability Growth Model for Managing And Assessing Corrective Actions, Reliability and Maintainability symposium, Los Angeles, CA,http://www.reliasoft.com/pubs/2004rm_06B_01pdf.
8. L.H. Crow, Reliability analysis for Complex, Repairable systems, in Reliability and Biometry, ed. By F. Proschan and R.J. Serfing, pp.379-410, 1974, Philadelphia, SIAM.
9. R. Bala Kumar & S. Lakshmi (2011), "Proportional Intensity model of non Homogenous Poisson Process for secretion of Aldosterone in unilateral Aldosteronoma Patients." Aryabhatta J. of Math. & Info. Vol. 3 (1) pp 1-6.
10. Senthil Kumar P and Umamaheswari N, " Stochastic Model for the Box-Cox Power transformation and estimation of the Ex-Gaussian and Estimation of Cortisol secretion of breast cancer due to smoking people, Antartica J. Math.,11(1), 99-108, 2014.
11. Stone AA, Schwartz JE, Smyth J, Kirschibaum C, Cohen S, Hellhammer D, Grossman S, Individual differences in the diurnal cycle of salivary free cortisol a replication of flattened cycles for some individuals Psychoneuroendocrinology 2001;26:295-306.
12. S. Lakshmi & N. Durga Devi (2014), "A Multi-variate model for the combined propranolol/TSST PARADIGM". Aryabhatta J. of Math. & Info. Vol. 6 (1) pp 103-108.

Aryabhatta Journal of Mathematics & Informatics Vol. 6, No. 2, July-Dec, 2014 ISSN : 0975-7139
Journal Impact Factor (2013) : 0.489

# A NEW APPROACH TO SOLVE INTERVAL LINEAR ASSIGNMENT PROBLEM

**Dr. A. Ramesh Kumar* & S. Deepa****

*Head, Department of Mathematics, Srimad Andavan Arts and Science College (Autonomous), T.V.Kovil, Trichy-5.
** Assistant professor, Department of mathematics, Srimad Andavan Arts and Science College (Autonomous), T.V.Kovil,Trichy-5
E-mail : *rameshmaths@ymail.com, **sdeepamathematics@yahoo.com

**ABSTRACT :**

***In this paper the new Technical method to solve interval linear Assignment Problem have been discussed which gives optimal solution directly within few steps. In a ground reality the entries of the cost matrix is not always crisp. In many application this parameters are uncertain and these uncertain parameters are represented by interval. Interval analysis concept for solving interval linear assignment problems seem to be easiest as compare to methods of assignment problem. The method of solving interval linear assignment problem is appropriate for both minimization and maximization cases.***

**Keywords :** ***Assignment Problems, Hungarian method, and Interval Analysis,***

***MSc code: 90B80***

## 1. INTRODUCTION

The assignment problem is one of the fundamental combinatorial optimization problems in the branch of Operations Research in Mathematics. In such problem, it is required to perform all tasks by assigning exactly one agent to each task in such a way that the overall cost of the assignment is minimized, if the numbers of agents and tasks are equal. The assignment for all tasks is equal to the sum of the costs for each agent then the problem is called linear assignment problem. In practical field we are sometime faced with type of problem which consists of jobs to machines, drivers to trucks, men to offices etc. in which the assignees possess varying degree of efficiency, called as cost or effectiveness .The basic assumption of this type of problem is that one person can perform one job at a time. An assignment plan is optimal if it minimizes the total cost or maximizes the profit.

This type of linear assignment problems can be solved by the very well-known Hungarian method which was derived by the two mathematician D. König and E.Egerváry. Authors have proposed different methods to handle different types of assignment problems. In this paper a general interval linear assignment problem is taken into consideration with basis assumption that one person can perform one job at a time. In this study, the new method has been proposed to handle such type of problem. We solved one example problem using this proposed method. Corresponding results are computed and has been reported.

In this paper, we have developed new Technical method for solving an interval linear assignment problem to achieve the optimal solution. It has been found that the optimal solution obtained in this method is same as that of Hungarian method.

## 2. DEFINITION

### 2.1 ARITHMETIC OPERATIONS

The interval form of the parameters may be written as where is the left value [ $\underline{x}$] and is the right value [ $\overline{x}$] of the interval respectively. We define the centre is m = $\frac{\overline{x}+\underline{x}}{2}$ and w = $\overline{x}-\underline{x}$ is the width of the interval [ $\overline{x},\underline{x}$]

Let [ $\overline{x}, \underline{x}$] and [ $\overline{y}, \underline{y}$] be two elements then the following arithmetic are well known

(i) [ $\overline{x}, \underline{x}$] +[ $\overline{y}, \underline{y}$] = [$\overline{x} + \overline{y}$, $\underline{x} + \underline{y}$]

(ii)[ $\overline{x}, \underline{x}$] - [ $\overline{y}, \underline{y}$] = [ $\overline{x}$-$\underline{y}$ , $\underline{x}$-$\overline{y}$] Provide if [$\overline{y}$ , $\underline{y}$] ≠ [0, 0],

## 3. ALGORITHM FOR MINIMIZATION CASE IN INTERVAL ASSIGNMENT PROBLEM

Let A, B, C ...Z denote resources and I, II, III, IV... denote the activities. To solve the interval linear assignment problems

**Step 1:** Find out the mid values of each interval in the cost matrix. Consider row as a worker and column as a job

**Step 2:** Under column1 write the resource say A,B,C,.........Z. Next find minimum Unit cost for each row whichever minimum value is available in the respecting column, select it and write it in term of activities under column 2 continue this process for all the Z rows and write the term of I,II......

**Step 3:** If there is unique activity then assign that activity for the corresponding resource hence we achieved our optimal solution. If there is no unique activity for corresponding resource then the assignment can be made using following given steps.

**Step 4:** If any one resource has unique activity, then assign that activity for the corresponding resource. Next delete that row and its corresponding column for which resource has already been assigned.

**Step 5:** Again find the minimum unit cost for the remaining rows. Check if it satisfies step 4 otherwise, check which rows have only one same activity. Next find difference b/w minimum and next minimum unit cost for all those rows which have same activity. Assign that activity which has maximum difference. Delete those rows and corresponding columns for which those resource have been assigned

**Step 6:** Repeat steps 4 to 5 till all jobs are assigned uniquely to the corresponding activity.

## 4. BALANCED INTERVAL ASSIGNMENT PROBLEM

### 4.1 EXAMPLE

The centre needs four application programs to be developed. The head of the computer centre, after studying carefully the programs to be developed. Estimate the computer time in minutes required by the experts for the application programs.

| Programmers | Programs | | | |
|---|---|---|---|---|
| | I | II | III | IV |
| A | 5 | 7 | 11 | 6 |
| B | 8 | 5 | 9 | 6 |
| C | 4 | 7 | 10 | 7 |
| D | 10 | 4 | 8 | 3 |

Assign the programmers to the programs in such a way that the total computer time is minimum.

**Solution:**

| Programmers | Programs | | | |
|---|---|---|---|---|
| | I | II | III | IV |
| A | [4,6] | [6,8] | [10,12] | [5,7] |
| B | [7,9] | [4,6] | [8,10] | [5,7] |
| C | [3,5] | [6,8] | [9,11] | [6,8] |
| D | [9,11] | [3,5] | [7,9] | [2,4] |

| | | | |
|---|---|---|---|
| [4,6] | [6,8] | [10,12] | [5,7] |
| [7,9] | [4,6] | [8,10] | [5,7] |
| [3,5] | [6,8] | [9,11] | [6,8] |
| [9,11] | [3,5] | [7,9] | [2,4] |

| Resource | Activity |
|---|---|
| A | I |
| B | II |
| C | I |
| D | IV |

| | | |
|---|---|---|
| [4,6] | [6,8] | [5,7] |
| [7,9] | [4,6] | [5,7] |
| [9,11] | [3,5] | [2,4] |

| Resource | Activity |
|---|---|
| A | I |
| B | II |
| D | IV |

| | |
|---|---|
| [4,6] | [5,7] |
| [3,5] | [2,4] |

| Resource | Activity |
|---|---|
| B | II |
| D | IV |

**Result:**

We are applying the proposed interval Hungarian method and solve this problem. We get an minimum assignment cost is [19,27] and optimal assignment as A ,B,C,D machines are assign to I,II, III ,IV operators respectively This answer is to be same as that of Hungarian method. Hence we can say that the minimum time as 23 in both the methods.

## 5. ALGORITHM FOR MAXIMIZATION CASE IN INTERVAL ASSIGNMENT PROBLEM

Let A, B, C, ..........Z denote resources and I,II,III........... denote the activities.

**Step 1:** Find out the mid values of each interval in the cost matrix. Consider row as a worker (resource) and column as a job (activity)

**Step 2:** Find the maximum unit cost for each row which ever maximum unit cost is available. in the respecting column. Select it and write it in term of activities under column 2 Continue this process for all the Z rows and write in term of I, II.........

**Step 3:** If there is Unique activity then assigned that activity for the corresponding resource, hence we achieved our optimal solutions. If there is no unique activity for corresponding resources then the assignment can be made using following given steps.

**Step 4:** If any one resource has unique activity then assign that activity for the corresponding resource. Next delete that row and its corresponding column for which resource has already been assigned.

**Step 5:** Again, find the maximum unit cost for the remaining rows. Check if it satisfies step 4 Otherwise, check which rows have only one same activity. Find difference between maximum and next maximum Unit cost for all those rows which have same activity. Assign that activity which has maximum difference. Delete these rows and corresponding columns for which those resources have been assigned.

**Step 6:** Repeat steps 4 to 5 till all jobs are assigned uniquely to the corresponding activity.

### 5.1 EXAMPLE:

A marketing manager has five salesman and five districts. The marketing manager estimates that sales per month (in hundred rupees)for each district would be as follows.

| | | A | B | C | D | E |
|---|---|---|---|---|---|---|
| Sales man | 1 | 32 | 38 | 40 | 28 | 40 |
| | 2 | 40 | 27 | 33 | 30 | 37 |
| | 3 | 41 | 38 | 41 | 36 | 36 |
| | 4 | 22 | 33 | 40 | 35 | 39 |
| | 5 | 29 | 33 | 40 | 35 | 39 |

Find the assignment of salesman to districts that will result in maximum sales.

**Solution:**

| | | A | B | C | D | E |
|---|---|---|---|---|---|---|
| Sales man | 1 | [31,33] | [37,39] | [39,41] | [27,29] | [39,41] |
| | 2 | [39,41] | [26,28] | [32,34] | [29,31] | [36,38] |
| | 3 | [40,42] | [37,39] | [40,42] | [35,37] | [35,37] |
| | 4 | [21,23] | [32,34] | [39,41] | [34,36] | [38,40] |
| | 5 | [28,30] | [32,34] | [39,41] | [34,36] | [38,40] |

| | | | |
|---|---|---|---|
| [37,39] | [39,41] | [27,29] | [39,41] |
| [26,28] | [32,34] | [29,31] | [36,38] |
| [37,39] | [40,42] | [35,37] | [35,37] |
| [32,34] | [39,41] | [34,36] | [38,40] |

| Col $_1$ | Col $_2$ |
|---|---|
| 1 | C, E |
| 3 | E |
| 4 | C |
| 5 | C |

| | | |
|---|---|---|
| [37,39] | [39,41] | [27,29] |
| [37,39] | [40,42] | [35,37] |
| [32,34] | [39,41] | [34,36] |

| Col $_1$ | Col $_2$ |
|---|---|
| 1 | C |
| 4 | C |
| 5 | C |

| | |
|---|---|
| [37,39] | [27,29] |
| [37,39] | [35,37] |

| Col $_1$ | Col |
|---|---|
| 1 | B |
| 4 | B |

**Result:**

We are applying the proposed interval Hungarian method and solve this problem. We get a maximum assignment cost is [186,196] and optimal assignment as 1, 2,3,4,5 machines are assign to B, A, E, D, C operators respectively. This answer is to be same as that of Hungarian method. Hence we can say that the maximum time as 191 in both the methods.

## 6. UNBALANCED INTERVAL ASSIGNMENT PROBLEM

Find out the mid values of each interval in the cost matrix. Consider row as a worker (resource) and column as a job (activity).The number of columns and rows in the assignment matrix be equal. However when the given cost matrix is not a square matrix, the assignment problem is called an unbalanced problem. Once the unbalanced assignment problem is converted into balanced assignment problem then we can follow usual algorithm to solve the assignment problem. In such cases a dummy row(s) (or) column(s) are added in the matrix with zero as the cost elements to make it a square matrix,

### 6.1 EXAMPLE PROBLEMS:

The corporation has floated tenders and five contractors have sent in their bids. In order to expedite work, one road will be awarded to only one contractor.

| | | Cost of repairs | (Rs. Lakh) | | |
|---|---|---|---|---|---|
| Contactors | $C_1$ | 9 | 14 | 19 | 15 |
| | $C_2$ | 7 | 17 | 20 | 19 |
| | $C_3$ | 9 | 18 | 21 | 18 |
| | $C_4$ | 10 | 12 | 18 | 19 |
| | $C_5$ | 10 | 15 | 21 | 16 |

Find the best way of assigning the repair work to the contractors and the costs.

**Solution:**

The given cost matrix is not balanced. Hence we add one dummy column (Road R5) with a zero cost in that column

| | $R_1$ | $R_2$ | $R_3$ | $R_4$ | $R_5$ |
|---|---|---|---|---|---|
| $C_1$ | [8,10] | [13,15] | [18,20] | [14,16] | [0,0] |
| $C_2$ | [6,8] | [16,18] | [19,21] | [18,20] | [0,0] |
| $C_3$ | [8,10] | [17,19] | [20,22] | [17,19] | [0,0] |
| $C_4$ | [9,11] | [11,13] | [17,19] | [18,20] | [0,0] |
| $C_5$ | [9,11] | [14,16] | [20,22] | [15,17] | [0,0] |

| | | | |
|---|---|---|---|
| [13,15] | [18,20] | [14,16] | [0,0] |
| [17,19] | [20,22] | [17,19] | [0,0] |
| [11,13] | [17,19] | [18,20] | [0,0] |
| [14,16] | [20,22] | [15,17] | [0,0] |

| Col 1 | Col2 | diff |
|---|---|---|
| $C_1$ | $R_1$ | [5, 5] |
| $C_2$ | $R_1$ | [10, 10] |
| $C_3$ | $R_1$ | [9, 9] |
| $C_4$ | $R_1$ | [2, 2] |
| $C_5$ | $R_1$ | [5, 5] |

| | | |
|---|---|---|
| [18,20] | [14,16] | [0,0] |
| [20,22] | [17,19] | [0,0] |
| [20,22] | [15,17] | [0,0] |

| Col 1 | Col2 | diff |
|---|---|---|
| $C_1$ | $R_4$ | [4, 4] |
| $C_3$ | $R_4$ | [3, 3] |
| $C_5$ | $R_4$ | [5, 5] |

| | |
|---|---|
| [18,20] | [0,0] |
| [20,22] | [0,0] |

| Col 1 | Col2 | diff |
|---|---|---|
| $C_1$ | $R_3$ | [18, 20] |
| $C_3$ | $R_3$ | [20, 22] |

**Result:**

We have applied the proposed interval Hungarian method to solve problem. We get minimum assignment cost is [50,58] and optimal assignment $C_1$, $C_2$, $C_3$, $C_4$, $C_5$ road are assigned to $R_3$,$R_1$,$R_5$,$R_2$,$R_4$ operators respectively This answer is same as that of Hungarian method. i.e. the minimum time as 54 in both the methods.

**CONCLUSION:**

Hungarian method is used to obtain optimal solution for an assignment problem. In this paper a new Technical method of interval linear Assignment Problem is proposed where it is shown that this method also gives optimal solution. The optimal solution obtained using this method is same as that of optimal solution obtained by Hungarian method. However the technique for solving an assignment problem using our method is more simpler and easy as it takes few steps for the optimal solution.

**References:**

1. Kellerer, H. and Wirsching, G. (1998) “Bottleneck quadratic assignment problems”
2. R.E. Macho1 and M. Wien, “A hard assignment problem”, Operations Research 24 (1976) 190-192.
3. H.J.Zimmermann, Rudrajit Tapador, Solving the assignment, third Ed. kluwer Academic, Boston, 1996.
4. Li, Y. and Pardalos, P. M. (1992) “Generating Quadratic Assignment Test Problems with known Optimal Permutations, “Computational Optimization and Applications”, 1(2),163–184.
5. Gabow, H. N. and Tarjan, R. E. (1988) “Algorithms for two bottleneck assignment problems,” Journal of Algorithms, 9 (3), 411–417
6. Kuhn, H. W. (1955) “The Hungarian method for the assignment problem,” Naval Research Logistics Quarterly, 2 (1–2), 83–87.
7. Pardalos, P. M. and Pitsoulis, L. (Eds.) (2000) Nonlinear Assignment Problems: Algorithms and Applications, Kluwer Academic Publishers.
8. Hamdy A. Taha, Operations Research (Eighth Edition)(2008), Pearson, ISBN -9780131889231.
9. Cloud, Michael J.; Moore, Ramon E.; Kearfott, R. Baker, Introduction to Interval Analysis. Philadelphia: Society for Industrial and Applied Mathematics. ISBN 0-89871 669-1(2009).

Aryabhatta Journal of Mathematics & Informatics Vol. 6, No. 2, July-Dec, 2014 ISSN : 0975-7139
Journal Impact Factor (2013) : 0.489

# AVAILABILITY ANALYSIS OF TWO DIFFERENT UNITS SYSTEM WITH A STANDBY HAVING IMPERFECT SWITCH OVER DEVICE IN BANKING INDUSTRY

**Surender Kumar*, Dr. Pardeep Goel****
*Research Scholar, JJT University, Jhunjhunu (Rajasthan)
**Associate Prof. M. M. (PG) College, Fatehabad -125050 (Haryana)
E-mail : *surendergarg26004@gmail.com, **pardeepgoel1958@gmail.com

**ABSTRACT :**
***In this paper the availability analysis of a banking industry with two unit redundant system having imperfect switch using Regenerative point Graphical Technique (RPGT) is discussed. Initially two non identical units are operative and one unit if kept as standby. The standby put online with the help of an imperfect switch. The failure and repair rates are taken as exponential and system is discussed with the help of tables.***
**Keywords: -** ***Availability, Reliability, Primary Circuits, Secondary Circuits, Tertiary Circuits, Base-State, Regenerative Point Graphical Technique (RPGT), MTSF, Busy Period of the Server, Expected No. of Server's Visits, Cold Stand-by, Imperfect Switch.***

## 1. INTRODUCTION

In this paper we have discussed the availability analysis of a banking industry which has two units of different efficiency. There is a third unit also which can be used as a standby to any of the two main units. The main units are christened as A and B and the standby unit as C. The system fails if any two units fail. The failure rates for three units are taken as $\lambda_1$, $\lambda_2$ and $\lambda_3$ respectively and the repair rates are taken as $w_1$,$w_2$, $w_3$ respectively. The failure and repair rates are taken exponentially distributed. Priority in repair is as A > B > C i. e. A is repaired first, then B and then C. single repairman facility is available. The availability and behavior analysis has been discussed assuming that the system is under steady conditions. The system is in good state if any two of the units are working and fails if more than two units fail. The standby unit C is switched in with the help of a switch which is imperfect i.e. it may fail even when it is not in use due to damage or aging or wearing out or any other reason. As per the hypothesis of the work four parameters i.e. MTSF. Availability, No. Of Server's Visits and the Busy Period of the sever are evaluated. Particular cases are discussed taking all the repair rates and failure rates equal followed by tables.

**2. Assumptions** : - The following assumptions and notations / symbols are used: -

1. The system consists of three non-identical units. Initially, two units are operative and other unit is kept in cold standby.
2. Switching over is imperfect.
3. There is a single repairman for repair of failed units, and switch over device.
4. The failure rates and repair rates are exponentially distributed and are independent and are different for different operative units i.e. main unit, standby and switch over device.
5. Repairs are perfect i.e. the repair facility never does any damage to the units.
6. Repaired unit is as good as new.
7. The order of priority of repair is: unit 'A', unit 'B', standby unit.
8. The system is discussed under steady state conditions.
9. Steady unit 'C' is steady to both nits 'A' & 'B'.

**3. Transition Diagram of the System**: - Following the above assumptions and notations, the transition diagram of the system is as shown in Figure 1.

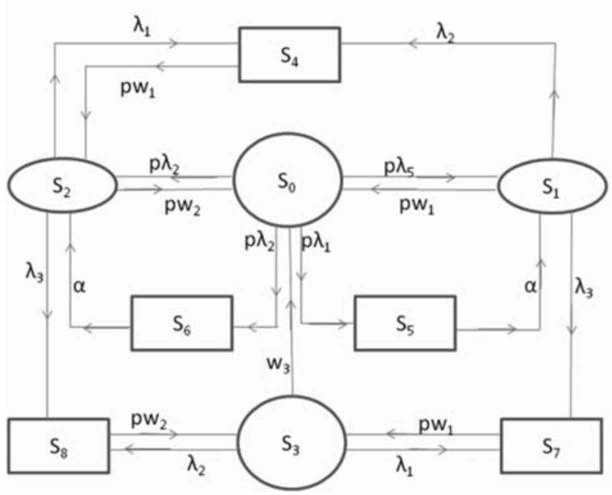



**Figure 1**

The system can be in any of the following states with respect to the above symbol.

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| $S_0$ | = | AB(C)S | $S_1$ | = | aBCS | $S_2$ | = | AbCS |
| $S_3$ | = | ABcS | $S_4$ | = | abCS | $S_5$ | = | aB(C)s |
| $S_6$ | = | Ab(C)s | $S_7$ | = | aBcS | $S_8$ | = | AbcS |

## 3. Analysis of System:

They key parameters (under steady state conditions) are evaluated by determining a base-state and applying RPGT. From the analysis of the diagram 1 it is concluded that the base state for the transition diagram 1 of the model is ‘0’ state.

**3.1 Transition Probabilities: -** The mean time to system failure and all the key parameters of the system (under steady state conditions) are evaluated by using Regenerative Point Graphical Technique (RPGT) and using ‘0’ as the base state of the system as under: -

**Table 1**

| $q_{i,j}(t)$ | $P_{ij} = q^*_{i,j}(0)$ |
|---|---|
| $q_{0,1} = p\lambda_1 \bar{e}^{(p\lambda_1+\bar{p}\lambda_1+p\lambda_2+\bar{p}\lambda_2)t}$ <br> $= p\lambda_1 \bar{e}^{(\lambda_1+\lambda_2)t}$ <br> $q_{0,2} = p\lambda_2 \bar{e}^{(\lambda_1+\lambda_2)t}$ <br> $q_{0,5} = p\lambda_1 \bar{e}^{(\lambda_1+\lambda_2)t}$ <br> $q_{0,6} = p\lambda_2 \bar{e}^{(\lambda_1+\lambda_2)t}$ | $p_{0,1} = p\lambda_1/(\lambda_1+\lambda_2)$ <br><br> $p_{0,2} = p\lambda_2/(\lambda_1+\lambda_2)$ <br> $p_{0,5} = \bar{p}\lambda_1/(\lambda_1+\lambda_2)$ <br> $p_{0,6} = \bar{p}\lambda_2/(\underline{\lambda_1}+\lambda_2)$ |
| $q_{1,0} = pw_1 \bar{e}^{(pw_1+\lambda_2+\lambda_3)t}$ <br> $q_{1,4} = \lambda_2 \bar{e}^{(pw_1+\lambda_2+\lambda_3)t}$ <br> $q_{1,7} = \lambda_3 \bar{e}^{(pw_1+\lambda_2+\lambda_3)t}$ | $p_{1,0} = pw_1/(pw_1+\lambda_2+\lambda_3)$ <br> $p_{1,4} = \lambda_2/(pw_1+\lambda_2+\lambda_3)$ <br> $p_{1,7} = \lambda_3/(pw_1+\lambda_2+\lambda_3)$ |
| $q_{2,0} = pw_2 \bar{e}^{(pw_2+\lambda_1+\lambda_3)t}$ <br> $q_{2,4} = \lambda_1 \bar{e}^{(pw_2+\lambda_1+\lambda_3)t}$ <br> $q_{2,8} = \lambda_3 \bar{e}^{(pw_2+\lambda_1+\lambda_3)t}$ | $p_{2,0} = pw_2/(pw_2+\lambda_1+\lambda_3)$ <br> $p_{2,4} = \lambda_1/(pw_2+\lambda_1+\lambda_3)$ <br> $p_{2,8} = \lambda_3/(pw_2+\lambda_1+\lambda_3)$ |
| $q_{3,0} = = w_3 \bar{e}^{(w_3+\lambda_1+\lambda_2)t}$ <br> $q_{3,7} = \lambda 1 \bar{e}^{(w_3+\lambda_1+\lambda_2)t}$ <br> $q_{3,8} = \lambda 2 \bar{e}^{(w_3+\lambda_1+\lambda_2)t}$ | $p_{3,0} = w_3/(\lambda_1+\lambda_2+w_3)$ <br> $p_{3,7} = \lambda_1/(\lambda_1+\lambda_2+w_3)$ <br> $p_{3,8} = \lambda_2/(\lambda_1+\lambda_2+w_3)$ |
| $q_{4,2} = pw_1 \bar{e}^{(pw_1)t}$ | $p_{4,2} = 1$ |
| $q_{5,1} = \alpha e^{-\alpha t}$ | $p_{5,1} = 1$ |
| $q_{6,2} = \alpha e^{-\alpha t}$ | $p_{6,2} = 1$ |
| $q_{7,3} = pw_1 \bar{e}^{(pw_1)t}$ | $p_{7,3} = 1$ |
| $q_{8,3} = pw_2 \bar{e}^{(pw_2)t}$ | $P_{8,3} = 1$ |

**Table 2 : Mean Sojourn Times**

| $R_i(t)$ | $\mu_i = R_i^*(0)$ |
|---|---|
| $R_0^{(t)} = \bar{e}^{(\lambda_1+\lambda_2)t}$ | $\mu_0 = 1/(\lambda_1+\lambda_2)$ |
| $R_1^{(t)} = \bar{e}^{(pw_1+\lambda_2+\lambda_3)t}$ | $\mu_1 = 1/(pw_1+\lambda_2+\lambda_3)$ |
| $R_2^{(t)} = \bar{e}^{(pw_2+\lambda_1+\lambda_3)t}$ | $\mu_2 = 1/(pw_2+\lambda_1+\lambda_3)$ |
| $R_3^{(t)} = \bar{e}^{(w_3+\lambda_1+\lambda_2)t}$ | $\mu_3 = 1/(\lambda_1+\lambda_2+w_3)$ |
| $R_4^{(t)} = \bar{e}^{pw_1 t}$ | $\mu_4 = 1/pw_1$ |
| $R_5^{(t)} = e^{-\alpha t}$ | $\mu_5 = 1/\alpha$ |
| $R_6^{(t)} = e^{-\alpha t}$ | $\mu_6 = 1/\alpha$ |
| $R_7^{(t)} = \bar{e}^{pw_1 t}$ | $\mu_7 = 1/pw_1$ |
| $R_8 = \bar{e}^{pw_2 t}$ | $\mu_8 = 1/pw_2$ |

The steady state equations of the system can be depited as :

$V_{0,0} = (pw_1+\lambda_1+\lambda_3)\lambda_1+\lambda_2(pw_1+\lambda_2+\lambda_3)/(\lambda_1+\lambda_2)(pw_1+\lambda_2+\lambda_3) = 1$

$V_{0,1} = (0,1)+(0,5,1)$

$V_{0,1} = p_{0,1}+p_{0,5}p_{5,1}$

$= p\lambda_1/(\lambda_1+\lambda_2)+ \lambda_1/(\lambda_1+\lambda_2) = \lambda_1/(\lambda_1+\lambda_2)$

$V_{0,2} = \lambda_2(pw_1+\lambda_1+\lambda_2+\lambda_3)(pw_2+\lambda_1+\lambda_3)/ (pw_2+\lambda_3)(\lambda_1+\lambda_2)(pw_1+\lambda_2+\lambda_3)$

$V_{0,3} = \lambda_3(\lambda_1+\lambda_2+w_3)/w_3(\lambda_1+\lambda_2)(pw_1+\lambda_2+\lambda_3)[p\, \lambda_1 w_2+\lambda_1\lambda_3+p\, \lambda_2 w_1+\lambda_1\lambda_2+\lambda_2+\lambda_2\lambda_3/(pw_2+\lambda_3)]$

$V_{0,4} = \lambda_1\lambda_2(pw_1+pw_2+\lambda_1+\lambda_2+2\lambda_3)/(\lambda_1+\lambda_2)(pw_2+\lambda_3)(pw_1+\lambda_2+\lambda_3)$

$V_{0,5} = \lambda_1/(\lambda_1+\lambda_2)$

$V_{0,6} = \lambda_2/(\lambda_1+\lambda_2)$

$V_{0,7} = \lambda_1\lambda_3/p\lambda_1 w_2+\lambda_1\lambda_3+pw_2w_3+\lambda_3w_3+p\lambda_2w_1+\lambda_1\lambda_2+\lambda_2+\lambda_2\lambda_3)/w_3(\lambda_1+\lambda_2)(pw_1+\lambda_2+\lambda_3)$
$(pw_2+\lambda_3)$

$V_{0,8} = 1/[1-\lambda_2/( \lambda_1+\lambda_2+w_3)-\lambda_1/(\lambda_1+\lambda_2+w_3)[p\lambda_1(\lambda_1+\lambda_2).\lambda_3/(pw_1+\lambda_2+\lambda_3).\lambda_2/(\lambda_1+\lambda_2+w_3)$
$+ \lambda_1/(\lambda_1+\lambda_2).\lambda_3/(pw_1+\lambda_2+\lambda_3).\lambda_2/(\lambda_1+\lambda_2+w_3)+1-\lambda_2(\lambda_1+\lambda_2+w_3)/[1-\lambda_1/(pw_1+\lambda_2+\lambda_3).$
$1-\lambda_1/(\lambda_1+\lambda_2+w_3)-\lambda_2/(\lambda_1+\lambda_2+w_3)[p\lambda_1(\lambda_1+\lambda_2).\lambda_2/(pw_1+\lambda_2+\lambda_3).\lambda_3/(pw_2+\lambda_1+\lambda_3)$
$+ \lambda_1/(\lambda_1+\lambda_2).\lambda_2/(pw_1+\lambda_2+\lambda_3).\lambda_3/(pw_2+\lambda_1+\lambda_3)+p\lambda_2(\lambda_1+\lambda_2).\lambda_3/(pw_2+\lambda_1+\lambda_3)+ \lambda_2/$
$(\lambda_1+\lambda_2).\ \lambda_3/(pw_2+\lambda_1+\lambda_3)$

**(a) MTSF($T_0$):** The regenerative un-failed states to which the system can transit(from initial state '0'), before entering any failed state are: 0,1,2. For '$\xi$' = '0', MTSF is given by

$$\textbf{MTFS}(\mathbf{T_0}) = \left[\sum_{i,s_r}\left\{\frac{\left\{pr\left(\xi \xrightarrow{s_r(s_{ff})} i\right)\right\}\mu_i}{\prod_{m_1\neq\xi}\{1-V_{\overline{m_1,m_1}}\}}\right\}\right] \div \left[1-\sum_{s_r}\left\{\frac{\left\{pr\left(\xi \xrightarrow{s_r(s_{ff})} \xi\right)\right\}}{\prod_{m_2\neq\xi}(1-V_{\overline{m_2,m_2}})}\right\}\right]$$

$= [(0,0)\mu_0+(0,1)\mu_1+(0,2)\mu_2\ ]/[1-(0,1,0)+(0,2,0)]$

$= [(pw_1+\lambda_2+\lambda_3)\ (pw_2+\lambda_1+\lambda_3)+p\lambda_1(\lambda_1+\lambda_3 + pw_2) + p\ \lambda_2\ (pw_1+\lambda_2+\lambda_3)]$
$/[(\lambda_1+\lambda_2)\ (pw_1+\lambda_2+\lambda_3\ )\ (pw_2+\lambda_1+\lambda_3) - p^2 w_1\ \lambda_1(pw_2+\lambda_1+\lambda_3) - p^2 w_2\ \lambda_2$
$(pw_1+\lambda_2+\lambda_3\ )\ ]$

**(b) Availability of the system ($A_0$)**: The regenerative states at which the system is available are j = 0,2,3,5 and the regenerative states are 'i' = 0 to 3. for '$\xi$' = '0', the total fraction of time for which the system is available is given by

$$A_0 = \left[\sum j,s_r\left\{\frac{\{pr(\xi^{sr}\to j)\}fj,\mu j}{\Pi_{m_1\neq\xi}\{1-V_{m_1,m_1}\}}\right\}\right] \div \left[\sum i,S_r\left\{\frac{\{pr(\xi^{s_r}\to i)\}\mu_i^1}{\Pi_{m_2\neq\xi}\{1-V_{m2,m2}\}}\right\}\right]$$

$$A_0 = \left[\sum_j V_{\xi,j}\ ,f_j,\mu_j\right] \div \left[\sum_i V_{\xi,i}\ ,\mu_i^1\right]$$

$= V_{0,0}f_0\mu_0+V_{0,1}f_1\mu_1+V_{0,2}f_2\mu_2+V_{0,3}f_3\mu_3/\ V_{0,0}\mu_0^1+V_{0,1}\mu_1^1+V_{0,2}\mu_2^1+\ldots\ldots\ldots+V_{0,8}\mu_8^1$

Taking $f_0 = f_1 = f_2 = f_3 = 1$ and $\mu_i^1 = \mu_1$

$A_0 = [\{1/(\lambda_1+\lambda_2)\} + \{\lambda_1/(\lambda_1+\lambda_2)\}\{1/(pw_1+\lambda_2+\lambda_3)\} +\{\lambda_3(\lambda_1+\lambda_2+w_3)/w_3(\lambda_1+\lambda_2)$
$(pw_1+\lambda_2+\lambda_3)[p\,\lambda_1w_2+\lambda_1\lambda_3+p\,\lambda_2w_1+\lambda_1\lambda_2+\lambda_2+\lambda_2\lambda_3/(pw_2+\lambda_3)]\}\{1/(\lambda_1+\lambda_2+w_3)\}$
$\{\lambda_2(pw_1+\lambda_1+\lambda_2+\lambda_3)(pw_2+\lambda_1+\lambda_3)/(pw_2+\lambda_3)(\lambda_1+\lambda_2)(pw_1+\lambda_2+\lambda_3)\}\{1/(pw_2+\lambda_1+\lambda_3)\}]\,/$
$[\{1/(\lambda_1+\lambda_2)\}+\{\lambda_1/(\lambda_1+\lambda_2)\}\{1/(pw_1+\lambda_2+\lambda_3)\} +\{\lambda_3(\lambda_1+\lambda_2+w_3)/w_3(\lambda_1+\lambda_2)(pw_1+\lambda_2+\lambda_3)$
$[p\,\lambda_1w_2+\lambda_1\lambda_3+p\,\lambda_2w_1+\lambda_1\lambda_2+\lambda_2+\lambda_2\lambda_3/(pw_2+\lambda_3)]\}\{1/(\lambda_1+\lambda_2+w_3)\}\{\lambda_2(pw_1+\lambda_1+\lambda_2+\lambda_3)$
$(pw_2+\lambda_1+\lambda_3)/(pw_2+\lambda_3)(\lambda_1+\lambda_2)(pw_1+\lambda_2+\lambda_3)\}\{1/(pw_2+\lambda_1+\lambda_3)\}$
$+\{\lambda_1\lambda_2(pw_1+pw_2+\lambda_1+\lambda_2+2\lambda_3)/(\lambda_1+\lambda_2)(pw_2+\lambda_3)(pw_1+\lambda_2+\lambda_3)\}\{1/pw_1\}$
$+\{\lambda_1/(\lambda_1+\lambda_2)\}\{1/\alpha\}+\{\lambda_2/(\lambda_1+\lambda_2)\}\{1/\alpha\}+\{\lambda_1\lambda_3/p\lambda_1w_2+\lambda_1\lambda_3+pw_2w_3+\lambda_3w_3+$
$p\lambda_2w_1+\lambda_1\lambda_2+\lambda_2+\lambda_2\lambda_3)/w_3(\lambda_1+\lambda_2)(pw_1+\lambda_2+\lambda_3)(pw_2+\lambda_3)\}\{1/pw_1\}$
$+\{1/[1-\lambda_2/(\lambda_1+\lambda_2+w_3)-\lambda_1/(\lambda_1+\lambda_2+w_3)[p\lambda_1(\lambda_1+\lambda_2).\lambda_3/(pw_1+\lambda_2+\lambda_3).\lambda_2/(\lambda_1+\lambda_2+w_3)$
$+\lambda_1/(\lambda_1+\lambda_2).\lambda_3/(pw_1+\lambda_2+\lambda_3).\lambda_2/(\lambda_1+\lambda_2+w_3)+1-\lambda_2(\lambda_1+\lambda_2+w_3)/[1-\lambda_1/(pw_1+\lambda_2+\lambda_3).$
$1-\lambda_1/(\lambda_1+\lambda_2+w_3)-\lambda_2/(\lambda_1+\lambda_2+w_3)[p\lambda_1(\lambda_1+\lambda_2).\lambda_2/(pw_1+\lambda_2+\lambda_3).\lambda_3/(pw_2+\lambda_1+\lambda_3)+\lambda_1/$
$(\lambda_1+\lambda_2).\lambda_2/(pw_1+\lambda_2+\lambda_3).\lambda_3/(pw_2+\lambda_1+\lambda_3)+\lambda_2(\lambda_1+\lambda_2).\lambda_3/(pw_2+\lambda_1+\lambda_3)+\lambda_2/(\lambda_1+\lambda_2).$
$\lambda_3/(pw_2+\lambda_1+\lambda_3)\}\{1/pw_2\}]$

**(c) Busy period of server ($B_0$): -** The regenerative states where the server is busy while doing repairs are j = 1,2,3,4,5, 6 7 8; the regenerative states are i = 0,1,2,3,4,5,6,7, 8 for '$\xi$' = '0', the total fraction of time the server remains busy is

$$B_0 = \left[\sum j, s_r\left\{\frac{\{pr(\xi^{sr}\to j)\},\eta j}{\Pi_{m_1\neq\xi}\{1-V_{m_1,m_1}\}}\right\}\right] \div \left[\sum i, S_r\left\{\frac{\{pr(\xi^{S_r}\to i)\},\mu_i^1}{\Pi_{m2\neq\xi}\{1-V_{m2,m2}\}}\right\}\right]$$

$$B_0 = \left[\sum_j V_{\xi,j}, \eta_j\right] \div \left[\sum_i V_{\xi,i}, \mu_i^1\right]$$

$= (V_{0,1}\eta_1+V_{0,2}\eta_2+V_{0,3}\eta_3+V_{0,4}\eta_4+V_{0,5}\eta_5+V_{0,6}\eta_6+V_{0,7}\eta_7+V_{0,8}\eta_8)/(V_{0,0}\mu_0{}^1+V_{0,1}\mu_1{}^1+$
$V_{0,2}\mu_2{}^1+V_{0,3}\mu_3{}^1+V_{0,4}\mu_4{}^1+V_{0,5}\mu_5{}^1+V_{0,6}\mu_6{}^1+V_{0,7}\mu_7{}^1+V_{0,8}\mu_8{}^1)$
$= [\{\lambda_1/(\lambda_1+\lambda_2)\}\{1/(pw_1+\lambda_2+\lambda_3)\} +\{\lambda_3(\lambda_1+\lambda_2+w_3)/w_3(\lambda_1+\lambda_2)(pw_1+\lambda_2+\lambda_3)$
$[p\,\lambda_1w_2+\lambda_1\lambda_3+p\,\lambda_2w_1+\lambda_1\lambda_2+\lambda_2+\lambda_2\lambda_3/(pw_2+\lambda_3)]\}\{1/(\lambda_1+\lambda_2+w_3)\}\{\lambda_2(pw_1+\lambda_1+\lambda_2+\lambda_3)$
$(pw_2+\lambda_1+\lambda_3)/(pw_2+\lambda_3)(\lambda_1+\lambda_2)(pw_1+\lambda_2+\lambda_3)\}\{1/(pw_2+\lambda_1+\lambda_3)\}$
$+\{\lambda_1\lambda_2(pw_1+pw_2+\lambda_1+\lambda_2+2\lambda_3)/(\lambda_1+\lambda_2)(pw_2+\lambda_3)(pw_1+\lambda_2+\lambda_3)\}\{1/pw_1\} +$
$\{\lambda_1/(\lambda_1+\lambda_2)\}\{1/\alpha\} + \{\lambda_2/(\lambda_1+\lambda_2)\}\{1/\alpha\}+\{\lambda_1\lambda_3/p\lambda_1w_2+\lambda_1\lambda_3+pw_2w_3+\lambda_3w_3$
$+p\lambda_2w_1+\lambda_1\lambda_2+\lambda_2+\lambda_2\lambda_3)/w_3(\lambda_1+\lambda_2)(pw_1+\lambda_2+\lambda_3)(pw_2+\lambda_3)\}\{1/pw_1\}$
$+\{1/[1-\lambda_2/(\lambda_1+\lambda_2+w_3)-\lambda_1/(\lambda_1+\lambda_2+w_3)[p\lambda_1(\lambda_1+\lambda_2).\lambda_3/(pw_1+\lambda_2+\lambda_3).\lambda_2/(\lambda_1+\lambda_2+w_3)$
$+\lambda_1/(\lambda_1+\lambda_2).\lambda_3/(pw_1+\lambda_2+\lambda_3).\lambda_2/(\lambda_1+\lambda_2+w_3)+1-\lambda_2(\lambda_1+\lambda_2+w_3)/[1-\lambda_1/(pw_1+\lambda_2+\lambda_3).$
$1-\lambda_1/(\lambda_1+\lambda_2+w_3)-\lambda_2/(\lambda_1+\lambda_2+w_3)[p\lambda_1(\lambda_1+\lambda_2).\lambda_2/(pw_1+\lambda_2+\lambda_3).\lambda_3/(pw_2+\lambda_1+\lambda_3)$
$+\lambda_1/(\lambda_1+\lambda_2).\lambda_2/(pw_1+\lambda_2+\lambda_3).\lambda_3/(pw_2+\lambda_1+\lambda_3)+p\lambda_2(\lambda_1+\lambda_2).\lambda_3/(pw_2+\lambda_1+\lambda_3)$
$+\lambda_2/(\lambda_1+\lambda_2).\lambda_3/(pw_2+\lambda_1+\lambda_3)\}\{1/pw_2\}]\,/\,[\{1/(\lambda_1+\lambda_2)\} + \{\lambda_1/(\lambda_1+\lambda_2)\}$
$\{1/(pw_1+\lambda_2+\lambda_3)\} +\{\lambda_3(\lambda_1+\lambda_2+w_3)/w_3(\lambda_1+\lambda_2)(pw_1+\lambda_2+\lambda_3)[p\,\lambda_1w_2+\lambda_1\lambda_3+p$
$\lambda_2w_1+\lambda_1\lambda_2+\lambda_2+\lambda_2\lambda_3/(pw_2+\lambda_3)]\}\{1/(\lambda_1+\lambda_2+w_3)\}\{\lambda_2(pw_1+\lambda_1+\lambda_2+\lambda_3)$
$(pw_2+\lambda_1+\lambda_3)/(pw_2+\lambda_3)(\lambda_1+\lambda_2)(pw_1+\lambda_2+\lambda_3)\}\{1/(pw_2+\lambda_1+\lambda_3)\} + \{\lambda_1\lambda_2$
$(pw_1+pw_2+\lambda_1+\lambda_2+2\lambda_3)/(\lambda_1+\lambda_2)(pw_2+\lambda_3)(pw_1+\lambda_2+\lambda_3)\}\{1/pw_1\}+\{\lambda_1/(\lambda_1+\lambda_2)\}$
$\{1/\alpha\} + \{\lambda_2/(\lambda_1+\lambda_2)\}\{1/\alpha\} + \{\lambda_1\lambda_3/p\lambda_1w_2+\lambda_1\lambda_3+pw_2w_3+\lambda_3w_3+p\lambda_2w_1+\lambda_1\lambda_2$
$+\lambda_2+\lambda_2\lambda_3)/w_3(\lambda_1+\lambda_2)(pw_1+\lambda_2+\lambda_3)(pw_2+\lambda_3)\}\{1/pw_1\}+\{1/[1-\lambda_2/(\lambda_1+\lambda_2+w_3)$
$-\lambda_1/(\lambda_1+\lambda_2+w_3)[p\lambda_1(\lambda_1+\lambda_2).\lambda_3/(pw_1+\lambda_2+\lambda_3).\lambda_2/(\lambda_1+\lambda_2+w_3)+\lambda_1/(\lambda_1+\lambda_2).\lambda_3/$
$(pw_1+\lambda_2+\lambda_3).\lambda_2/(\lambda_1+\lambda_2+w_3)+1-\lambda_2(\lambda_1+\lambda_2+w_3)/[1-\lambda_1/(pw_1+\lambda_2+\lambda_3).1-\lambda_1/(\lambda_1+\lambda_2+w_3)$
$-\lambda_2/(\lambda_1+\lambda_2+w_3)[p\lambda_1(\lambda_1+\lambda_2).\lambda_2/(pw_1+\lambda_2+\lambda_3).\lambda_3/(pw_2+\lambda_1+\lambda_3)+\lambda_1/(\lambda_1+\lambda_2).\lambda_2/$

$(pw_1+\lambda_2+\lambda_3).\lambda_3/(pw_2+\lambda_1+\lambda_3)+p\lambda_2(\lambda_1+\lambda_2).\lambda_3/(pw_2+\lambda_1+\lambda_3)+ \lambda_2/(\lambda_1+\lambda_2).$
$\lambda_3/(pw_2+\lambda_1+\lambda_3)\}\ \{1/pw_2\}]$

**(d) Expected number of server's visits ($V_0$)**: - The regenerative states where the server visits afresh for repair of the system are j = 1,2,5,6 the regenerative states are i = 0 to 8 for $\xi$ = '0', the expected number of server's visits per unit time are given by

$$V_0=\left[\sum_{j,s_r}\left\{\frac{\{pr(\xi\xrightarrow{s_r}j)\}}{\prod_{k_1\neq\xi}\{1-V_{k_1,k_1}\}}\right\}\right] \div \left[\sum_{i,s_r}\left\{\frac{\{pr(\xi\xrightarrow{s_r}i)\}\mu_i^1}{\prod_{k_2\neq\xi}(1-V_{k_2,k_2})}\right\}\right]$$

$$V_0=\left[\sum_j V_{\xi,j}\right] \div \left[\sum_i V_{\xi,i}\,.\,\mu_i^1\right]$$

$= V_{0,1}+V_{0,2}+V_{0,5}+V_{0,6}/V_{0,0}\,\mu_0^1+V_{0,1}\,\mu_1^1+V_{0,2}\mu_2^1+V_{0,3}\mu_3^1+V_{0,4}\mu_4^1+V_{0,5}\mu_5^1+$
$V_{0,6}\mu_6^1+V_{0,7}\mu_7^1+V_{0,8}\mu_8^1$ Taking $\mu_i^1 = \mu_1$

$= [\lambda_1/(\lambda_1+\lambda_2)+\lambda_2(pw_1+\lambda_1+\lambda_2+w_3)(pw_2+\lambda_1+\lambda_2)+ \lambda_1/(\lambda_1+\lambda_2)+ \lambda_2/(\lambda_1+\lambda_2)]/ [\{1/(\lambda_1+\lambda_2)\}$
$+\{ \lambda_1/(\lambda_1+\lambda_2)\}\ \{1/(pw_1+\lambda_2+\lambda_3)\}\ +\{ \lambda_3(\lambda_1+\lambda_2+w_3)/w_3(\lambda_1+\lambda_2)(pw_1+\lambda_2+\lambda_3)$
$[p\ \lambda_1w_2+\lambda_1\lambda_3+p\ \lambda_2w_1+\lambda_1\lambda_2+\lambda_2+\lambda_2\lambda_3/(pw_2+\lambda_3)]\}\{1/(\lambda_1+\lambda_2+w_3)\}\{\lambda_2(pw_1+\lambda_1+\lambda_2+\lambda_3)$
$(pw_2+\lambda_1+\lambda_3)/ (pw_2+\lambda_3)(\lambda_1+\lambda_2)(pw_1+\lambda_2+\lambda_3)\}\{1/(pw_2+\lambda_1+\lambda_3)\}+ \{\lambda_1\lambda_2$
$(pw_1+pw_2+\lambda_1+\lambda_2+2\lambda_3) /(\lambda_1+\lambda_2)\ (pw_2+\lambda_3)(pw_1+\lambda_2+\lambda_3)\}\ \{1/pw_1\}+\{ \lambda_1/(\lambda_1+\lambda_2)\}$
$\{1/\alpha\}+\{ \lambda_2/(\lambda_1+\lambda_2)\}\{1/\alpha\}+\{\lambda_1\lambda_3/ p\lambda_1w_2+\lambda_1\lambda_3+pw_2w_3+\lambda_3w_3+p\lambda_2w_1+\lambda_1\lambda_2+\lambda_2+\lambda_2\lambda_3)$
$/w_3(\lambda_1+\lambda_2)(pw_1+\lambda_2+\lambda_3)(pw_2+\lambda_3)\}\{1/pw_1\}+\{1/[1-\lambda_2/( \lambda_1+\lambda_2+w_3)-\lambda_1/(\lambda_1+\lambda_2+w_3)$
$[p\lambda_1(\lambda_1+\lambda_2)\ .\lambda_3/(pw_1+\lambda_2+\lambda_3).\lambda_2/(\lambda_1+\lambda_2+w_3) + \lambda_1/ (\lambda_1+\lambda_2).\lambda_3/(pw_1+\lambda_2+\lambda_3).\lambda_2/$
$(\lambda_1+\lambda_2+w_3)+1-\lambda_2(\lambda_1+\lambda_2+w_3)/[1-\lambda_1/(pw_1+\lambda_2+\lambda_3).1-\lambda_1/ (\lambda_1+\lambda_2+w_3)-$
$\lambda_2/(\lambda_1+\lambda_2+w_3)[p\lambda_1(\lambda_1+\lambda_2).\lambda_2/(pw_1+\lambda_2+\lambda_3).\lambda_3/(pw_2+\lambda_1+\lambda_3) + \lambda_1/$
$(\lambda_1+\lambda_2).\lambda_2/(pw_1+\lambda_2+\lambda_3).\lambda_3/(pw_2+\lambda_1+\lambda_3)+p\lambda_2(\lambda_1+\lambda_2).\lambda_3/(pw_2+\lambda_1+\lambda_3)$
$+ \lambda_2/(\lambda_1+\lambda_2).\ \lambda_3/(pw_2+\lambda_1+\lambda_3)\}\ \{1/pw_2\}]$

## 4. Particular cases :

If we put $\lambda_1 = \lambda_2 = \lambda_3 = \lambda$ and $w_1 = w_2 = w_3 = w$ , $\alpha = 1$, then we get,

**MTFS($T_0$)** $= [1/2\lambda+(p/2)(1/pw+2\lambda)+(p/2)(1/pw+2\lambda)]/$
$[1-(p/2)(pw/pw+2\lambda)(p/2)(pw/pw+2\lambda)]$
$= [1/2\lambda+(p/pw+2\lambda)]/[1-(p^2w/pw+2\lambda)]$
$= [(pw+2\lambda+4\lambda p)/2\lambda(pw+2\lambda)]/[(pw+2\lambda-p^2w)/(pw+2\lambda)]$
$= [(pw+2\lambda+4\lambda p)/2\lambda/[(pw+2\lambda-p^2w)]$

**Availability ( $A_0$) =** $pw[w(pw+2\lambda)(pw+3\lambda)+[2\lambda^2(pw+2\lambda)]/ [pw^2(pw+\lambda)(pw+2\lambda)+$
$[2p\lambda w^2(pw+2\lambda)]+ [2\lambda^2wp(pw+2\lambda)]+[2\lambda^2w(pw+2\lambda)]+[2\lambda w^2p$
$(pw+\lambda)(pw+2\lambda)]+[2\lambda^2(pw^2+2\lambda w+2p\lambda w+4\lambda^2)]$

**Busy period of server ($B_0$) =** $[pw^2(p+1)+pw^2(p+3)+2\lambda w(p+1)(p+2)+2p\ \lambda w^2(p+1)$
$(p+2)+p\lambda w(2wp+4p\lambda+4w+5\lambda)]/pw^2(p+3)\ (p+2)+2\lambda w(p+1)$
$(p+2)+2p\ \lambda w^2(p+1)(p+2)+p\lambda w(2wp+4p\lambda+4w+5\lambda)]$

**Expected number of server's visits ($V_0$)**$= [p\lambda w^2(p+2)(2p+4+ (p+1)]/pw^2(p+1)(p+3)$
$+pw^2(p+3)+2p\lambda w(p+2)+2p\ \lambda w^2(p+1)(p+2+$
$P\lambda w(2pw+4p\lambda+4w+5\lambda)]$

**MTFS ($T_0$) :** for different values of failure rates and repair rates.

MTFS ($T_0$) = $[(pw+2\lambda+4\lambda p)/2\lambda/[(pw+2\lambda-p^2w)]$

The MTFS of the system is calculated for different values of failure rates by taking failure rates $\lambda$ = 0.06, .08, .009 and .010 and for different values of repair rate by taking w = 0.80,0.85, 0.90, 0.95 , and 1.0. The data so obtained is given in table 3 .

**Table 3 : MTSF of the System**

| Λ | w = 0.80 | w = 0.85 | w = 0.90 | w = 0.95 | w = 1.00 |
|---|---|---|---|---|---|
| 0.06 | 76.38 | 79.86 | 83.33 | 86.80 | 90.27 |
| 0.08 | 46.87 | 48.82 | 50.78 | 52.73 | 54.68 |
| 0.09 | 38.58 | 40.12 | 41.66 | 43.20 | 44.75 |
| 0.10 | 32.50 | 33.75 | 35.00 | 36.25 | 37.50 |

The table 3 shows that behavior of MTFS Vs repair rate of the unit of the system for different values of failure rate λ. From the above table we can conclude that MTFS is increasing as the failure rates decrease and repair rates increase which should be so.

**Availability Vs repair rate (w) and failure rate (λ) :** The availability of the system is calculated for different values of of the failure rate λ by taking λ = 0.06, 0.08, 0.09 and 0.010 for different values of repair rate w by taking w = 0.80, 0.85, 0.90, 0.95, and 1.0 . The data so obtained is given in the table 3.

**Table 4 : Availability of the System**

| Λ | w = 0.80 | w = 0.85 | w = 0.90 | w = 0.95 | w = 1.00 |
|---|---|---|---|---|---|
| 0.06 | 0.9808 | 0.9828 | 0.9845 | 0.9860 | 0.9873 |
| 0.08 | 0.9677 | 0.9710 | 0.9738 | 0.9762 | 0.9784 |
| 0.09 | 0.9603 | 0.9643 | 0.9677 | 0.9707 | 0.9732 |
| 0.10 | 0.9549 | 0.9571 | 0.9611 | 0.9646 | 0.9677 |

**Conclusion:-** from the above discussion it is clear that mean time to system failure increases with increase in repair rates and decreases with increase in failure rate. Similarly availability increases with increase in repair rates which is in agreement with the expectations.

**References :-**


1. Goel, P. & Singh, J. (1997) : Availability Analysis of A Thermal Power Plant Having Two Imperfect Switches, Proc. (Reviewed) of 2nd Annual Conference Of ISITA.
2. Gupta, P., Singh, J.& Singh, I.P. (2004) : Availability Analysis of Soap Cakes Production System- A Case Study, Proc. National Conference On Emerging Trends In Manufacturing Systems, SLIET, Longowal (Punjab) Pp 283-295.
3. Goel, P. & Singh, J. (1998) : Availability Analysis of Butter Manufacturing System In A Diary Plant, Proc. Of International Conference on Operational Research For A Better Tomorrow, pp 109-116.
4. Gupta, P., Lal, A.K., Sharma, R.K.& Singh, J. (2005) : Behavioral Study of The Cement Manufacturing Plant- A Numerical Approach, Journal of Mathematics And Systems Science(JMASS), Vol.1,No.1, pp 50-70.
5. Gupta, V.K. & Singh, J. (2008) : Behavior And Profit Analysis of A Soap Industry, Journal of Mathematics And Systems Science(JMASS), Vol.4, No.1, pp 41-62.
6. Sumit Kumar & Rajeev Khanduja (2013), "Availability Analysis of sheet formation system of Utensils Manufacturing Plant." Aryabhatta J. of Maths & Info. Vol. 5 (2) pp 217-226.
7. Gupta V. K. (2011). "Analysis of a single unit system using a base state." Aryabhatta J. of Maths & Info. Vol. 3 (1) pp 59-66.

# A STOCHASTIC VASICEK MODEL FOR THE GLUCOSE –INDUCED GLUCAGON – LIKE PEPTIDE 1 SECRETION IS DEFICIENT IN PATIENTS WITH NON- ALCOHOLIC FATTY LIVER DISEASE.

**Dr. P. Senthil Kumar* & D. Sarguna Sundari****

*Assistant Professor, Department of Mathematics, Rajah Serfoji Government College, Thanjavur, India
** Assistant Professor, Department of Mathematics, Bharath College of Science & Management, Thanjavur, India
Email: senthilscas@yahoo.com, laerieljudson@gmail.com

**ABSTRACT :**

***The incretins glucagon like peptide -1 (GLP-1) is one of the gastrointestinal peptide hormones regulating postprandial insulin release secreted from pancreatic - cells. GLP-1 agonism is a treatment strategy in Type 2 diabetes and is estimated in Non- alcoholic fatty liver disease NAFLD. Since GLP1 can stimulate insulin Secretion from the pancreas and alleviate type 2 diabetes. In this paper we determine the secretion of incretins after oral glucose administration in Non- diabetic NAFLD patients.***

***In risk management and in the rating practice it is desirable to grasp the essential statistical features of a time series representing a risk factor to begin a detailed technical analysis of the product or the entity under scrutiny. This paper introduces a number of risk management applications. The model developed here suffice for the first step in Quantitative analysis.***

***The broad qualitatives features here is mean reversion under the mean reverting processes the Vasicek model is one of the earliest stochastic models of the short term interest rate.***

$$S_t = b + aS_s + c \in (t)$$

**Keywords :** ***Stochastic processes, mean reversion vasicek model, glucagon like peptide-1, Non alcoholic fatty liver disease.***

***2000 Mathematics subject Classification: 60H10, 60H05***

## INTRODUCTION

Non alcoholic fatty liver disease (NAFLD) has become the most frequent chronic liver disease. The incretin glucagon like peptide-1 is gastrointestinal peptide hormone regulating postprandial insulin released from pancreatic β- cells. GLP-1 is released postprandially form endocrine L- cells into the splanchnic and portal circulation. It lowers plasma glucose and improves insulin sensitivity by increasing postprandial insulin release, decreasing glucagon secretion and delaying gastric emptying [9,7]. The important criteria of the study was to determined incretin secretion in patients with NAFLD.

**Notations:**

$S_t$ - Stochastic process

$W_t$ - Standard wiener process

$\beta$ - long term mean reversion level

$\alpha$ - Long term mean reversion speed

$\sigma$ - Long term mean reversion volatility.

$u$ - Drift parameter

## THE GEOMETRIC BROWNIAN MOTION

The Stochastic process $S_t$ is said to follow a Geometric Brownian motion describes the Random behavior of the asset price level $S_t$ over time. The GBM satisfies the stochastic differential equation,

$$dS_t = uS_t\ dt + \sigma S_t\ dW_t \quad \ldots\ldots\ldots\ldots\ldots\ldots\ (1)$$

Here $W_t$ is a standard Brownian motion, a Wiener process that is characterized by independent identically distributed (IID) increments that are normally (or Gaussian) distributed with Zero mean and a standard deviation equal to the square root of the time step. Independence in the increments implies that the model is a Markov Process, which is a particular type of process for which only the current asset price is relevant for computing probabilities of events involving future asset prices. In other terms, to compute probabilities involving future asset prices, knowledge of the whole past does not add anything to knowledge of the present[1][6].

The d terms indicate that the equation is given in its continuous time stochastic process is one where the value of the price can change at any point in time. The property of independent identically distributed increments is very important and will be exploited in the calibration as well as in the simulation of the random behavior of asset prices. By techniques of the separation of variables, then the equation (1) becomes.

$$\frac{dS_t}{S_t} = udt + \sigma dW_t \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(2)$$

The take the integration of both sides

$$\int \frac{dS_t}{S_t} = \int (udt + \sigma dW_t)dt \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(3)$$

Since $\frac{dS_t}{S_t}$ relates to derivative of ln $(S_t)$

By involving Ito calculus

$$\ln\left(\frac{dS_t}{S_t}\right) = \left(u - \frac{1}{2}\sigma^2\right)t + \sigma W_t \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(4)$$

Taking the exponential of both side and plugging the initial condition $S_0$,
then the analytical solution of this geometric Brownian motion[1] is given by,

$$S_t = S_0 \exp\left[\left(u - \frac{1}{2}\sigma^2\right)t + \sigma W_t)\right] \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ (5)$$

This equation shows that the asset price in the GBM follows a log-normal distribution, while the logarithmic returns log $(S_{t+\Delta t}/S_t)$ are normally distributed.

**THE MEAN REVERTING BEHAVIOUR**

One can define mean reversion as the property to always revert to a certain constant or time varying level with limited variance around it. This property is true for an autoregressive process if the absolute value of the auto regression coefficient is less than one ($|\alpha|<1$). To be precise, the formulation of the first order autoregressive process is:

$$X_{t+1} = u + \alpha x_t + \sigma \in_{t+1} \Rightarrow \Delta x_{t+1}(1-\alpha)\left(\frac{u}{1-\alpha} - x_t\right) + \sigma \in_{t+1} \quad \ldots\ldots(6)$$

All the mean reverting behaviour in the processes that are introduced in this section is due to an autoregressive process feature in the discretised version of the relevant SDE [4],[6].

In general, before thinking about fitting a specific mean reverting process to available data, it is important to check the validity of the mean reversion assumption. A simple way to do this is test for stationarity. Since for the autoregressive process $|\alpha| <1$ is also a necessary and sufficient condition for stationarity, testing for mean reversion is equivalent to testing for stationarity. Note that in the special case where $\alpha$ =1, the process behaves like a pure random walk with constant drift, $\Delta x_{t+1} = u + \sigma \in_{t+1}$

**MEAN REVERSION : THE VASICEK MODEL**

Mean Reversion is included in many finance models for the interest rate one of the earliest models is the Vasicek Model, which describes the evolution of an unspecified short-term interest rate in a SDE, [3],[8].

$$ds_t = \alpha(\beta - s_t)dt + \sigma dW_t \quad \ldots\ldots (7)$$

with $\alpha$ , $\beta$ and $\sigma$ are non – negative constants, $s_t$ is the current level of interest rate and $dW_t$ a standard Wiener process. Under the risk neutral measure used for valuation and pricing one may assume a similar parametrisation for the same same process but with one more parameter in the drift modeling the market price of risk.

This model assumes a mean- reverting stochastic behaviour of interest rates. The Vasicek model applies to the evolution of the short-term interest rate. The Vasicek model has a major shortcoming that is the non null probability of negative rates. This is an unreaslistic property for the modeling of positive entities like interest rates or credit spreads. The explicit solution to the SDE between any two instants s and t, with $0 \le s < t$, can be easily obtained form the solution to the Ornstein – Uhlenbeck SDE, namely;

$$s_t = \beta(1 - e^{-\alpha(t-s)}) + s_s e^{-\alpha(t-s)} + \sigma e^{-\alpha t} \int_s^t e^{\alpha u} dW_u \quad \ldots\ldots (8)$$

The discrete time version of this equation, on a time grid $0 = t_0, t_1, t_2$ ……… With (assume constant for simplicity) time step $\Delta t = t_i - t_{i-1}$, $(\delta = t\text{-}s)$ is

$$s_t = b + a s_s + c \in (t) \quad \ldots\ldots (9)$$

Where the coefficients are:

$$a = e^{-\alpha\delta} \quad \ldots\ldots (10)$$

$$b = \beta(1 - e^{-\alpha\delta}) \quad \ldots\ldots (11)$$

And $\in(t)$ is a Gaussian white noise ($\in \sim N(0,1)$). The Volatility of the innovations can be deduced by the Ito isometry [5][6]

$$c = \sigma\sqrt{\frac{1 - e^{-2\alpha\delta}}{2\alpha}} \quad \ldots\ldots (12)$$

Whereas if one used the Euler Scheme to discretise the equation this would simply be $\sigma\sqrt{\Delta t}$ which is the same ( first order in $\Delta$ t) as the exact one above.

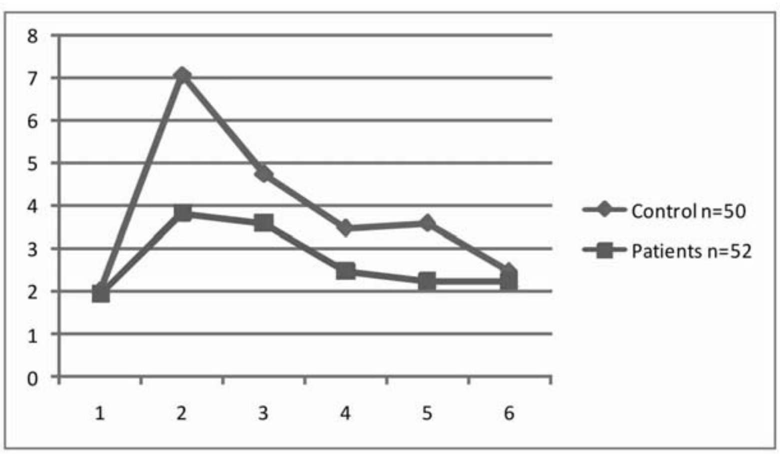



Figure 1.

### Example

Here 52 non- diabetic patients with biopsy proven NAFLD including patients with simple steatosis (n=16) or NASH (n=36) and 50 healthy controls. Standardized oral glucose tolerance test was performed in all subjects after an overnight fast according to a standardized protocol using 75g of glucose in 300ml tap water. Blood samples were drawn before and [9] sequentially at 15,30,60,90 and 120 minutes after glucose administration. Plasma samples were stored at – $70^0$C. The GLP-1 levels are measured sequentially for 120 minutes after glucose administration. Glucose induced GLP-1 secretion was significantly decreased in patients compared to controls is in fig 2.

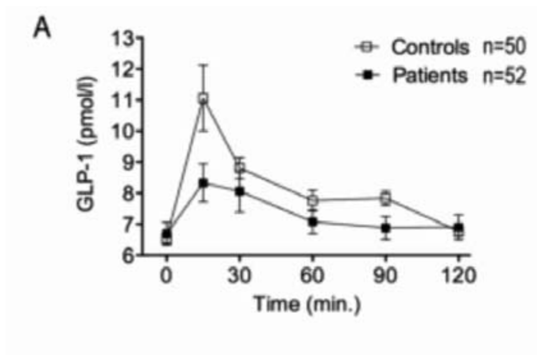



Figure 2

### CONCLUSION :

The Mathematical model also stresses the same cumulative effects of patients $V_s$ controls which are beautifully fitted with Vasicek model as shown in figure 1 and 2. The Results of these analyses indicates that the Glucagon like peptide-1 secretion was significantly decreased in patients compared to controls after oral glucose. The results of the Mathematical and medical reports are beautifully coincide.

### REFERENCES

1. G.E. Uhlenbeck 2 L.S. Ornstein , on the theory of Brownian motion, phys, Rev 36 (1930) 823-841.
2. S. Lakshmai & Durga Devi (2014) “A multi variate model for combined propranolol/TSST paradigm.” Aryabhatta J. of Math & Info. Vol. 6 (1) pp 103-108.
3. Vasicek, O.A. (1977). An equilibrium characterization of the term structure. *Journal of F*inancial Economics 5, 177-188,27.
4. Bollerslev, T(1986), Generalized autoregressive conditional heteroskedasticity. *Journal of Econometrics 31, 307-327, 13.*
5. Brigo, D and F. Mercurio (2006). *Interest Rate Models;Theory and Practice-With Smile, Inflation and Credit.* New York,NY:Springer Verlag. 29, 32,33
6. D. Brigo, A. Dalessandro, M. Neugebauer, F. Triki, 15 November 2007, A stochastic process toolkit for Management.
7. Holst JJ(2007) The physiology of glucagons-like peptide 1, Physiol Rev 87: 1409-1439, doi:10,1152/physrev, 00034, 2006.
8. Calibrating the Ornstein – Uhlenbeck(Vasicek) model by Thijs Van den Berg / May 28, 2011.
9. Christine Bernmeier, Anne C.Meyer Gerspach, Lea S. Blaser,Lia Jeker, Robert E.Steinert, Markus H. Heim, Christoph Beglinger, Glucose-induced Glucagon –like peptide 1 secretion is deficient in patients with Non-Alcoholic Fatty Liver Disease, (January 2014), Vol 9, issue 1 e87488.

Aryabhatta Journal of Mathematics & Informatics Vol. 6, No. 2, July-Dec, 2014 ISSN : 0975-7139
Journal Impact Factor (2013) : 0.489

# COST ANALYSIS OF A QUEUE SYSTEM WITH IMPATIENT CUSTOMERS

**T.P. Singh*, Arti Tyagi****
*Professor, Deptt. of Mathematics, Yamuna Instt. of Engg. & Tech., Gadholi-Yamuna Nagar
**Assistant Professor, M.M. University Sadopur, Ambala, Haryana

**ABSTRACT :**
***The paper deals the cost analysis of M/M/1/N queuing system with impatient customers. We assume that the arriving customers balk with some probability and reneging follow negative exponential distribution. The Markov process techniques have been applied to develop the steady state probability. Matrix form solution is given along with some performance measures. A cost model has been developed to find the optimal service rate. To demonstrate the use of the model, different parametric values have been specified and lucid illustration has been presented with the help of table and graphs.***
**Key words:** ***Balking, Reneging, Cost analysis, Exponential distribution, Steady state probability.***

## INTRODUCTION

For any Queuing system, Cost Analysis constitutes a very important aspect of its investigation. A study of queue system seems to be adequate unless comprises of economic analysis. In practical situation, it is nothing but a cost factor which enables to survive a queuing system. Various Researchers discussed the Equilibrium level queue decision model (cost model). Under the analysis of the cost model, they have discussed the two conflicting costs offering the service i.e. the cost of providing the service with the cost of delay in offering the services (waiting time cost) and computed the expected total cost per unit time for the system.

Many researchers find insignificant place for impatient customers with random service. But in modern society & in many real life problems the impatient customers play a meaningful role in a service system and can't be ignored. Such situations involving impatient telephone switch board customers, hospital emergency room which handles critical patient, the inventory system with storage of valuable goods etc.. In this paper, we discussed the performance analysis & the cost model of M/M/1/ N queue system taking the human behavior balking and reneging in account.

Haight (1957, 1959) first of all considered the effect of balking and reneging phenomenon in queuing problem. Ancker & Gafarin (1963) and Abou & Hariri (1992) studied the balking and reneging effect on some queuing problems. Singh T.P (1985) made an attempt to incorporate the reneging concept (reluctant a customer remains in line after joining & waiting) in serial queue network. Man Singh & Umed Singh (1994) studied the steady state behavior for impatient customers. Ashok & Taneja (1983) studied the cost analysis of Multi channel Queue system wherein both the arrival & service intensities are subject to alterations. Neetu Gupta (2009) etal. made an effort of the balking and reneging effect on the performance of a queue system under certain constraints. Recently, Singh T.P & Arti etal. (2014) discussed transient as well as steady state behavior of a serial queue system with reneging.

This work is further, an extended work of Singh T.P. etal. (2014) and it combines the work done by Haight & Neetu Gupta in the sense that both concepts balking and reneging have been included and in addition the cost analysis for the system has been explored with impatient customers through numerically example as well as graphically to demonstrate how the various parameters of the model influence the behavior of the system.

Activating and Deactivating the server may involve power charges equipment or man power charges. The cost can be contributed as a fixed cost which we are not going to consider in our model. When the server is running or busy, fuel and other costs may be charged called running cost or operating cost but these cost do not represent the entire picture of an operation. Consider the case of an air craft repair. The air planes are not productive during they stay in the repair depot and this loss time represents a cost to the owner i.e. we may consider a penalty for delaying the customer in the system, the penalty may be termed as a waiting cost. If the customer balks or renege from the system it is again a loss.

**MODEL DESCRIPTION:**

Model for impatient customers i.e. for balking & reneging stage can be depicted as follow

**ASSUMPTIONS:**

1. Customers arrive at the system one by one in a Poisson stream with mean arrival rate $\lambda$. On arrival a customer either decides to join the queue with probability $q_n$ or balk with probability $1-q_n$ when n customers are ahead (n=1,2,3,4………N-1) where N is maximum number of customers in the system. i.e.

$0\leq q_{n-1} \leq q_n <1 \qquad\qquad 1 \leq n \leq N-1$

$q_n = 0 \qquad\qquad n\geq N$

2. After joining queue each customer has to wait a certain time period T for service to start. If the service is not started, he feels irritated and gets impatient and reneges from queue without being served. The time T is a random variable follow exponential distribution and its probability density function is given by, $d(t) = \alpha e^{-\alpha t}$, $t \geq 0$, $\alpha\geq 0$

Here, $\alpha$ is the rate of waiting time T. as the arrival and departure of impatient customers without service are independent, the function of customer's average reneging rate is given by

$$\begin{cases} r(n) = (n-i)\alpha, & i \leq n \leq N \\ \quad = 0, & n > N \end{cases} \qquad i= 0,1,2,3\ldots\ldots$$

3. Queue discipline is FIFO, once service starts, it is without interruption i.e. it precedes till its completion.
4. We assume service times to be distributed as per exponential distribution whose density function are given by $S(t) = \mu e^{-\mu t}$, $t \geq 0$, $\mu > 0$ Where $\mu$ is service rate

**Steady state differential difference equations:**

Define, $P_n$= probability that there are n customers in the system,

$q_n$ = probability that on arrival a customer decides to join the queue or Balk with probability $1-q_n$ when n customers are ahead.

On Appling the Markov process techniques and elementary probability reasoning the following set of steady state equations has been observed,

$\mu P(1) = \lambda P(0)$ for n=0

$\lambda q_{n-1}P(n-1) + (\mu+n\alpha)P(n+1) = [\lambda q_n+\mu+(n-1)\alpha)]P(n)$ for n=1,2,3, ….., N-1.

$\lambda q_{N-1}P(N-1) = [\mu+(N-1)\alpha)]P(N)$, for n = N

Now, we discuss the cost analysis of our system For N=4

The following equations are obtained:

$\mu P(1) = \lambda P(0)$ for n=0 (1)

$\lambda P(0) + (\mu+\alpha)P(2) = [(\lambda q_1+\mu)]P(1)$ for n = 1 (2)

$\lambda q_1P(1) + (\mu+2\alpha)P(3) = [(\lambda q_2+\mu+\alpha)]P(2)$ for n = 2 (3)

$\lambda q_2P(2) + (\mu+3\alpha)P(4) = [\lambda q_3+\mu+2\alpha)]P(3)$ for n = 3 (4)

$\lambda q_3P(3) = [\mu+3\alpha)]P(4)$, for n = 4 (5)

**Solution methodology:**

Writing the above equations in matrix form:

$$\begin{bmatrix} -\lambda & \mu & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \lambda & -(\lambda q_1+\mu) & (\mu+\alpha) & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \lambda q_1 & -(\lambda q_2+\mu+\alpha) & (\mu+2\alpha) & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \lambda q_2 & (\lambda q_3+\mu+2\alpha) & (\mu+3\alpha) \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \lambda q_3 & (\mu+3\alpha) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

By solving as usual method, we get,

$P(0) = k$

$P(1) = \frac{\lambda}{\mu}k$

$P(2) = \lambda^2 \frac{q_1 k}{\mu(\mu+\alpha)}$

$P(3)= \lambda^3 \frac{q_1 q_2 k}{\mu(\mu+\alpha)(\mu+2\alpha)}$

$P(4) = \lambda^4 \frac{q_1 q_2 q_3 k}{\mu(\mu+\alpha)(\mu+2\alpha)(\mu+3\alpha)}$

Applying initial conditions

$\sum_{i=1}^{4} P_i = 1$

i.e. $P(0) + P(1) + P(2) + P(3) + P(4) = 1$

Putting the values we get,

$K=1+\frac{\lambda}{\mu} + \lambda^2 \frac{q_1}{\mu(\mu+\alpha)} + \lambda^3 \frac{q_1 q_2}{\mu(\mu+\alpha)(\mu+2\alpha)} + \lambda^4 \frac{q_1 q_2 q_3}{\mu(\mu+\alpha)(\mu+2\alpha)(\mu+3\alpha)}$

Which is the value of P(0)

**Cost Analysis and Performance measures:**

Expected number of customers in queue i.e. waiting customer

$E(N_q) = \sum_{n=1}^{N}(n-1)P(n)$

$E(N) = \sum_{n=1}^{N} nP(n)$

Balking (B.R.) = $\sum_{n=1}^{N} \lambda(1-q_n)P(n)$

Reneging (R.R.) = $\sum_{n=1}^{N}(n-1)\alpha P(n)$

L.R. = B.R. + R.R.

Where L.R. is cost incurred due to customers loss.

μ= control variable,

Object is to control the service rate to minimize system's total average cost per unit.

$C_1$ = cost per unit time when server is busy.

$C_2$ = cost per unit time when a customer joins in the queue and waits for service.

$C_3$ = cost per unit time when a customer balks or reneges

$F(\mu^*)$= expected functional cost of the system per unit time

Total expected functional cost of the system per unit time

$TF(\mu^*) = C_1P(B) + C_2E(N_q) + C_3L.R.$

P(B) = busy probability of server.

**NUMERICAL ILLUSTRATION**:

Consider maximum number of customer in system N =4,

The probability $q_n$=1/n+1 & cost elements $C_1$=16, $C_2$ =10, $C_3$= 20

**Table-1 for α =.1**

| λ → | .4 | .5 | .6 | .7 | .8 |
|---|---|---|---|---|---|
| $E(N_q)$ | 0.06114 | 0.091609 | 0.125640 | 0.16526 | 0.207158 |
| E(N) | 0.3860 | 0.47839 | 0.56904 | 0.65800 | 0.7452126 |
| L.R. | 0.07542 | 0.114223 | 0.158101 | 0.211769 | 0.27639 |
| $TF(\mu^*)$ | 8.09076 | 10.6553 | 13.3549 | 16.2949 | 19.460916 |
| $\mu^*$ | 0.93458 | 0.88577 | 0.8418 | 0.78834 | 0.72361 |

**Table -2 for λ= .5**

| α → | .1 | .2 | .3 | .4 | .5 |
|---|---|---|---|---|---|
| $E(N_q)$ | 0.091609 | 0.0813 | 0.073405 | 0.0667 | 0.0661 |
| E(N) | 0.47839 | 0.46308 | 0.45102 | 0.4412 | 0.43297 |
| L.R. | 0.114223 | 0.118914 | 0.1228 | 0.1260 | 0.131157 |
| $TF(\mu^*)$ | 11.20055 | 11.19128 | 11.19005 | 11.1817 | 11.28 |
| $\mu^*$ | .885777 | 0.8810 | 0.8772 | 0.874 | 0.8688 |

**Analysis of the Table:**

1. We select the fix rate of waiting time α =.1 & change value of arrival rate of customers **λ**, result can be summarized in table-1.
2. We take fix value of **λ**= .5 & change the value of α, the numerical results are shown in table-2.

Table -1 shows that the total expected cost $F(\mu^*)$ increase with increase of **λ**.The expected number of customers in system as well as expected number of waiting customers and average rate of customer loss all increase with increase of **λ**.

Table -2 shows that the total expected cost increases with the increase of α. The expected number of waiting customers in system and average number of customer loss as well as the optimal service rate all decreases with increase in the value of α.

## CONCLUSION

We have discussed the simple queue system with impatient customers and developed the steady state probability equations. The Matrix form of the solution has been derived. We formulate a cost model to determine the optimal service rate and total expected cost of the system per unit time. Although this function is too complicated to derive the explicit expression for optimal service rate, even than we have made an attempt to evaluate numerically the performance measures & the optimal service rate for the system. We have also presented how the various parameters of the model influence the behavior of the system.

**REFERENCES:**


1. Haight F.A (1957) "Queuing with Balking" Biometric, vol.44, pp 360- 369.
2. Haight F.A (1959) "Queuing with Reneging" Biometric, vol.52, pp 186- 197.
3. Ancker Jr & Gaforiam. Ar. (1963) "some queuing problems with balking & Reneging" operation Research. Vol. 11 pp 921-937.
4. Robert E. (1979) "Reneging phenomenon of single channel queue" Mathematics of the Operations-Research. Vol. 27 pp 162- 178.
5. Rajeev Kumar & R. K. Taneja ( 1983 ) " Cost Analysis Of Heterogeneous Multichannel Queueing System." PAMS vol.XVIII No.1-2 pp35-48.
6. Singh T.P. & Ayu Kumar (1985) "On Two queues In series with reneging" JISSOR Vol. 6 (1-4)
7. Aboue E.I. & Harri S.M. (1997) "M/M/C/N Queue with Balking and Reneging" Computer & operational Research 19 pp.713-716.
8. Singh Man & Singh Umeed (1994) "Network of Serial & non serial queuing processes with Impatient Customers" JISSOR 15 (1-4).
9. Nitu Gupta etal. (2009) "Performance Measure of M/M/1 Queue with Balking and Reneging" Pure and applied Mathematical Science. Vol. LXX, pp 59-65.
10. Singh T. P. & Kusum (2011), "Trapezoidal fuzzy Network queue model with Blocking" Aryabhatta J. of Maths & Info. Vol. 3 (1) 185-192
11. Arti Tyagi, T.P. Singh & M.S. Saroa (2014) "Stochastic analysis of a queue Network with Impatient Customers." International Journal of Math. Sci. & Engg. Appl. Vol. 8 (1) pp 337-347.

Aryabhatta Journal of Mathematics & Informatics Vol. 6, No. 2, July-Dec, 2014 ISSN : 0975-7139
Journal Impact Factor (2013) : 0.489

# COMPARATIVE ANALYSIS OF TWO STOCHASTIC MODELS FOR A BASE TRANSCEIVER SYSTEM CONSIDERING HARDWARE AND SOFTWARE INTERACTION FAILURES

**Sunny Kapoor*, Rajeev Kumar****
* &** Deptt. of Mathematics, M.D. University, Rohtak, Haryana

**ABSTRACT :**
***This paper deals with comparative analysis of two stochastic models for a base transceiver system given in [5] and [6] to judge which and when one model is better than the other in terms of different measures of system performance and costs. A base transceiver system may fail or may not work satisfactorily on occurrence of some minor or major faults in its hardware and software components, catastrophic failures or calls congestion. Further in hardware and software components of the system there can be two types of hardware and software interaction failures viz. hardware based softwar failures and software based hardware failures. In the first model given in [5], only software based hardware failures are taken whereas in the model given in [6], hardware based software failures are taken apart from considering occurrence of some minor or major faults in hardware/ software components, catastrophic failures and calls congestion. In these models it is assumed that the minor fault leads to partial failure or degradation state whereas a major fault and catastrophic failure leads to complete failure of the system. On failures, the available technicians first inspect whether fault is due to hardware, software or hardware and software interaction failures then recovery of the infected component is done. On the basis of computed measures of system performance, comparison of the models with respect to their mean times to failures, expected uptimes/degradation times and profits is made using graphs for some particular case and conclusions are drawn.***

**Index Terms :** *Base Transceiver System (BTS), catastrophic failure, congestion of calls, hardware and software interaction failures reliability, mean up/degradation times, profit, Markov process, regenerative point technique.*

## 1. INTRODUCTION

In the field of stochastic modeling, a large number of researchers developed and analysed models for several one/two-unit systems considering various different aspects. For instance, Arora [1] discussed enhancement in the system reliability by assigning priority repair discipline. Goel et. al. [2] carried out reliability analysis of a system with preventive maintenance and two types of repair. Recently Anita Taneja [10] discussed a two unit cold stand by system with inspection and replacement. Gopalan et. al. [3] did cost analysis of a system subject to on-line preventive maintenance and repair. Singh and Mishra [11] evaluated profit for a two-unit standby system with two operating modes. Kumar et. al. [9] analysed a two-unit redundant system with degradation and replacement policy. Kumar and Mor [8] have given a probabilstic analysis of a sophisticated system with some warranted components considering two types of service facility and delay in warranty claims. Taneja et. al. [12] studied a single unit PLC considering the four types of failure. Kumar and Bhatia [4] presented economic and reliability analysis of a centrifuge system with rest period, neglected faults and stoppage on minor faults, etc.Beside these a few researchers like Teng et. al. [13], Tumer and Smidts [14], Welke et. al. [15], Kumar and Kumar [7] etc. discussed various types of hardware/software failures with different recovery policies while analyzing systems having both hardware and software components.

Recently, Kumar and Kapoor [5] analysed a model on a base transceiver system considering hardware, software,

software based hardware, catastrophic failures and congestion of calls.Thereafter, for the system Kumar and Kapoor [6] discussed a model considering hardware based software failures in place of software based hardware failures.

It is pertinent to mentioned that the model developed for a system can not be good for all its operational conditions/situations. However stakeholders are interested to judge which and when one model is better for the system with respect to different measures of system performance.

Keeping this fact in view, in the present paper a comparative analysis has been done between the models for a base transceiver system given in [5] and [6] to investigate which and when a model is better for the system, respectively, in terms of their reliability, expected uptime/degradation times and profit. A base transceiver system may fail or may not work satisfactorily on occurrence of some minor or major faults in its hardware and software components, catastrophic failures or calls congestion. Further in hardware and software components of the system there can be two types of hardware and software interaction failures viz. hardware based software failures and software based hardware failures. In the first model given in [5], say model-I , only software based hardware failures are taken whereas in the model given in [6], say model-II, hardware based software failures are taken apart from considering occurrence of some minor or major faults in hardware/ software components, catastrophic failures and calls congestion. In these models it is assumed that the minor fault leads to partial failure or degradation state whereas a major fault and catastrophic failure leads to complete failure of the system. On failures, the available technicians first inspect whether fault is due to hardware, software or hardware and software interaction failures then recovery of the infected component is done. On the basis of computed measures of system performance given in [5] and [6], comparison of the models with respect to their mean times to failures, expected uptimes/degradation times and profits is made using graphs for some particular case and conclusions are drawn.

**STATES OF THE SYSTEM**

| | |
|---|---|
| $O$ | Operative state |
| $O_c$ | Congestion state |
| $O_i$ / $F_i$ | Degradation state/Failed state under inspection |
| $O_{h_r}$ / $O_{s_r}$ / $O_{hs_r}$ | Degradation state due to hardware/software/hardware based software fault under repair |
| $O_{sh_{rp}}$ / $F_{sh_{rp}}$ | Degradation state/failed state due to software based minor/major hardware fault under replacement |
| $F_{h_r}$ / $F_{s_r}$ / $F_{hs_r}$ | Failed state due to hardware/software/hardware based software fault under repair |

**NOTATIONS**

| | |
|---|---|
| $\lambda_1 / \lambda_2$ | Rate of occurrence of major/minor faults |
| $\lambda_3 / \lambda_4$ | Rate of occurrence of software based major/minor hardware faults |
| $\lambda_5 / \lambda_6$ | Rate of occurrence of hardware based major/minor software faults |
| $\eta$ | Rate of congestion of calls |
| $\delta_1$ | Rate with which system restored automatically after congestion |
| $a_1 / a_2$ | Probability that the major/minor hardware fault occurs in the system |
| $b_1 / b_2$ | Probability that the major/minor software fault occurs in the system |
| $c_1 / c_2$ | Probability that the hardware based major/minor software fault occurs in the system |
| $d_1$ | Probability that the catastrophic fault occurs in the system |
| $q_{ij}(t) / Q_{ij}(t)$ | P.d.f./C.d.f. of first passage time from state 'i' to state 'j' |
| $g_{h_1}(t) / g_{h_2}(t)$ | P.d.f. of repair time of major/minor hardware fault |

| | |
|---|---|
| $G_{h_1}(t)/G_{h_2}(t)$ | C.d.f. of repair time of major/minor hardware fault |
| $g_{s_1}(t)/g_{s_2}(t)$ | P.d.f. of repair time of major/minor software fault |
| $G_{s_1}(t)/G_{s_2}(t)$ | C.d.f. of repair time of major/minor software fault |
| $g_{h_3}(t)/g_{h_4}(t)$ | P.d.f. of repair time of hardware based major/minor software fault |
| $G_{h_3}(t)/G_{h_4}(t)$ | C.d.f. of repair time of hardware based major/minor software fault |
| $g_{c_f}(t)/G_{c_f}(t)$ | P.d.f./C.d.f of repair time of catastrophic fault |
| $h_{h_3}(t)/h_{h_4}(t)$ | P.d.f. of replacement time of software based major/minor hardware fault |
| $H_{h_3}(t)/H_{h_4}(t)$ | C.d.f. of replacement time of software based major/minor hardware fault |
| $i_1(t)/i_2(t)$ | P.d.f. of inspection time of major/minor fault |
| $I_1(t)/I_2(t)$ | C.d.f. of inspection time of major/minor fault |

## 2. STATE TRANSITION DIAGRAM

For the model-I, a diagram presenting various states of transition of the system is shown in fig. 1 whereas for the model-II, various states are given in fig. 2. In both the diagrams the epochs of entry in to various states are regenerative points and hence all the states are regenerative states.

**Model-I**

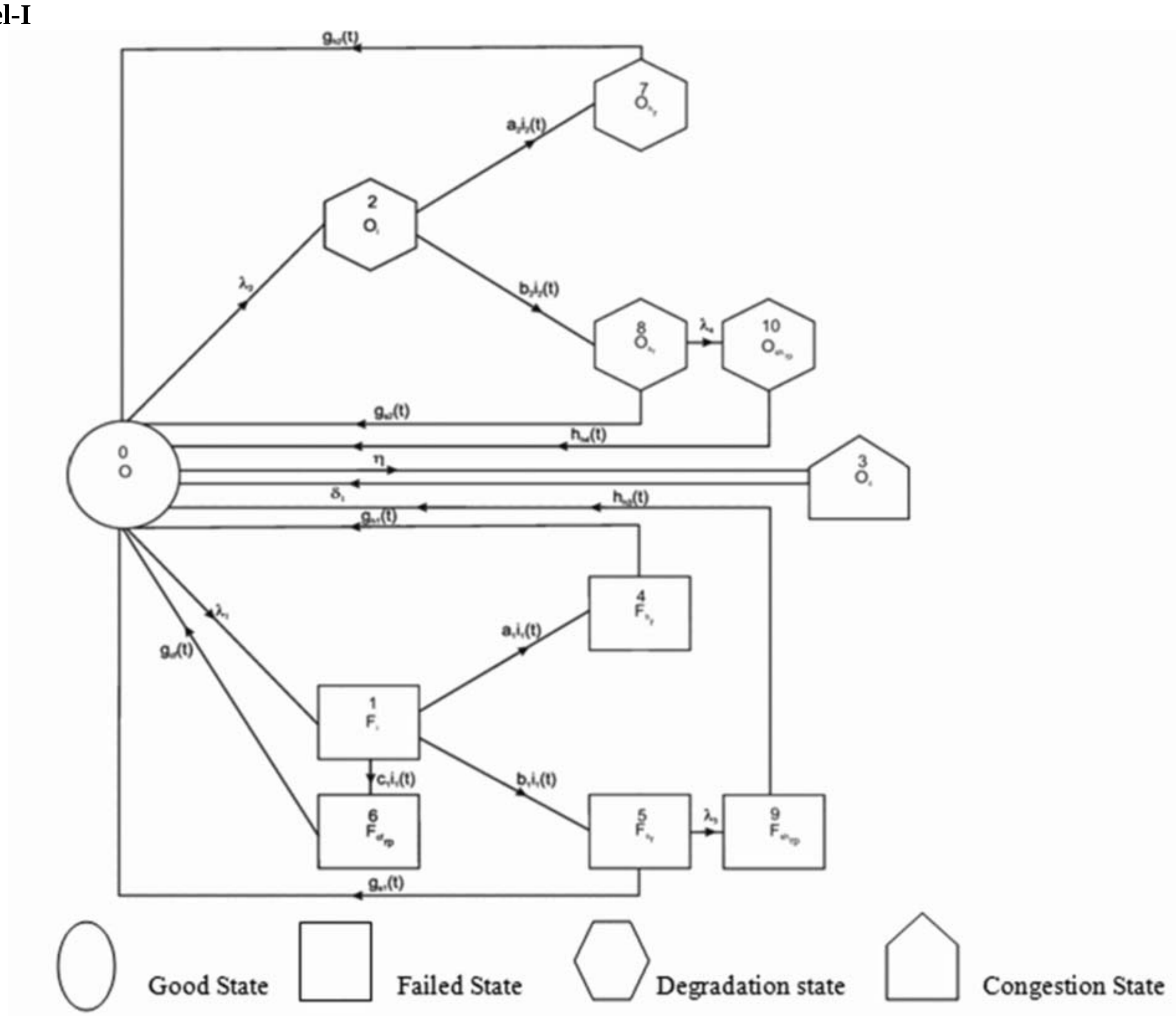
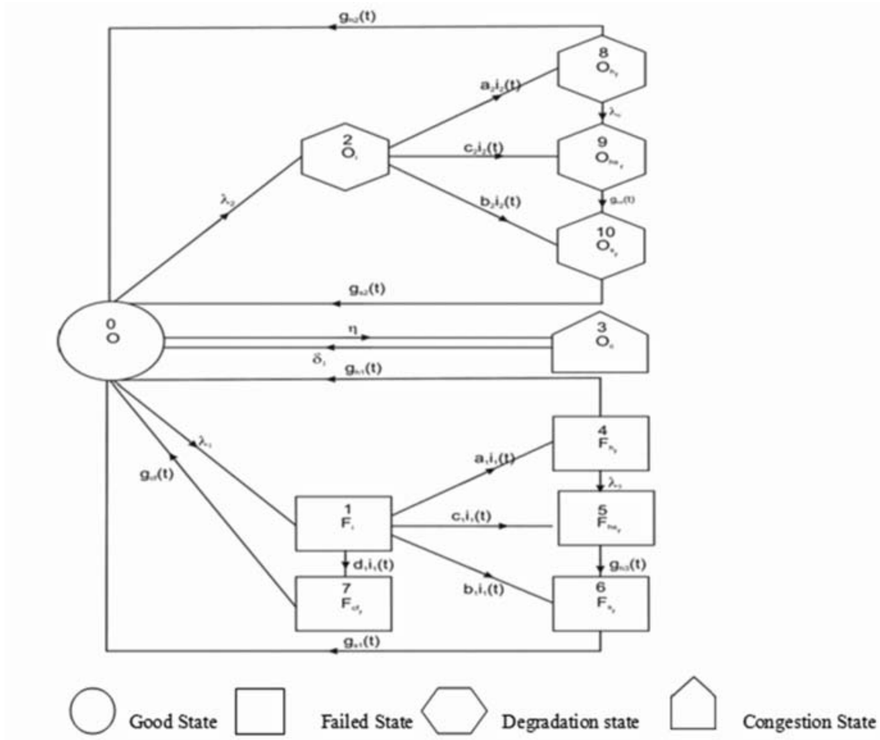



**Fig.1**

**Model-II**




## 3. TRANSITION PROBABILITIES AND MEAN SOJOURN TIMES

Various transition probabilities and mean sojourn times respectively for the Model-I and Model-II as given in [5] and [6] are as under

**Model-I**

$p_{01} = \dfrac{\lambda_1}{\lambda_1 + \lambda_2 + \eta}$ $\quad p_{02} = \dfrac{\lambda_2}{\lambda_1 + \lambda_2 + \eta}$ $\quad p_{03} = \dfrac{\eta}{\lambda_1 + \lambda_2 + \eta}$ $\quad p_{14} = a_1 i_1^*(0)$

$p_{15} = b_1 i_1^*(0)$ $\quad p_{16} = d_1 i_1^*(0)$ $\quad p_{27} = a_2 i_2^*(0)$ $\quad p_{28} = b_2 i_2^*(0)$

$p_{30} = 1$ $\quad p_{40} = g_{h_1}^*(0)$ $\quad p_{50} = g_{s_1}^*(\lambda_3)$ $\quad p_{59} = 1 - g_{s_1}^*(\lambda_3)$

$p_{60} = g_{c_f}^*(0)$ $\quad p_{70} = g_{h_2}^*(0)$ $\quad p_{80} = g_{s_2}^*(\lambda_4)$ $\quad p_{810} = 1 - g_{s_2}^*(\lambda_4)$

$p_{90} = h_{h_3}^*(0)$ $\quad p_{100} = h_{h_4}^*(0)$

By these transition probabilities, it can be verified that

$p_{01}+p_{02}+p_{03}= p_{14}+p_{15}+p_{16}=p_{27}+p_{28}= p_{50}+p_{59} =p_{80}+p_{810}=1$

$p_{30} = p_{40}= p_{60}= p_{70}= p_{90}= p_{100}=1$

The mean sojourn time ($\mu_i$) in the regenerative state i is defined as the time of stay in that state before transition to any other state. If T denotes the sojourn time in regenerative state i, then

$\mu_0 = \dfrac{1}{\lambda_1 + \lambda_2 + \eta}$ $\quad \mu_1 = -i_1^{*'}(0)$ $\quad \mu_2 = -i_2^{*'}(0)$ $\quad \mu_3 = \dfrac{1}{\delta_1}$

$\mu_4 = -g_{h_1}^{*'}(0)$ $\quad \mu_5 = \dfrac{1}{\lambda_3}(1 - g_{s_1}^*(\lambda_3))$ $\quad \mu_6 = -g_{c_f}^{*'}(0)$ $\quad \mu_7 = -g_{h_2}^{*'}(0)$

$\mu_8 = \dfrac{1}{\lambda_4}(1 - g_{s_2}^*(\lambda_4))$ $\quad \mu_9 = -h_{h_3}^{*'}(0)$ $\quad \mu_{10} = -h_{h_4}^{*'}(0)$

The unconditional mean time taken by the system to transit for any regenerative state j, when it is counted from epoch of entrance into that state i, is mathematically stated as

$m_{ij} = \int t\, q_{ij}(t)\, dt$

Thus,

$m_{01} + m_{02} + m_{03} = \mu_0$ $\quad m_{14}+m_{15}+m_{16}=\mu_1$ $\quad m_{27} + m_{28} = \mu_2$ $\quad m_{30} = \mu_3$

$m_{40} = \mu_4$ $\quad m_{50} +m_{59}= \mu_5$ $\quad m_{60} = \mu_6$ $\quad m_{70} = \mu_7$

$m_{80} +m_{810}= \mu_8$ $\quad m_{90} = \mu_9$ $\quad m_{100} = \mu_{10}$

**Model-II**

$$p_{01} = \frac{\lambda_1}{\lambda_1+\lambda_2+\eta} \quad p_{02} = \frac{\lambda_2}{\lambda_1+\lambda_2+\eta} \quad p_{03} = \frac{\eta}{\lambda_1+\lambda_2+\eta} \quad p_{14} = a_1 i_1^*(0)$$

$$p_{15} = c_1 i_1^*(0) \quad p_{16} = b_1 i_1^*(0) \quad p_{17} = d_1 i_1^*(0) \quad p_{28} = a_2 i_2^*(0)$$

$$p_{29} = c_2 i_2^*(0) \quad p_{210} = b_2 i_2^*(0) \quad p_{30} = 1 \quad p_{40} = g_{h_1}^*(\lambda_5)$$

$$p_{45} = 1 - g_{h_1}^*(\lambda_5) \quad p_{56} = g_{h_3}^*(0) \quad p_{60} = g_{s_1}^*(0) \quad p_{70} = g_{c_f}^*(0)$$

$$p_{80} = g_{h_2}^*(\lambda_6) \quad p_{89} = 1 - g_{h_2}^*(\lambda_6) \quad p_{910} = g_{h_4}^*(0) \quad p_{100} = g_{s_2}^*(0)$$

By these transition probabilities, it can be verified that

$p_{01}+p_{02} +p_{03}= p_{14}+p_{15}+ p_{16} + p_{17} = p_{28}+p_{29} + p_{210}= p_{40}+p_{45} =p_{80}+p_{89}= 1$

$p_{30} = p_{56}= p_{60}= p_{70}= p_{910}= p_{100}=1$

The mean sojourn time ($\mu_i$) in the regenerative state i is defined as the time of stay in that state before transition to any other state. If T denotes the sojourn time in regenerative state i, then

$$\mu_0 = \frac{1}{\lambda_1+\lambda_2+\eta} \quad \mu_1 = -i_1^{*'}(0) \quad \mu_2 = -i_2^{*'}(0) \quad \mu_3 = \frac{1}{\delta_1}$$

$$\mu_4 = \frac{1}{\lambda_5}(1 - g_{h_1}^*(\lambda_5)) \quad \mu_5 = -g_{h_3}^{*'}(0) \quad \mu_6 = -g_{s_1}^{*'}(0) \quad \mu_7 = -g_{c_f}^{*'}(0)$$

$$\mu_8 = \frac{1}{\lambda_6}(1 - g_{h_2}^*(\lambda_6)) \quad \mu_9 = -g_{h_4}^{*'}(0) \quad \mu_{10} = -g_{s_2}^{*'}(0)$$

Thus,

$m_{01} + m_{02} + m_{03} = \mu_0$ $\quad m_{14}+m_{15}+m_{16}+m_{17}= \mu_1$ $\quad m_{28} + m_{29}+ m_{210} = \mu_2$ $\quad m_{30} = \mu_3$

$m_{40} + m_{45}= \mu_4$ $\quad m_{56} = \mu_5$ $\quad m_{60} = \mu_6$ $\quad m_{70} = \mu_7$

$m_{80} +m_{810}= \mu_8$ $\quad m_{910} = \mu_9$ $\quad m_{100} = \mu_{10}$

## 4. MEASURES OF SYSTEM PERFORMANCE

For the purpose of comparison between the models following values of measures of performance of the system are taken from [5] and [6]:

**For Model-I**

Mean time to system failure ($T_1$) $= N_1/D_1$

Expected up time of the system ($UT_1$) $= N_{11}/ D_{11}$

Expected degradation time of the system ($DT_1$) $= N_{21}/ D_{11}$

Expected congestion time of the system ($CT_1$) $=N_{31}/ D_{11}$

Busy period of the repairman ($BI_1$) $=N_{41}/ D_{11}$
(Inspection time only)

Busy period of the repairman ($BR_1$) $=N_{51}/ D_{11}$
(Repair time only)

Busy period of the repairman ($BRP_1$) $=N_{61}/ D_{11}$
(Replacement time only)

**For Model-II**

Mean time to system failure ($T_2$) $= N_2/D_1$

Expected up time of the system ($UT_2$) $= N_{11}/ D_{12}$

Expected degradation time of the system ($DT_2$) $= N_{22}/ D_{12}$

Expected congestion time of the system ($CT_2$) $=N_{31}/ D_{12}$
Busy period of the repairman ($BI_2$) $=N_{41}/ D_{12}$
(Inspection time only)
Busy period of the repairman ($BR_2$) $=N_{52}/ D_{12}$
(Repair time only)
where

$N_1= \mu_0 + p_{02}\, \mu_2+ p_{03}\, \mu_3+ p_{02}\, (p_{27}\, \mu_7 + \mu_8 p_{28} + p_{28}\, p_{810}\, \mu_{10})$

$N_2= \mu_0 + p_{02}\, \mu_2+ p_{03}\, \mu_3+ p_{02}\, p_{28}\, \mu_8 + (p_{02}\, p_{28}\, p_{89} + p_{02}\, p_{29})\, \mu_9 + (p_{02}\, p_{28}\, p_{89} + p_{02}\, p_{29} + p_{02}\, p_{210})\, \mu_{10}$

$N_{11}= \mu_0$ $N_{21}= p_{02}\mu_2+p_{02}\, p_{27}\mu_7+p_{02}\, p_{28}\mu_8 +p_{02}\, p_{28}\, p_{810}\mu_{10}$

$N_{31} = p_{03}\mu_3$ $N_{41}= p_{01}\mu_1+p_{02}\mu_2$,

$N_{51}= p_{01}\, p_{14}\mu_4 + p_{01}p_{15}\mu_5 + p_{01}p_{16}\, \mu_6 + p_{02}p_{27}\mu_7+ p_{02}p_{28}\mu_8$,

$N_{61}= p_{01}\, p_{15}\, p_{59}\, \mu_5 + p_{02}\, p_{28}\, p_{810}\, \mu_6$

$N_{22} = p_{02}\mu_2 + p_{02}\, p_{28}\mu_8 + p_{02}\, (\, p_{28}\, p_{89} + p_{29}\, )\, \mu_9 + p_{02}(\, p_{28}\, p_{89} + p_{29} \, + p_{210})\, \mu_{10}$ ,

$N_{52}= p_{01}p_{14}\mu_4 + (p_{01}p_{14}\, p_{45}+ p_{01}p_{15})\, \mu_5+ (p_{01}p_{14}p_{45} + p_{01}p_{15} +p_{01}p_{16})\, \mu_6 + p_{01}p_{17}\mu_7 + (p_{01}p_{14}p_{45} + p_{01}p_{15} + p_{01}p_{16})\, \mu_6 + p_{01}p_{17}\mu_7 + p_{02}p_{28}\mu_8 + (p_{02}p_{28}p_{89} + p_{02}p_{29})\mu_9 + (p_{02}p_{28}p_{89} + p_{02}p_{29} + p_{02}p_{210})\, \mu_{10}$

$D_1 = p_{01}$

$D_{11}= \mu_0+ p_{01}\mu_1+ p_{02}\mu_2 + p_{03}\, \mu_3+ p_{01}p_{14}\mu_4 + p_{01}p_{15}\mu_5+ p_{01}\, p_{16}\, \mu_6 + p_{02}\, p_{27}\mu_7+ p_{02}\, p_{28}\, \mu_8+p_{01}\, p_{15}\, p_{59}\, \mu_9 +p_{02}\, p_{28}\, p_{810}\, \mu_{10}$

$D_{12}= \mu_0 + p_{01}\mu_1 + p_{02}\mu_2 + p_{03}\mu_3 + p_{01}p_{14}\mu_4 + p_{01}\, (p_{14}p_{45} + p_{15})\, \mu_5 + p_{01}\, (p_{14}p_{45} + p_{15} + p_{16})\, \mu_6 + p_{01}\, p_{17}\mu_7 + p_{02}p_{28}\, \mu_8 + p_{02}\, (p_{28}p_{89} + p_{29})\, \mu_9 + p_{02}\, (p_{28}p_{89} + p_{29} + p_{210})\, \mu_{10}$

## 5. PROFIT ANALYSIS

The expected profits of the system corresponding to model-I and model-II are:

$P_1= C_0\, UT_1 + C_1 DT_1 + C_2\, CT_1 – C_3\, BI_1 –C_4 BR_1\text{-}C_5\, BRP_1\text{-}C$

$P_2 = C_0\, UT_2 + C_1 DT_2 + C_2\, CT_2 – C_3\, BI_2 –C_4 BR_2\text{--}C$

Where

$C_0$ = revenue per unit uptime of the system
$C_1$= revenue per unit degradation time of the system
$C_2$ = revenue per unit congestion time of the syste
$C_3$ = cost per unit time of inspection
$C_4$ = cost per unit time of repair
$C_5$ = cost per unit time of replacement
C = cost of installation of the system

## 6. GRAPHICAL ANALYSIS

For graphical analysis the following particular case is considered:

$g_{h_1}(t) = \beta_{h_1} e^{-\beta_{h_1} t}$; $g_{h_2}(t) = \beta_{h_2} e^{-\beta_{h_2} t}$; $g_{s_1}(t) = \beta_{s_1} e^{-\beta_{s_1} t}$; $g_{s_2}(t) = \beta_{s_2} e^{-\beta_{s_2} t}$;

$i_1(t) = \alpha_1 e^{-\alpha_1 t}$; $i_2(t) = \alpha_2 e^{-\alpha_2 t}$; $g_{h_3}(t) = \beta_{h_3} e^{-\beta_{h_3} t}$; $g_{h_4}(t) = \beta_{h_4} e^{-\beta_{h_4} t}$;

$g_{c_f}(t) = \beta_{c_f} e^{-\beta_{c_f} t}$; $h_{h_3}(t) = \gamma_{h_3} e^{-\gamma_{h_3} t}$ $h_{h_4}(t) = \gamma_{h_4} e^{-\gamma_{h_4} t}$

Various graphs for comparing performance of two models viz. differences of mean times to system failure , expected uptimes/expected degradation times/expected congestion times and profits are plotted for different values of rates of occurrence of fault, ($\lambda_1$, $\lambda_2$, $\lambda_3$, $\lambda_4$, $\lambda_5$, $\lambda_6$), probabilities of occurrence of hardware/software/hardware based software/catastrophic failure ($a_1,a_2,b_1,b_2,c_1,c_2,d_1$), inspection rates($\alpha_1,\alpha_2$), hardware/software /hardware based software repair rates ($\beta_{h_1}, \beta_{h_2}, \beta_{s_1}, \beta_{s_2}, \beta_{h_3}\ \beta_{h_4}$), software based hardware replacement rates ($\gamma_{h_3}, \gamma_{h_4}$), repair rate for catastrophic failures ($\beta_{c_f}$), calls congestion and system restoration rates($\eta$, $\delta_1$).

Fig. 3 shows the behavior of difference between the mean times to system failure of the models ($T_2$-$T_1$) with respect to the rate of occurrence of hardware based minor software faults ($\lambda_6$) for different values rate of occurrence of software based minor hardware faults ($\lambda_4$). The graph reveals that the difference of mean times to system failure of two models decreases with increase in the values of rate of occurrence of hardware based software minor faults but increases with increase in rate of software based minor hardware faults. It may also be observed that for $\lambda_4$=0.0003, the difference of mean times to system failure is > or = or < 0 according as $\lambda_6$ is < or = or < 0.000313.Hence the model-II is better or equally good or worse than model-I whenever $\lambda_6$ is > or = < 0.000313. Similarly, for $\lambda_4$= 0.0006 and $\lambda_4$=0.0009, the difference of mean times to system failure is > or = or < 0 according as $\lambda_6$ is < or = or > 0.000653 and 0.001023 respectively. Thus, in these cases, model-II is better or equally good or worse than model-I in terms of mean times to system failures whenever $\lambda_6$<or=or> 0.000653and$\lambda_6$<or=or>0.001023.

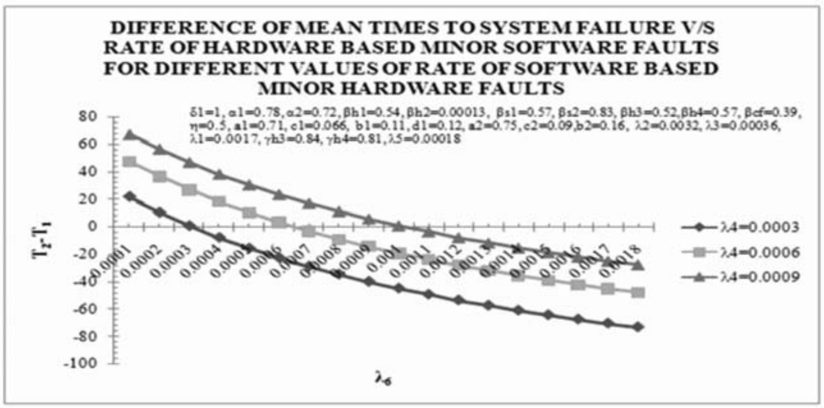



**Fig. 3**

The curves in the fig.4 shows the behavior of difference of mean times to system failure of the models ($T_2$-$T_1$) with respect to the repair rate of minor software faults ($\beta_{s_2}$) of the system for the different values of repair rate of minor hardware faults ($\beta_{h_2}$).It is evident from the graph that difference of mean times to system failure decreases with the increase in the values of repair rate of minor software faults but increases with increase in the values of repair rate of minor faults. It can also conclude that model-II has higher reliability than model I.

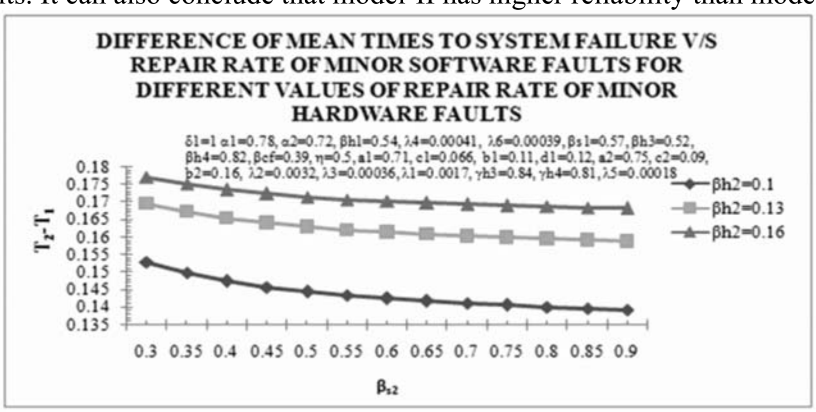



**Fig. 4**

The curves in the fig.5 show the behavior of difference of the expected uptimes of the models ($UT_2$-$UT_1$) with respect to the rate of occurrence of software based major hardware faults of the system ($\lambda_3$) for different values of rate of occurrence of major faults ($\lambda_1$). It is evident from the graph that difference of expected uptimes increases with the increase in the values of rate of occurrence of software based major hardware fault and has higher value for higher values of rate of occurrence of major faults ($\lambda_1$). From the fig.5 it may also be observed that for $\lambda_1$= 0.0015, difference of the expected uptime is < or = or > 0 according as $\lambda_3$ is < or = or > 0.00138. Hence the model-I is better or equally good or worse than model-II whenever $\lambda_3$ is < or = > 0.00138. Similarly, for $\lambda_1$= 0.002 and $\lambda_5$=0.0025, difference of the expected uptimes is < or = or > 0 according as s $\lambda_3$ < or = or > 0.00116 and 0.00103 respectively. Thus, in these cases, model-I is better or equally good or worse than model-II in terms of expected uptime whenever $\lambda_3$ < or = or > 0.00116 and $\lambda_3$ < or = or > 0.00103.

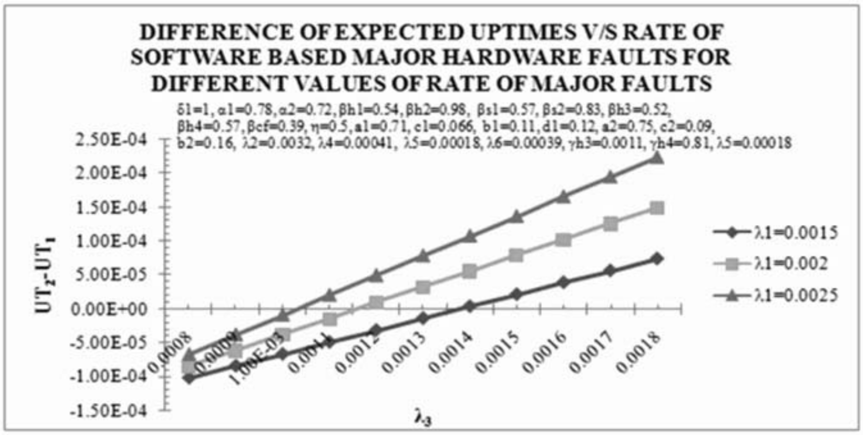



**Fig. 5**

Fig. 6 shows the behavior of difference of the expected degradation times of the models ($DT_2$-$DT_1$) with respect to the rate of occurrence of hardware based minor software faults ($\lambda_6$) for different values of rate of occurrence of minor faults ($\lambda_2$). The graph reveals that the difference of expected degradation times of the two models increases with increase in the values of rate of occurrence of hardware based minor software faults and has higher value for larger values of rate of occurrence of minor faults. This shows that model-II has more expected degradation time than model-I for given values of parameters.

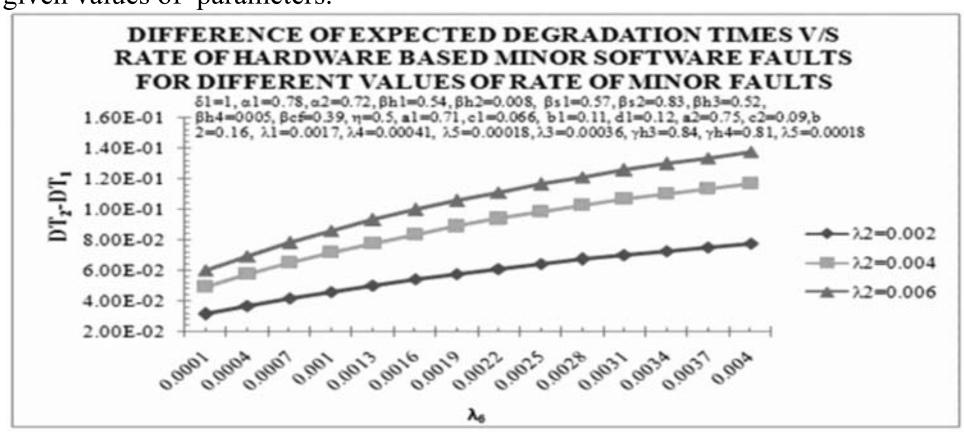



**Fig. 6**

The curves in the fig. 7 shows the behavior of the difference of profits of the models ($P_2$-$P_1$) with respect to the rate of occurrence of software based major hardware faults of the system ($\lambda_3$) for different values of repair rate of hardware based major software faults $(\beta_{h_3})$ .The graph indicates that difference of profits increases with the increase in the values of rate of occurrence of software based major hardware faults. From the fig. 7 it may also be observed that for $\beta_{h_3}$ =0.25, the difference of profits is < or = or > 0 according as $\lambda_3$ is < or = or > 0.00102. Hence the model-I is better or equally good or worse than model-II whenever $\lambda_3$ < or = or > 0.000102. Similarly, for $\beta_{h_3}$ =0.35 and $\beta_{h_3}$ =0.45, the difference of profits is < or = or > 0 according as is $\lambda_3$ < or = or > 0.00085 and 0.00075 respectively.

Thus, in these cases, model-I is better or equally good or worse than model-II in terms of profit incurred whenever $\lambda_3$ < or = or > 0.00085 and $\lambda_3$ < or = or >0.00075.

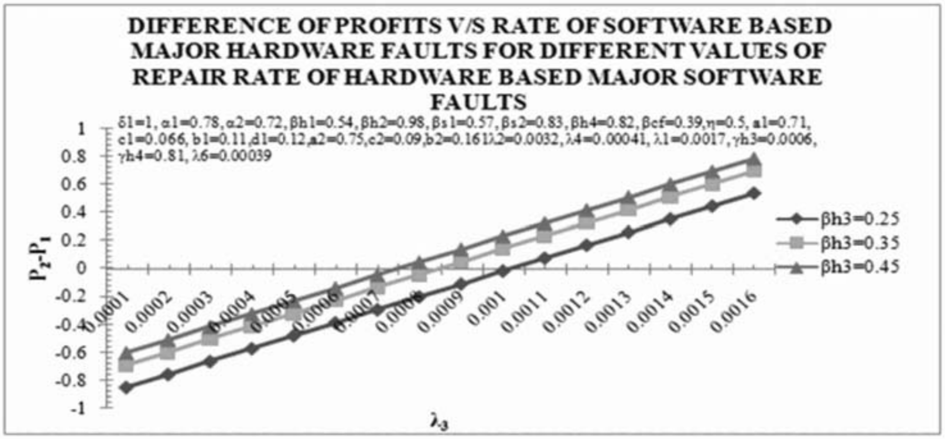
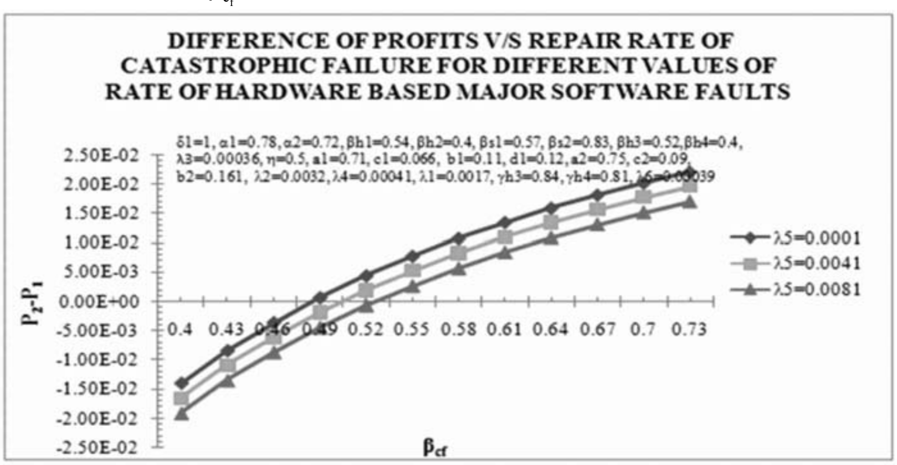



**Fig. 7**

The curves in the fig.8 shows the behavior of difference of the profits ($P_2$-$P_1$) with respect to the repair rate of catastrophic failure ($\beta_{c_f}$) for the different values of rate of occurrence of hardware based minor software faults ($\lambda_5$).

It is evident from the graph that difference of profits increases with the increase in the values of repair rate of catastrophic failure but decreases with increase in rate of occurrence of hardware based minor software faults. From the fig. 8 it may also be observed that for $\lambda_5$ = 0.0001, the profit difference is < or = or > 0 according as is $\beta_{c_f}$ < or = or > 0.485. Hence the model-I is better or equally good or worse than model-II in terms of profits whenever $\beta_{c_f}$ < or = or >0.485. Similarly, for $\lambda_5$=0.0041 and $\lambda_5$=0.0081, the difference of profits is < or = or > 0 according as is $\beta_{c_f}$ < or = or > 0.505 and 0.525 respectively. Thus, in these cases, model-I is better or equally good or worse than model-II whenever $\beta_{c_f}$ < or = or > 0.505 and $\beta_{c_f}$ < or = or > 0.525.




## 7. CONCLUSION

From the comparative analysis it can be concluded that the difference of mean times to system failure of the models decreases with increase in the rate of occurrence of hardware based minor faults as well as with rate of occurrence of software based minor hardware faults. It is also concluded that the model given in [5] has higher reliability than model given in [6] for some considered values of the parameters. It also revealed from the analysis that difference of mean times to system failure decreases with increase in the repair rate of minor software faults and has higher values for higher values of repair rate of minor hardware faults. It is also concluded that the difference of the expected uptimes increases with increase in the values of rate of occurrence of software based minor hardware faults as well as with rate of major faults. Further it is observed that difference of degradation times increases with increase in rate of occurrence of hardware based minor software faults as well as with rate of minor faults. It may also be concluded that expected degradation time of model given in [5] is more than model given in [6] for the given values of parameters.

Also cut-off points for various parameters of importance can be obtained for differences of mean times to system failure, expected uptimes, expected degradation times and profits of the models given in [5] and [6] to suggest the stakeholders which model is better and under what conditions.

## REFERENCES


1. Arora, J.R. (1977), “Reliability of several standby priority redundant systems”, IEEE Trans. Reliab., 26, 290-293.
2. Goel et al. (1986), “Reliability analysis of a system with preventive maintenance and two types of repair”, Microelectron. Reliab. 26, 429-433.
3. Gopalan, M.N. and Murlidhar, N.N. (1991), “Cost analysis of a one unit repairable system subject to online preventive maintenance and/or repair”, Microelectron Reliability, 31 (2/3), 223-228
4. Kumar, R. and Bhatia, P. (2011) “Reliability and cost analysis of one unit centrifuge system single repairman and inspection”, Pure and Applied Mathematika Sciences, 74 (1-2), 113-121.
5. Kumar, R. and Kapoor , S. (2013) “Profit evaluation of a stochastic model on Base Transceiver system considering software based hardware failures and congestion of calls”, International Journal of Application or Innovation in Engineering and Management, 2 (3), 554-562.
6. Kumar, R. and Kapoor , S. (2013) “Economic and performance evaluation of stochastic model on a Base Transceiver system considering various operational modes and catastrophic failures”, Journal of Mathematics and Statistics, 9 (3), 198-207.
7. Kumar, R. and Kumar, M. (2012), “Performance and cost-benefit analysis of a hardware-software system considering hardware based software interaction failures and different types of recovery”, International Journal of Computer Application, 78(2), 29-35.
8. Kumar, R. and Mor, S. (2011), “Probabilistic analysis of a sophisticated system with some warranted components considering two types of service facility and delay in warranty claims”, Caledonian Journal of Engineering, 7(2),21-27.
9. Kumar, R. and Vashistha, (2001) “A two-unit redundant system with degradation replacement, Pure and Applied Mathematika Sciences, LIV(1-2), 27-38.
10. Taneja Anita (2014) “Reliability & Profit evaluation of a Two unit cold standby system with inspection & chances of Replacement.” Aryabhatta J. of Mathematics & Informatics Vol. 6 (1) pp 211-218.
11. Singh, S.K. and Mishra, A.K. (1994), “Profit evaluation of a two unit standby redundant system with two operating modes”, Microelectron. Reliab., 34(4), 747-750.
12. Taneja, G., Tayagi, V.K and Bhardwaj, P. (2004), “Profit analysis of single unit programmable logic controller”, Pure and Applied Mathematica Sciences, LX (1-2), 55-71.
13. Teng, X., Pham, H. and Jeske, D.R. (2006) “Reliability modeling of hardware and software interactions, and its applications”, IEEE Transactions on Reliability, 55( 4), 571-57
14. Tumer, I.Y. and Smidts, C.S. (2011) “Integrated design-stage failure analysis of software driven hardware systems”, IEEE Trans on Computers, 60 (8), 1072-1084.
15. Welke, S.R., Johnson, B.W. and Aylor, J.H.(1995) “Reliability modeling of hardware/software systems”, IEEE Trans of Reliability, 44 (.3),413-418.

Aryabhatta Journal of Mathematics & Informatics Vol. 6, No. 2, July-Dec, 2014 ISSN : 0975-7139
Journal Impact Factor (2013) : 0.489

# STOCHASTIC MODEL TO FIND THE EFFECT OF GALLBLADDER CONTRACTION RESULT USING UNIFORM DISTRIBUTION

**P. Senthil Kumar*, A. Dinesh Kumar** & M. Vasuki*****

*Assistant Professor, Department of Mathematics, Rajah Serfoji Government College (Autonomous), Thanjavur, Tamilnadu, India
** Assistant Professor, Department of Mathematics, Dhanalakshmi Srinivasan Engineering College, Perambalur, Tamilnadu, India. Email:
*** Assistant Professor, Department of Mathematics, Srinivasan College of Arts and Science, Perambalur, Tamilnadu, India.
Email: senthilscas@yahoo.com, dineshkumarmat@gmail.com, vasuki.maths@gmail.com

**ABSTRACT :**

***The cholecystokinin is a 33 amino acid polypeptide. The C-terminal octapeptide fragment (OP-CCK) reproduces all the known biologic activities of the intact molecule. The purpose of this study was to determine the effect on gallbladder contraction of $20\ ng/kg$ of OP-CCK intravenously in patients undergoing cholecystography.***

***If $a \geq 1$ and $b \geq 1$, let us define $\Theta(a,b)$ as the smallest $r = 0, 1, 2, \ldots\ldots$ for which either $\xi_r = a$ or $\xi_r = -b$. We can interpret $\Theta(a,b)$ as the duration of games in the classical ruin problem. We have***

$$E\{\omega^{\Theta(a,b)}\} = \frac{[\gamma(\omega)]^a + [\gamma(\omega)]^b}{1 + [\gamma(\omega)]^{a+b}} \text{ if } |\omega| \leq 1, \text{ where } \gamma(\omega) = \frac{(1-\sqrt{1-\omega^2})}{\omega}$$

**Key Words:** ***Gallbladder, Brownian Motion, Cholecystokinin and Uniform Distribution.***
***2010 Mathematics Subject Classification: 60J65, 60G57***

## 1. INTRODUCTION

The diagnostic value of gallbladder contraction during cholecystography is uncertain possible advantages include: visualization of gallstones not seen without contraction; visualization of bile ducts and choledocholithiasis; reproduction of the spontaneous pain of biliary dyskinesia, and correlation of the pain with roentgenographic features, and possible prediction of future cholelithiasis.

The common bile duct was visualized in 47 percent of our patients after OP-CCK, Gallbladder contraction occurs at a more rapid rate after OP-CCK than after a fat meal, use of OP-CCK can shorten the time and probably decrease the number of film exposures and amount of radiation required to perform oral cholecystography [12].

Let $\{\xi(t), t \geq 0\}$ be a standard Brownian motion process. We have $P\{\xi(t) \leq x\} = \varphi\left({x}/{\sqrt{t}}\right)$ for $t > 0$, where $\varphi(x)$ is the normal distribution function from the random walk $\{\xi_r, r \geq 0\}$ of Brownian motion process. If $a \geq 1$ and $b \geq 1$, let us define $\Theta(a,b)$ as the smallest $r = 0, 1, 2, \ldots\ldots$ for which either $\xi_r = a$ or $\xi_r = -b$. We can interpret $\Theta(a,b)$ as the duration of games in the classical ruin problem. We have from the random walk of standard Brownian motion,

$$E\{\omega^{\Theta(a,b)}\} = \frac{[\gamma(\omega)]^a + [\gamma(\omega)]^b}{1 + [\gamma(\omega)]^{a+b}}$$

if $|\omega| \leq 1$, where $\gamma(\omega)$ is defined by $\gamma(\omega) = \frac{(1-\sqrt{1-\omega^2})}{\omega}$ and it is fitted with uniform distribution.

## 2. NOTATIONS:

$\varphi(x)$ - Normal Density Function
$\tau(\alpha)$ - Local Time
$\delta(S)$ - Indicator Variable

| | | |
|---|---|---|
| $\omega(\alpha)$ | - | Nonnegative Random Variable |
| $F_\alpha(x)$ | - | Distribution Function |
| $M_r(\alpha)$ | - | Moments |
| $C_n$ | - | Catalan Number |
| $\omega$ | - | Assuming Time |
| $a$ | - | Scale Parameter |
| $b$ | - | Shape Parameter |

## 3. MOMENTS:

Let $\{\xi(t), t \geq 0\}$ be a standard Brownian motion process. We have $P\{\xi(t) \leq x\} = \Phi\left({x}/{\sqrt{t}}\right)$ for $t > 0$ where

$$\Phi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{-u^2/2}\, du \tag{1}$$

is the normal distribution function. We also use the notation

$$\Phi(x) = \frac{1}{\sqrt{2\pi}} e^{-x^2/2}$$

for the normal density function. Let us define

$$\tau(\alpha) = \lim_{\epsilon \to 0} \frac{1}{\epsilon} measure\ \{t: \alpha \leq \xi(t) < \alpha + \epsilon, 0 \leq t \leq 1\} \tag{2}$$

for any real $\alpha$. The limit (2) exists with probability one and $\tau(\alpha)$ is a non negative random variable which is called the local time at level $\alpha$. We define also

$$\omega(\alpha) = \int_0^1 \delta(\xi(t) > \alpha) dt \tag{3}$$

for $\alpha \geq 0$ where $\delta(S)$ denotes the indicator variable of any event $S$, that is, $\delta(S) = 1$ if $S$ occurs and $\delta(S) = 0$ if $S$ does not occur. The integral (3) exists with probability one and $\omega(\alpha)$ is a non negative random variable which is called the Sojourn time of the process $\{\xi(t), t \geq 0\}$ spent in the set $(\alpha, \infty)$ in the time interval $(0,1)$. We also consider the reflecting Brownian motion process $\{|\xi(t)|, t \geq 0\}$ and define

$$\omega^*(\alpha) = \int_0^1 \delta(|\xi(t)| > \alpha) dt \tag{4}$$

for $\alpha \geq 0$ as the Sojourn time of the process $\{|\xi(t)|, t \geq 0\}$ spent in the set $(\alpha, \infty)$ in the time interval $(0,1)$.

Our main object is to determine the distribution and the moments of $\omega^*(\alpha)$ for $\alpha > 0$. In principle, we can apply the method of [2] to find the distribution of $\omega^*(\alpha)$. His method requires the inversion of a double Laplace transform which can be obtained by solving a certain Sturm-Liouville differential equation. Our approach is combinatorial and we shall find explicit formulas for the distribution function and the moments of $\omega^*(\alpha)$

Let us define
$$E\{[\tau(\alpha)]^r\} = m_r(\alpha),$$
$$E\{[\omega(\alpha)]^r\} = M_r(\alpha), \tag{5}$$
and
$$E\{[\omega^*(\alpha)]^r\} = M_r^*(\alpha) \tag{6}$$

for $r = 1,2,3, \dots\dots\dots$ and $\alpha > 0$. We shall prove the following surprisingly simple formulas for the moments (5) and (6):

$$M_r(\alpha) = m_{2r}(\alpha)/(2^r r!) \tag{7}$$

And
$$M_r^*(\alpha) = \frac{(r-1)!}{2^{r-1}} \sum_{k=1}^{r} \frac{m_{2r}((2k-1)\alpha)}{(r-k)!(r+k-1)!} \tag{8}$$

if $r = 1,2,3, \dots\dots\dots$ and $\alpha > 0$. Equations (7) and (8) make it possible to determine the distribution function $P\{\omega^*(\alpha) \leq x\} = G_\alpha(x)$ explicitly. We shall prove that

$$G_\alpha(x) = 2F_\alpha(x) - 1 + 2\sum_{k=2}^{\infty} \sum_{j=2}^{k} \frac{(-1)^j j!}{(k+j-1)!} \binom{k-2}{j-2} \frac{d^{k-1} x^{k-1} [1 - F_{(2j-1)\alpha}(x)]}{dx^{k-1}} \tag{9}$$

if $0 \leq x < 1$ and $\alpha > 0$, and $G_\alpha(1) = 1$. In (9), $F_\alpha(x) = P\{\omega(\alpha) \leq x\}$. We have

$$F_\alpha(x) = 1 - \frac{1}{\pi}\int_0^{1-x} \frac{e^{-\alpha^2/(2u)}}{\sqrt{u(1-u)}}du \text{ for } 0 < x \le 1, \text{ and } \alpha \ge 0, \text{ and}$$

$$F_\alpha(x) = 2\Phi(\alpha) - 1$$

for $\alpha \ge 0$. The distribution function $F_\alpha(x)$ was found by [1].

If, in particular, $x = 0$ in (9), we obtain that

$$G_\alpha(x) = 1 + 4\sum_{k=1}^{\infty}(-1)^k\left[1 - \Phi\big((2k-1)\alpha\big)\right]$$

$$= \frac{4}{\pi}\sum_{j=1}^{\infty}\frac{(-1)^{j-1}}{2j-1}e^{\frac{-(2j-1)^2\pi^2}{(8\alpha^2)}}$$

for $\alpha > 0$. We note that $\quad P\{\tau(\alpha) \le x\} = 2\Phi(\alpha + x) - 1$

if $x \ge 0$ and $\alpha \ge 0$, and $\quad m_r(\alpha) = 2r\int_0^{\infty} x^{r-1}\,[1 - \Phi(\alpha + x)]dx \qquad (10)$

if $\alpha \ge 0$ and $r \ge 1$ where $\Phi(x)$ is defined by (1). Explicitly,

$$m_r(\alpha) = 2(-1)^r\{a_r(\alpha)[1 - \Phi(\alpha)] - b_r(\alpha)\Phi(\alpha)\}$$

for $r = 1,2,3, \ldots\ldots\ldots\ldots\ldots$ where $a_r(\alpha) = r!\sum_{j=0}^{[r/2]}\frac{\alpha^{r-2j}}{2^j j!(r-2j)!}$

and $\quad b_r(\alpha) = \sum_{j=0}^{[(r-1)/2]}\binom{r-1-j}{j}\frac{j!\alpha^{r-1-2j}}{2^j}\sum_{v=0}^{j}\binom{r}{v}$

for $r \ge 1$. See [10]

Our approach is based on a symmetric random walk $\{\xi_r, r \ge 0\}$ where $\xi_r = \xi_1 + \xi_1 + \xi_1 \ldots\ldots\ldots\ldots\ldots\ldots\xi_r$ for $r \ge 1$, $\xi_0 = 0$, and $\{\xi_r, r \ge 1\}$ is a sequence of independent and identically distributed random variables for which

$$P\{\xi_r = 1\} = P\{\xi_r = -1\} = {}^1\!/_2$$

Let us define $\tau_n(\alpha)$ as the number of subscripts $r = 0,1,2, \ldots\ldots\ldots\ldots n$ for which $\xi_r = a$ where $a = 0,1,2, \ldots\ldots\ldots\ldots\ldots$ Furthermore, define $\omega_n(a)$ as the number of subscripts $r = 0,1,2, \ldots\ldots\ldots\ldots\ldots n$ for which $\xi_r \ge a$ where $a = 0,1,2, \ldots\ldots\ldots\ldots$ and $\omega_n^*(a)$ as the number of subscripts $r = 0,1,2, \ldots\ldots\ldots\ldots n$ for which $|\xi_r| \ge a$ where $a = 0,1,2, \ldots\ldots\ldots\ldots\ldots$

By the results of [4], if $n \to \infty$, the process $\left\{\xi_{|nt|}\,\middle|\,\sqrt{n}, 0 \le t \le 1\right\}$ converges weakly to the Brownian motion $\{\xi(t),\ 0 \le t \le 1\}$. See also [7].

In [6] proved that $\quad \lim_{n\to\infty} P\left\{\frac{\tau_n([\alpha\sqrt{n}])}{\sqrt{n}} \le x\right\} = P\{\tau(\alpha) \le x\}$

for $\alpha \ge 0$ and $x > 0$. Since the integrals (3) and (4) are continuous functional of the process $\{\xi(t),\ 0 \le t \le 1\}$, we can conclude that

$$\lim_{n\to\infty} P\big\{\omega_n\big([\alpha\sqrt{n}]\big) \le nx\big\} = P\{\omega(\alpha) \le x\}$$

and

$$\lim_{n\to\infty} P\big\{\omega_n^*\big([\alpha\sqrt{n}]\big) \le nx\big\} = P\{\omega^*(\alpha) \le x\}$$

for $\alpha > 0$ and $x \ge 0$.

We shall determine the distributions and the moments of the random variables $\tau_n(a), \omega_n(a)$ and $\omega_n^*(a)$, and their asymptotic behavior in the case where $a = [\alpha\sqrt{n}], \alpha > 0$, and $n \to \infty$. We shall prove that

$$\lim_{n\to\infty} E\left\{\left(\frac{\tau_n([\alpha\sqrt{n}])}{\sqrt{n}}\right)^r\right\} = m_r(\alpha)$$

for $r \ge 1$ and $\alpha \ge 0$ where $m_r(\alpha)$ is given by (10). Furthermore, we shall determine (5) and (6) by calculating the following limits

$$\lim_{n\to\infty} E\left\{\left(\frac{\omega_n([\alpha\sqrt{n}])}{\sqrt{n}}\right)^r\right\} = M_r(\alpha)$$

and

$$\lim_{n\to\infty} E\left\{\left(\frac{\omega_n^*([\alpha\sqrt{n}])}{\sqrt{n}}\right)^r\right\} = M_r^*(\alpha)$$

for $r \ge 1$ and $\alpha \ge 0$. The moments $M_r(\alpha), (r \ge 1\ )$ and $M_r^*(\alpha), (r \ge 1\ )$ uniquely determine the distribution functions $P\{\omega(\alpha) \le x\}$ and $P\{\omega^*(\alpha) \le x\}$

## 4. THE RANDOM WALK $\{\xi_r, r \geq 0\}$

Let us recall some results for $\{\xi_r, r \geq 0\}$ which we need in this paper. See [9], We have $P\{\xi_n = 2j - n\} = \binom{n}{j}\frac{1}{2^n}$

for $j = 0,1,2, \ldots \ldots \ldots, n$ and by the central limit theorem

$$\lim_{n\to\infty} P\left\{\frac{\xi_n}{\sqrt{n}} \leq x\right\} = \Phi(x)$$

where $\Phi(x)$ is defined by (1).

Let us define $\rho(a)$ as the first passage time through $a$ $(a = 0, \pm1, \pm2, \ldots \ldots \ldots$ that is,

$$\rho(a) = inf\{r: \xi_r = a \text{ and } r \geq 0\}$$

We have
$$P\{\rho(a) = a + 2j\} = \frac{a}{a+2j}\binom{a+2j}{j}\frac{1}{2^{a+2j}} \tag{11}$$

for $a \geq 1$ and $j \geq 0$. If $1 \leq a \leq n$, then

$$P\{\rho(a) \leq n\} = P\{\xi_n \geq a\} + P\{\xi_n > a\} \tag{12}$$

By (11),
$$\sum_{n=0}^{\infty} P\{\rho(a) = n\}\omega^n = [\gamma(\omega)]^a$$

for $a \geq 1$ and $|\omega| \leq 1$ where $\gamma(0) = 0$ and

$$\gamma(\omega) = \frac{(1-\sqrt{1-\omega^2})}{\omega} \tag{13}$$

for $0 < |\omega| \leq 1$. The identity

$$\sum_{j=0}^{n} P\{\rho(a) = j\}P\{\rho(a) = n - j\} = P\{\rho(a + b) = n\} \tag{14}$$

is valid for any $a \geq 1, b \geq 1$ and $n \geq 1$.

We note that
$$P\{\rho(1) = 2n + 1\} = \frac{C_n}{2^{2n+1}} \tag{15}$$

for $n = 0,1,2, \ldots \ldots \ldots \ldots \ldots$ where $C_n = \binom{2n}{n}\frac{1}{n+1}$ is the n[th] Catalan number.

Let us define
$$\sum_{\substack{\alpha_1+\alpha_2+\cdots+\alpha_n=v \\ \alpha_1+2\alpha_2+\cdots+n\alpha_n=n}} \frac{v!}{\alpha_1!\alpha_2!\ldots\alpha_n!} C_0^{\alpha_1} C_1^{\alpha_2} \ldots \ldots \ldots \ldots C_{n-1}^{\alpha_n} \tag{16}$$

for $1 \leq v \leq n$. To evaluate (16) let us express each Catalan number in (16) by (15). By the repeated applications of (14) we obtain that

$$P(n, v) = 2^{2n-v} P\{\rho(v) = 2n - v\} = \binom{2n-v}{n}\frac{v}{2n-v}$$

By (12) we obtain that
$$\sum_{s=j}^{r} P\{\rho(2j) = 2s\} = P\{\rho(2j) \leq 2r + 1\}$$
$$= 2P\{\xi_{2r+1} \geq 2j + 1\} = \sum_{s=j}^{r}\binom{2r+1}{r-s}\frac{1}{2^{2r}}$$

for $j = 0,1,2, \ldots \ldots \ldots \ldots \ldots r$.

If $a \geq 1$ and $b \geq 1$, let us define $\Theta(a, b)$ as the smallest $r = 0,1,2, \ldots \ldots \ldots \ldots \ldots$ for which either $\xi_r = a$ or $\xi_r = -b$. We can interpret $\Theta(a, b)$ as the duration of games in the classical ruin problem. See [8]. By the results of [5], we have

$$E\{\omega^{\Theta(a,b)}\} = \frac{[\gamma(\omega)]^a + [\gamma(\omega)]^b}{1 + [\gamma(\omega)]^{a+b}}$$

If $|\omega| \leq 1$ where $\gamma(\omega)$ is defined by (13). See also [3].

## 5. EXAMPLE:

The effect of an intravenous injection of $20\ ng/kg$ of the C-terminal octapeptide of cholecystokinin (OP-CCK) on gallbladder contraction was investigated in 30 adult patients undergoing oral cholecystography. The figure shows the mean reduction in gallbladder size, $5, 10$ and $15$ mintues after injection of $20\ ng/kg$ of OP-CCK. The mean peak reduction in gallbladder size was 48 percent. Less than $40\%$ decrease in gallbladder size occurred in $4$ patients. An additional injection of $40\ ng/kg$ of OP-CCK in these $4$ patients produced further contraction in only one patients produced further contraction in only one patient. The common bile duct was visualized in $47\%$ of patients after $20\ ng/kg$ of OP-CCK. This figure is comparable to that obtain with a standard fat meal [11] & [12].

**Figure (1):** Mean reduction in gallbladder size at $5, 10$ and $15$ mintues after intravenous injection of $20\ ng/kg$

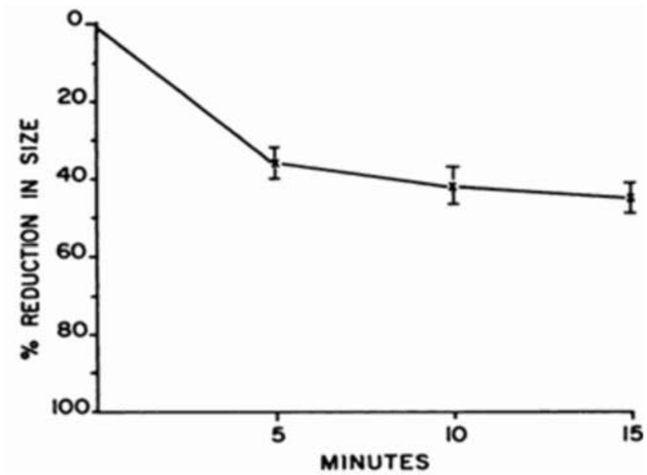
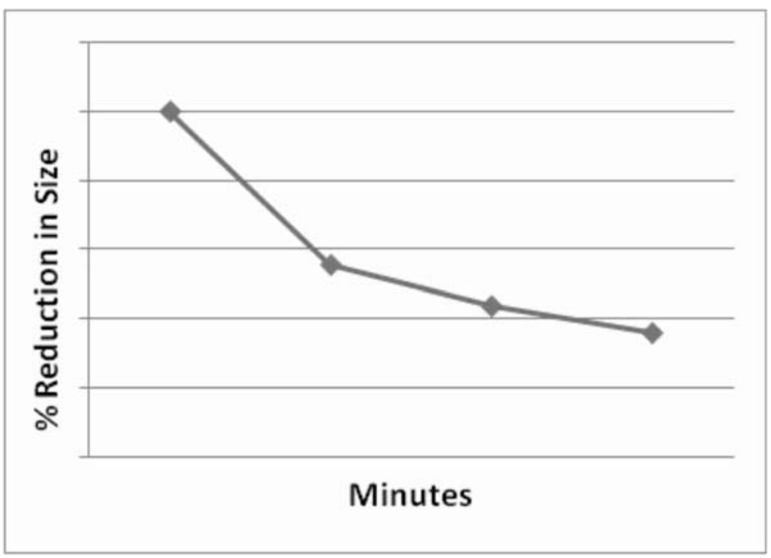



**Figure (2):**




## 6. CONCLUSION:

This study provides evidence that OP-CCK, $20\ ng/kg$, injected intravenously over 30 seconds is safe and effective as a gallbladder contracting agent in man, while side effects were reported by $25$ of $30$ patients, these were mild, transient and easily tolerated. There were no significant changes in any of the laboratory tests after OP-CCK. The random walk of Brownian motion process gives the same as the medical report.The medical reports are beautifully fitted with the mathematical model. Hence the mathematical report {Figure (2)} is coincide with the medical report {Figure (1)}.

## REFERENCES:

1. Levy P, “Sur Certain Processes Stochastic Homogenous”, Composition Mathematical 7 (1939), 283-339.
2. Kac M, “On distribution of certain Wiener Functional”, Trans. of the AMS 65 (1949),1-13.
3. Todhunter I, “A History of the Mathematical Theory of Probability from the Time of Pascal to that of Laplace”, Cambridge University Press 1865. [Reprinted by Chelsea, New York 1949].
4. Donsker, “An invariance principle for certain probability limit theorems, four papers on probability”, Memories of the AMS 6 (1951), 1-12.
5. Laplace P S, “Theorize Analytique des Probabilities”, Courcier, Paris 1812. [Reprinted by Culture et Civilisation, Bruxelles 1962]
6. Knight F B, “Random Walk and a Sojourn density process of Brownian motion”, Trans. of the AMS 109 (1965), 56-86.
7. Gikhman I I and Skorokhod A V, “Introduction to the Theory of Random Processes”, W.B. Saunders, Philadelphia 1969.
8. Takacs L, “On the Classical Ruin Problems”, Journal of the American Statistical Associations 64 (1969), 889-906.
9. Takacs L, “Fluctuation Problems for Bernoulli Trials”, SIAM Review 21 (1979), 222-228.
10. Takacs L, “On the local time of the Brownian Motion”, The Annals of Applied Probability 5 (1995), 741-756.
11. Senthil Kumar P & Umamaheswari N, “Stochastic Model for the Box Cox Power Transformation and Estimation of the Ex-Gaussian Distribution of Cortisol Secretion of Breast Cancer due to Smoking People”, Antarctica Journal of Mathematics, Volume 11, 99-108, 2014.
12. Jonathan A. Levant and Richard A L Sturdevant, “Use of C-Terminal Octopeptide of Cholecystokikin in Cholecystography”, Department of Medicine, UCLA School of Medicine, Los Angeles, California.

Aryabhatta Journal of Mathematics & Informatics Vol. 6, No. 2, July-Dec, 2014 ISSN : 0975-7139
Journal Impact Factor (2013) : 0.489

# CONTRIBUTION OF RAMANUJAN IN MODERN MATHEMATICS

**Dr. Pushpander Kadian*, Dr. Parvesh Kumar**, Mr. Jai Bhagwan*****
*Asst. Professor, Department of Mathematics, Govt. College, Badli (Jhajjar)
**Asst. Professor, Department of Mathematics, Govt. College, Mokhra (Rohtak)
***Asst. Professor, Department of Mathematics, Govt. College, Chhachhrauli (Yamunanagar)
E-mail: pushpanderkadian@gmail.com*, parveshiitd@gmail.com**, jai_maths05@yahoo.co.in**

**ABSTRACT :**
***In this article, the contribution of Great Indian Mathematician Srinivasa Ramanujan in the development of modern mathematics is described. Starting with the early life, his journey from a small village in Kumbakonam to the Fellow of the Royal Society is described in detail. Particular attention is given to his extraordinary research work which formed a strong foundation in the development of analysis, number theory, infinite series and continued fractions.***

## INTRODUCTION

Srinivasa Ramanujan (22 December 1887 – 26 April 1920) was an Indian mathematician who, with almost no formal training in pure mathematics, made extraordinary contributions to mathematical analysis, number theory, infinite series, and continued fractions. Living in India with no access to the larger mathematical community, Ramanujan developed his own mathematical research in isolation. As a result, he rediscovered known theorems in addition to producing new ones. Ramanujan was said to be a natural genius by the English mathematician G. H. Hardy, in the same league as mathematicians such as Euler and Gauss. His introduction to formal mathematics began at age 10 when he was given books on advanced trigonometry written by S. L. Loney that he mastered by the age of 12. He even discovered theorems of his own, and re-discovered Euler's identity independently. Ramanujan received a scholarship to study at Government College in Kumbakonam, which was later withdrawn when he failed in his non-mathematical coursework. He joined another college to pursue independent mathematical research, working as a clerk in the Accountant-General's office at the Madras Port Trust Office to support himself. In 1912–1913, he sent samples of his theorems to three academics at the University of Cambridge. G. H. Hardy, recognizing the brilliance of his work, invited Ramanujan to visit and work with him at Cambridge. He became a Fellow of the Royal Society and a Fellow of Trinity College, Cambridge. Ramanujan died of illness, malnutrition, and possibly liver infection in 1920 at the age of 32.

During his short lifetime, Ramanujan independently compiled nearly 3900 results and nearly all his claims have now been proven correct. He stated results that were both original and highly unconventional, such as the Ramanujan prime and the Ramanujan theta function, which have inspired and inspiring a vast amount of further research. In December 2011, in recognition of his contribution to mathematics, the Government of India declared that Ramanujan's birthday (22 December) should be celebrated every year as **National Mathematics Day**, and also declared 2012 the **National Mathematics Year.**

## THE LIFE OF SRINIVASA RAMANUJAN

### EARLY LIFE

Ramanujan was born in Erode, Madras Presidency (now Tamil Nadu), at the residence of his maternal grandparents. His father, K. SrinivasaIyengar, worked as a clerk in a sari shop and his mother was a housewife and also sang at a local temple. In December 1889, Ramanujan had smallpox and recovered. He moved with his mother to her parents' house in Kanchipuram, near Madras (now Chennai). On 1 October 1892, Ramanujan was formally enrolled at the local school. Just before the age of 10, in November 1897, he passed his primary examinations in English,Tamil, Geography and Arithmetic. With his scores, he stood first in the district and entered Town Higher Secondary School where he encountered formal mathematics for the first time.

In 1903 when he was 16, Ramanujan obtained from a friend a library-loaned copy of the book titled *A Synopsis of Elementary Results in Pure and Applied Mathematics* by G. S. Carrandcontaining 5000 theorems. The book is generally acknowledged as a key element in awakening the genius of Ramanujan. He had independently developed and investigated the Bernoulli numbers and had calculated the Euler–Mascheroni constant up to 15 decimal places. When he graduated from Town Higher Secondary School in 1904, Ramanujan was awarded the K. RanganathaRao prize for Mathematics by the school's headmaster by introducing him as an outstanding student who deserved scores higher than the maximum possible marks. He received a scholarship to study at Government Arts College, Kumbakonam, However, Ramanujan was so intent on studying mathematics that he could not focus on any other subjectsand failed most of them, losing his scholarship in the process. Ramanujan failed his Fellow of Arts exam in December 1906 and left the college without a degree but continued his independent research in mathematics. At this point in his life, he faced extreme poverty and starvation.

## ATTENTION TOWARDS MATHEMATICS

On 14 July 1909, Ramanujan was married to a ten-year old bride, Janakiammal (21 March 1899 – 13 April 1994). After the marriage, Ramanujanfaced surgical operation and subsequent health problems. Recovering all these, he met deputy collector V. RamaswamyAiyer, who had recently founded the Indian Mathematical Society. Ramanujan, with the help of RamaswamyAiyer, had his work published in the *Journal of the Indian Mathematical Society*.

One of the first problems he posed in the journal was to find out the value of:

$$\sqrt{1+2\sqrt{1+3\sqrt{1+\cdots\ldots\ldots}}}$$

He waited for a solution to his problem in three issues, but failed to receive any. At the end, Ramanujan supplied the solution to the problem himself. He formulated an equation that could be used to solve the infinitely nested radicals' problem as

$$x+n+a=\sqrt{ax+(n+a)^2+x\sqrt{a(x+n)+(n+a)^2+(x+n)\sqrt{\ldots\ldots}}}$$

Using this equation, the answer to the question posed in the *Journal* was simply 3. Ramanujan wrote his first formal paper for the *Journal* on the properties of Bernoulli numbers. In his 17-page paper, "Some Properties of Bernoulli's Numbers", Ramanujan gave three proofs, two corollaries and three conjectures.

Ramanujan later wrote another paper and also continued to provide problems in the *Journal*. In early 1912, he got a temporary job in the Madras Accountant General's office, with a salary of 20 rupees per month which lasted for only a few weeks. Then, he applied for a position under the Chief Accountant of the Madras Port Trust. In a letter dated 9 February 1912, Ramanujan wrote: "Sir, I understand there is a clerkship vacant in your office, and I beg to apply for the same. I have passed the Matriculation Examination and studied up to the F.A. but was prevented from pursuing my studies further owing to several untoward circumstances. I have, however, been devoting all my time to Mathematics and developing the subject. I can say I am quite confident I can do justice to my work if I am appointed to the post. I therefore beg to request that you will be good enough to confer the appointment on me." Three weeks after he had applied, he was accepted as a Grade IV accounting clerk with a salary of 30 rupees per month. At his office, Ramanujan easily and quickly completed the assigned work and spent his spare time doing mathematical research.

## CONTACTING ENGLISH MATHEMATICIANS

On 16 January 1913, Ramanujan wrote to G. H. Hardy. Coming from an unknown mathematician, the nine pages of mathematics made Hardy initially view Ramanujan's manuscripts as a possible "fraud". Hardy recognized some of Ramanujan's formulae but others seemed scarcely possible to believe. One of the theorems Hardy found so incredible was found on the bottom of page three (valid for $0 < a < b + 1/2$):

$$\int_0^{\infty} \frac{1+x^2/(b+1)^2}{1+x^2/a^2} \times \frac{1+x^2/(b+2)^2}{1+x^2/(a+1)^2} \times \dots\dots\dots\, dx = \frac{\sqrt{\pi}}{2} \times \frac{\sqrt{(a+\frac{1}{2})}\sqrt{(b+1)}\sqrt{(b-a+\frac{1}{2})}}{\sqrt{(a)}\sqrt{(b+\frac{1}{2})}\sqrt{(b-a+1)}}$$

Hardy was also impressed by some of Ramanujan's other work relating to infinite series:

$$1 - 5\left(\frac{1}{2}\right)^3 + 9\left(\frac{1\times 3}{2\times 4}\right)^3 - 13\left(\frac{1\times 3\times 5}{2\times 4\times 6}\right)^3 + \cdots \dots\dots\dots = \frac{2}{\pi}$$

$$1 + 9\left(\frac{1}{4}\right)^4 + 17\left(\frac{1\times 5}{4\times 8}\right)^4 + 25\left(\frac{1\times 5\times 9}{4\times 8\times 12}\right)^4 + \cdots \dots\dots\dots = \frac{2^{\frac{3}{2}}}{\pi^{\frac{1}{2}}2\sqrt{\left(\frac{3}{4}\right)}}$$

The first result had already been determined by a mathematician named Bauer. The second one was new to Hardy, and was derived from a class of functions called a hypergeometric series which had first been researched by Leonhard Euler and Carl Friedrich Gauss. After he saw Ramanujan's theorems on continued fractions on the last page of the manuscripts, **Hardy commented that "they (theorems) defeated me completely; I had never seen anything in the least like them before"**. He figured that Ramanujan's theorems "must be true, because, if they were not true, no one would have the imagination to invent them". Hardy asked a colleague, J. E. Littlewood, to take a look at the papers. Littlewood was amazed by the mathematical genius of Ramanujan. After discussing the papers with Littlewood, Hardy commented that Ramanujan was "a mathematician of the highest quality, a man of altogether exceptional originality and power". One colleague, E. H. Neville, later commented that "not one (theorem) could have been set in the most advanced mathematical examination in the world".On 8 February 1913, Hardy wrote a letter to Ramanujan, expressing his interest for his work. After continued attempts both by Hardy and the renowned Mathematicians in India, Ramanujan agreed to visit England to work with Hardy and Littlewood.

## LIFE IN ENGLAND

Ramanujan reached London on 14 April and immediately began his work with Littlewood and Hardy. Hardy and Ramanujan began to take a look at Ramanujan's notebooks. Hardy had already received 120 theorems from Ramanujan in the first two letters, but there were many more results and theorems to be found in the notebooks. Hardy saw that some were wrong, others had already been discovered, while the rest were new breakthroughs. Ramanujan left a deep impression on Hardy and Littlewood. Littlewood commented, "I can believe that he's at least a Jacobi", while Hardy said he "can compare him only with Euler or Jacobi."

Ramanujan spent nearly five years in Cambridge collaborating with Hardy and Littlewood and published a part of his findings there. While in England, Hardy tried his best to fill the gaps in Ramanujan's education without interrupting him.Ramanujan was awarded a B.A. degree by research (this degree was later renamed Ph.D.) in March 1916 for his work on highly composite numbers, the first part of which was published as a paper in the *Proceedings of the London Mathematical Society*. The paper was over 50 pages with different properties of such numbers proven. Hardy remarked that this was one of the most unusual papers seen in mathematical research at that time and that Ramanujan showed extraordinary ingenuity in handling it. On 6 December 1917, he was elected to the London Mathematical Society. He became a Fellow of the Royal Society in 1918 having the proud of being one of the youngest Fellows in the history of the Royal Society. On 13 October 1918, he was elected a Fellow of Trinity College, Cambridge for his investigation in Elliptic functions and the Theory of Numbers.

## CONTRIBUTION OF RAMANUJAN IN MATHEMATICS

In mathematics, there is a distinction between having an insight and having a proof. Ramanujan's talent suggested a plethora of formulae that provided a strong base for further mathematical research. It is said that Ramanujan's discoveries are unusually rich and that there is often more to them than initially seen. As a by-product, new directions of research were opened up. Examples of the most interesting of these formulae include the intriguing infinite series for π, one of which is given below

$$\frac{1}{\pi} = \frac{2\sqrt{2}}{9801}\sum_{k=0}^{\infty}\frac{(4k)!\,(1103+26390k)}{(k!)^4 396^{4k}}.$$

This result is based on the negative fundamental discriminant $d = -4\times 58 = -232$ with class number $h(d) = 2$ (note that 5×7×13×58 = 26390 and that 9801=99×99; 396=4×99) and is related to the fact that

$$e^{\pi\sqrt{58}} = 396^4 - 104.000000177\ldots\ldots\ldots$$

Ramanujan's series for π converges extraordinarily rapidly (exponentially) and forms the basis of some of the fastest algorithms currently used to calculate π. Truncating the sum to the first term also gives the approximation $\frac{9801\sqrt{2}}{4412}$ for π, which is correct to six decimal places.

One of his remarkable capabilities was the rapid solution for problems. He was sharing a room with P. C. Mahalanobis who had a problem, "Imagine that you are on a street with houses marked 1 through n. There is a house in between (x) such that the sum of the house numbers to left of it equals the sum of the house numbers to its right. If n is between 50 and 500, what are n and x?" This is a bivariate problem with multiple solutions. Ramanujan thought about it and gave the answer with a twist: He gave a continued fraction. The unusual part was that it was the solution to the whole class of problems. Mahalanobis was astounded and asked how he did it. "It is simple. The minute I heard the problem, I knew that the answer was a continued fraction. Which continued fraction, I asked myself. Then the answer came to my mind," Ramanujan replied. His intuition also led him to derive some previously unknown identities, such as

$$\left[1+2\sum_{n=1}^{\infty}\frac{\cos(n\theta)}{\cosh(n\pi)}\right]^{-2} + \left[1+2\sum_{n=1}^{\infty}\frac{\cosh(n\theta)}{\cosh(n\pi)}\right]^{-2} = \frac{2^4\left(\frac{3}{4}\right)}{\pi}$$

for all $\theta$, where $(z)$ is the gamma function. Expanding into series of powers and equating coefficients of $\theta^0$,$\theta^4$ and $\theta^8$ gives some deep identities for the hyperbolic secant.

In 1918, Hardy and Ramanujan studied the partition function $P(n)$ extensively and gave a non-convergent asymptotic series that permits exact computation of the number of partitions of an integer. Hans Rademacher, in

1937, was able to refine their formula to find an exact convergent series solution to this problem. Ramanujan and Hardy's work in this area gave rise to a powerful new method for finding asymptotic formulae, called the circle method. He discovered mock theta functions in the last year of his life. For many years these functions were a mystery, but they are now known to be the holomorphic parts of harmonic weak Maass forms.

## THE RAMANUJAN CONJECTURE

Although there are numerous statements that could have borne the name *Ramanujan conjecture*, there is one statement that was very influential on later work. In particular, the connection of this conjecture with conjectures of André Weil in algebraic geometry opened up new areas of research. That Ramanujan conjecture is an assertion on the size of the Tau-function, which has as generating function the discriminant modular form $\Delta(q)$, a typical cusp form in the theory of modular forms. It was finally proven in 1973, as a consequence of Pierre Deligne's proof of the Weil conjectures. The reduction step involved is complicated. Deligne won a Fields Medal in 1978 for his work on Weil conjectures.

## RAMANUJAN'S NOTEBOOKS

Ramanujan recorded the bulk of his results in four notebooks of loose leaf paper. These results were mostly written up without any derivations. This is probably the origin of the misperception that Ramanujan was unable to prove his results and simply thought up the final result directly. Mathematician Bruce C. Berndt, in his review of these notebooks and Ramanujan's work, says that Ramanujan most certainly was able to make the proofs of most of his results, but chose not to do so.This style of working may have been for several reasons. Since paper was very expensive, Ramanujan would do most of his work and perhaps his proofs on slate, and then transfer only the results to paper. He was also quite likely to have been influenced by the style of G. S. Carr's book studied in his youth, which stated results without proofs. Finally, it is possible that Ramanujan considered his workings to be for his personal interest alone; and therefore recorded only the results. Professor Bruce C. Berndt of the University of Illinois, during a lecture at IIT Madras in May 2011, stated that over the last 40 years, almost all of Ramanujan's theorems have been proven right.

The first notebook has 351 pages with 16 somewhat organized chapters and some unorganized material. The second notebook has 256 pages in 21 chapters and 100 unorganised pages, with the third notebook containing 33 unorganised pages. The results in his notebooks inspired numerous papers by later mathematicians trying to prove what he had found. Hardy himself created papers exploring material from Ramanujan's work as did G. N. Watson, B. M. Wilson, and Bruce Berndt. A fourth notebook with 87 unorganised pages, the so-called "lost notebook", was rediscovered in 1976 by George Andrews. Notebooks 1, 2 and 3 were published as a two-volume set in 1957 by the Tata Institute of Fundamental Research (TIFR), Mumbai, India. This was a photocopy edition of the original manuscripts, in his own handwriting.

## RAMANUJAN-HARDY NUMBER 1729

The number 1729 is known as the Hardy–Ramanujan number after a famous anecdote of the British mathematician G. H. Hardy regarding a visit to the hospital to see Ramanujan. In Hardy's words: I remember once going to see him when he was ill at Putney. I had ridden in taxi cab number 1729 and remarked that the number seemed to me rather a dull one. "No," he replied, "it is a very interesting number; it is the smallest number expressible as the sum of two cubes in two different ways." The two different ways are

$1729 = 1^3 + 12^3 = 9^3 + 10^3$. This shows that how Ramanujan used to love with the numbers and to what extent he was a magician of number theory.

## CONCLUSIONS

Throughout the discussion, it is very clear that the genius like SrinivasaRamanujan is rarely born. He was a person with inbuilt talent with no comparisons with anyone else. In a very short span of 32 years of life, his contribution in the development of Mathematics not only surprised the Mathematicians of that time but also provided a path to future researchers. The Great Ramanujan will always be remembered for his extraordinary contribution in the field of mathematical analysis, number theory, infinite series and continued fractions.

## REFERENCES


1. Berndt, Bruce C.; Andrews, George E. (2005). *Ramanujan's Lost Notebook*. Part I. New York: Springer. ISBN 0-387-25529-X.
2. Berndt, Bruce C.; Andrews, George E. (2008). *Ramanujan's Lost Notebook*. Part II. New York: Springer. ISBN 978-0-387-77765-8.
3. Berndt, Bruce C.; Andrews, George E. (2012). *Ramanujan's Lost Notebook*. Part III. New York: Springer. ISBN 978-1-4614-3809-0.
4. Berndt, Bruce C.; Andrews, George E. (2013). *Ramanujan's Lost Notebook*. Part IV. New York: Springer. ISBN 978-1-4614-4080-2.
5. Henderson, Harry (1995). *Modern Mathematicians*. New York: Facts on File Inc. ISBN 0-8160-3235-1.
6. Kanigel, Robert (1991). *The Man Who Knew Infinity: a Life of the Genius Ramanujan*. New York: Charles Scribner's Sons. ISBN 0-684-19259-4.
7. Nita Shah & J.C. Prajapati (2012) “Calculus war between Newton and Leiniz” Aryabhatta J. of Maths & Info. Vol. 4 (2) pp. 175-184.
8. Editor (2012) Srinivasa Ramanujan “A tribute Aryabhatta J. of Maths & Info. Vol. 4 (2).

# CHARACTERISTICS OF PETERSON GRAPH IN CYCLE MATRIX WITH ALGEBRAIC GRAPH THEORY

**G.Nirmala*, M.Murugan****
*Head and Associate Professor , PG & Research department of Mathamatics,
K.N.Govt.Arts College for women (Autonomous), Thanjavur -7 (Tamilnadu)
**Research Scholar, Department of Mathematics, Periyar Maniyammai University, Vallam,Thanjavur.

**ABSTRACT :**
***In this work basic concepts of algebraic graph theory and its properties are reviewed and extended to the related concepts of cycle matrix in Peterson graph and its properties. The relation between cycle matrix and incidence matrix is introduced. Rank of the cycle matrix is also reviewed.***

## INTRODUCTION

Algebraic graph theory can be viewed as an extension to graph theory in which algebraic methods are applied to graph theory problems.

They are many physical systems whose performance depends not only on the characteristics of their relative location. As an example in a structure, if the properties of a member are altered, the overall behaviour of the structure will be changed. This indicates that the performance of a structure depends on the characteristic of its member. On the other hand if the location of a member is changed, the properties of the structure will also be different. Therefore the connectivity of the structure influences the performance of the entire structure. The graph model of a system provides a powerful means for this purpose.

This paper deals with Peterson graph and its properties with cycle matrix and finding the different cycles in a Peterson graph. Relating to the cycle matrix with incidence matrix, Rank of the cycle matrix in Peterson graph are dealt with.

**Definition 1**

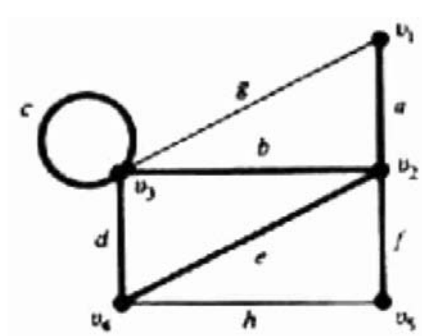
A walk of a graph G is alternating sequence of points and lines $V_0x_1$, $V_1x_2$, ……..$V_{n,}\ x_nV_{n.}$ beginning and ending with points such that each line $x_i$ is indicate with $V_{i-1}$ and $V_{i.}$

The walk joins $V_0$ and $V_n$ and its said to be $V_0$-$V_n$.

$V_0$ is said to be initial point.

$V_n$ is said to be terminal point.

**Example:**




(a) An open walk $v_1$ –$v_5$

**Definition 2**

The number of the lines in the walk is said to be the length of this walk.

**Definition 3**

For any graph G we define,

$\delta(G) = \min \{\deg v \,/\, v \in V(G)\}$

$\Delta(G) = \max \{\deg v \,/\, v \in V(G)\}$

If all the points of G have the same degree than $\delta(G)=\Delta(G)=r$ Then G is said to be a regular graph of degree r. Peterson graph is a regular graph of degree 3.

**Definition 4**

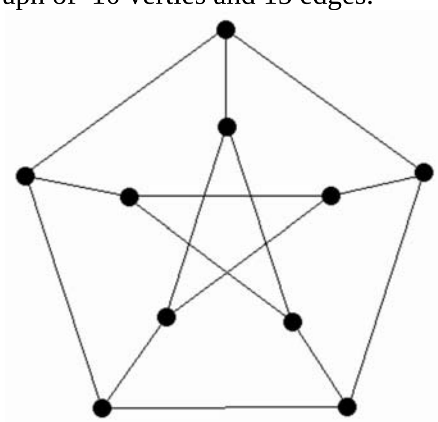
The degree of a point $V_i$ in a graph G is the number of lines incident with $V_i$ is denoted by $\deg(V_i)$.

A point v of degree 0 is said to be as isolated point.

A point v of degree 1 is said to be the pendent vertex.

**Definition 5**

Peterson graph is a 3 – regular graph of 10 verties and 15 edges.



**Definition 6**

Let the graph G have m edges and let q be the number of different cycles in G. The cycle matrix B is given by $B = [b_{ij}]_{qxm}$

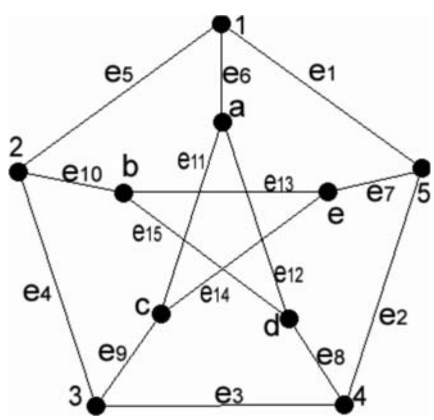
$$b_{ij} = \begin{cases} 1 & \text{if the ith cycle include jth edge} \\ 0 & \text{otherwise} \end{cases}$$

The cycle matrix of G is denoted by B(G).

**Cycle matrix of the Peterson graph**

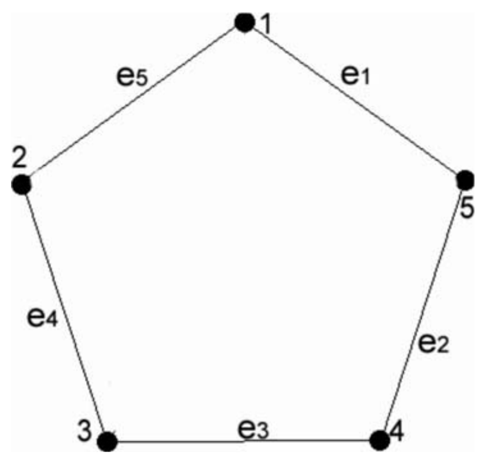
Whose order is 42 x 15.

Peterson graph has 42 different cycles



They are $Z_1, Z_2 \ldots\ldots\ldots\ldots Z_{42}$

| S.NO | Cycle length | Number of different cycles in a Peterson graph |
|---|---|---|
| 1 | 1 | 0 |
| 2 | 2 | 0 |
| 3 | 3 | 0 |
| 4 | 4 | 0 |
| 5 | 5 | 12 |
| 6 | 6 | 10 |
| 7 | 7 | 0 |
| 8 | 8 | 10 |
| 9 | 9 | 10 |
| 10 | 10 | 0 |
| 11 | 11 | 0 |
| 12 | 12 | 0 |
| 13 | 13 | 0 |
| 14 | 14 | 0 |
| 15 | 15 | 0 |
| | **Total** | **42** |




The different cycles are

$Z_1 = \{e_1,e_2,e_3,e_4,e_5\}$

$Z_2 = \{e_1,e_7,e_{13},e_{10},e_5\}$

$Z_3 = \{e_1,e_7,e_{14},e_{11},e_6\}$

$Z_4 = \{e_1,e_2,e_8,e_{12},e_6\}$

$Z_5 = \{e_{11},e_{12},e_{15},e_{13},e_{14}\}$

$Z_6 = \{e_6,e_{12},e_{15},e_{10},e_5\}$

$Z_7 = \{e_6,e_{11},e_9,e_4,e_5\}$

$Z_8 = \{e_7,e_{13},e_{15},e_8,e_2\}$

$Z_9 = \{e_7, e_{14}, e_9, e_3, e_2\}$

$Z_{10} = \{e_3, e_4, e_{10}, e_{15}, e_8\}$

$Z_{11} = \{e_3, e_9, e_{11}, e_{12}, e_8\}$

$Z_{12} = \{e_4, e_{10}, e_{13}, e_{14}, e_9\}$

$Z_{13} = \{e_1, e_2, e_8, e_{15}, e_{10}, e_5\}$

$Z_{14} = \{e_1, e_2, e_3, e_9, e_{11}, e_6\}$

$Z_{15} = \{e_1, e_7, e_{13}, e_{15}, e_{12}, e_6\}$

$Z_{16} = \{e_1, e_7, e_{14}, e_9, e_4, e_5\}$

$Z_{17} = \{e_2, e_3, e_4, e_{10}, e_{13}, e_7\}$

$Z_{18} = \{e_{11}, e_{12}, e_{15}, e_{10}, e_4, e_9\}$

$Z_{19} = \{e_{12}, e_8, e_3, e_4, e_5, e_6\}$

$Z_{20} = \{e_{13}, e_{14}, e_{11}, e_6, e_5, e_{10}\}$

$Z_{21} = \{e_{14}, e_{11}, e_{12}, e_8, e_2, e_7\}$

$Z_{22} = \{e_{15}, e_{13}, e_{14}, e_9, e_3, e_8\}$

$Z_{23} = \{e_1, e_2, e_8, e_{15}, e_{13}, e_{14}, e_{11}, e_6\}$

$Z_{24} = \{e_1, e_7, e_{13}, e_{15}, e_8, e_3, e_4, e_5\}$

$Z_{25} = \{e_1, e_7, e_{14}, e_9, e_3, e_8, e_{12}, e_6\}$

$Z_{26} = \{e_1, e_2, e_8, e_{12}, e_{11}, e_9, e_4, e_5\}$

$Z_{27} = \{e_1, e_2, e_3, e_9, e_{14}, e_{13}, e_{10}, e_5\}$

$Z_{28} = \{e_1, e_2, e_3, e_4, e_{10}, e_{15}, e_{12}, e_6\}$

$Z_{29} = \{e_{11}, e_{12}, e_{15}, e_{13}, e_7, e_1, e_3, e_9\}$

$Z_{30} = \{e_{12}, e_{15}, e_{13}, e_{14}, e_9, e_4, e_5, e_6\}$

$Z_{31} = \{e_{13}, e_{14}, e_{11}, e_{12}, e_8, e_3, e_4, e_{10}\}$

$Z_{32} = \{e_{14}, e_{11}, e_{12}, e_{15}, e_{10}, e_5, e_1, e_7\}$

$Z_{33} = \{e_1, e_7, e_{13}, e_{15}, e_{12}, e_{11}, e_9, e_4, e_5\}$

$Z_{34} = \{e_2, e_8, e_{15}, e_{13}, e_{14}, e_9, e_4, e_5, e_1\}$

$Z_{35} = \{e_3, e_9, e_{14}, e_{13}, e_{15}, e_{12}, e_6, e_1, e_2\}$

$Z_{36} = \{e_4, e_{10}, e_{13}, e_{14}, e_{11}, e_6, e_1, e_2, e_3\}$

$Z_{37} = \{e_5, e_6, e_{12}, e_{15}, e_{13}, e_7, e_2, e_3, e_4\}$

$Z_{38} = \{e_1, e_7, e_{14}, e_{11}, e_{12}, e_8, e_3, e_4, e_5\}$

$Z_{39} = \{e_2, e_8, e_{12}, e_{11}, e_{14}, e_{13}, e_{10}, e_5, e_1\}$

$Z_{40} = \{e_3, e_9, e_{11}, e_{12}, e_{15}, e_{10}, e_5, e_1, e_2\}$

$Z_{41} = \{e_4, e_{10}, e_{15}, e_{12}, e_{11}, e_{14}, e_7, e_2, e_3\}$

$Z_{42} = \{e_5, e_6, e_{11}, e_{14}, e_{13}, e_{15}, e_8, e_3, e_4\}$

The cycle matrix of the Peterson graph whose order is 42 x 15 is denoted by B(G).

**Cycle matrix B(G) of the Peterson graph :**

B(G)=

| | $e_1$ | $e_2$ | $e_3$ | $e_4$ | $e_5$ | $e_6$ | $e_7$ | $e_8$ | $e_9$ | $e_{10}$ | $E_{11}$ | $e_{12}$ | $e_{13}$ | $e_{14}$ | $e_{15}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $Z_1$ | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $Z_2$ | 1 | 0 | 0 | 0 | 1 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 1 | 0 | 0 |
| $Z_3$ | 1 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 1 | 0 |
| $Z_4$ | 1 | 1 | 0 | 0 | 0 | 1 | 0 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 0 |
| $Z_5$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 |
| $Z_6$ | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 1 | 0 | 1 | 0 | 0 | 1 |
| $Z_7$ | 0 | 0 | 0 | 1 | 1 | 1 | 0 | 0 | 1 | 0 | 1 | 0 | 0 | 0 | 0 |
| $Z_8$ | 0 | 1 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 1 | 0 | 1 |
| $Z_9$ | 0 | 1 | 1 | 0 | 0 | 0 | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 1 | 0 |
| $Z_{10}$ | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 1 |
| $Z_{11}$ | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 1 | 1 | 0 | 0 | 0 |
| $Z_{12}$ | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 1 | 1 | 0 |
| $Z_{13}$ | 1 | 1 | 0 | 0 | 1 | 0 | 0 | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 1 |
| $Z_{14}$ | 1 | 1 | 1 | 0 | 0 | 1 | 0 | 0 | 1 | 0 | 1 | 0 | 0 | 0 | 0 |
| $Z_{15}$ | 1 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 1 |
| $Z_{16}$ | 1 | 0 | 0 | 1 | 1 | 0 | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 1 | 0 |
| $Z_{17}$ | 0 | 1 | 1 | 1 | 0 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 1 | 0 | 0 |
| $Z_{18}$ | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 0 | 0 | 1 |
| $Z_{19}$ | 0 | 0 | 1 | 1 | 1 | 1 | 0 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 0 |
| $Z_{20}$ | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 1 | 1 | 0 | 1 | 1 | 0 |
| $Z_{21}$ | 0 | 1 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 1 | 1 | 0 | 1 | 0 |
| $Z_{22}$ | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 1 | 1 | 1 |
| $Z_{23}$ | 1 | 1 | 0 | 0 | 0 | 1 | 0 | 1 | 1 | 0 | 0 | 0 | 1 | 1 | 1 |
| $Z_{24}$ | 1 | 0 | 1 | 1 | 1 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 1 | 0 | 1 |
| $Z_{25}$ | 1 | 0 | 1 | 0 | 0 | 1 | 1 | 1 | 1 | 0 | 0 | 1 | 0 | 1 | 0 |
| $Z_{26}$ | 1 | 1 | 0 | 1 | 1 | 0 | 0 | 1 | 1 | 0 | 1 | 1 | 0 | 0 | 0 |
| $Z_{27}$ | 1 | 1 | 1 | 0 | 1 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 1 | 1 | 0 |
| $Z_{28}$ | 1 | 1 | 1 | 1 | 0 | 1 | 0 | 0 | 0 | 1 | 0 | 1 | 0 | 0 | 1 |
| $Z_{29}$ | 1 | 0 | 1 | 0 | 0 | 0 | 1 | 0 | 1 | 0 | 1 | 1 | 1 | 0 | 1 |
| $Z_{30}$ | 0 | 0 | 0 | 1 | 1 | 1 | 0 | 0 | 1 | 0 | 0 | 1 | 1 | 1 | 1 |
| $Z_{31}$ | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 1 | 0 | 1 | 1 | 1 | 1 | 1 | 1 |
| $Z_{32}$ | 1 | 0 | 0 | 0 | 1 | 0 | 1 | 0 | 0 | 1 | 1 | 1 | 0 | 1 | 0 |
| $Z_{33}$ | 1 | 0 | 0 | 1 | 1 | 0 | 1 | 0 | 1 | 0 | 1 | 1 | 1 | 0 | 1 |
| $Z_{34}$ | 1 | 1 | 0 | 1 | 1 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 1 | 1 | 1 |
| $Z_{35}$ | 1 | 1 | 1 | 0 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 1 | 1 | 1 | 1 |
| $Z_{36}$ | 1 | 1 | 1 | 1 | 0 | 1 | 0 | 0 | 0 | 1 | 1 | 0 | 1 | 1 | 0 |
| $Z_{37}$ | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 1 |
| $Z_{38}$ | 1 | 0 | 1 | 1 | 1 | 0 | 1 | 1 | 0 | 0 | 1 | 1 | 0 | 1 | 0 |
| $Z_{39}$ | 1 | 1 | 0 | 0 | 1 | 0 | 1 | 1 | 0 | 1 | 1 | 1 | 1 | 0 | 0 |
| $Z_{40}$ | 1 | 1 | 1 | 0 | 1 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 0 | 0 | 1 |
| $Z_{41}$ | 0 | 1 | 1 | 1 | 0 | 0 | 1 | 0 | 0 | 1 | 1 | 1 | 0 | 1 | 1 |
| $Z_{42}$ | 0 | 0 | 1 | 1 | 1 | 1 | 0 | 1 | 0 | 0 | 1 | 0 | 1 | 1 | 1 |

$42 \times 15$

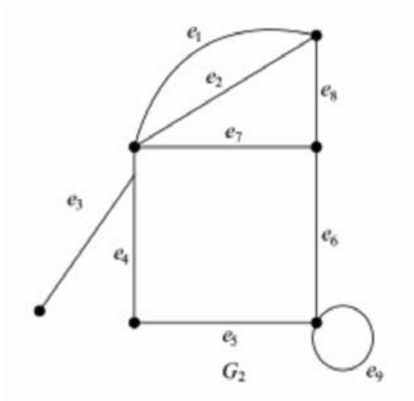
**Results**

(1) A column of all zeros corresponds to a non cycle edge that is an edge which does not belong to any cycle Peterson graph have no non cycle edge.

(2) Each row of B(G) is a cycle vector

(3) A cycle matrix has the property of representing a self loop and the corresponding row has a single one.

**Example**




The graph $G_2$ has 7 different cycles namely $Z_1 = \{e_1,e_2\}$ $Z_2 = \{e_2,e_7,e_8\}$ $Z_3 = \{e_1,e_7,e_8\}$ $Z_4 = \{e_4,e_5,e_6,e_7\}$ $Z_5 = \{e_2,e_4,e_5,e_6,e_8\}$ $Z_6 = \{e_1,e_4, e_5,e_6,e_8\}$ $Z_7 = \{e_9\}$ . Cycle Matrix of $G_2$ is given by,

$$B(G_2) = \begin{array}{c} \\ z_1 \\ z_2 \\ z_3 \\ z_4 \\ z_5 \\ z_6 \\ z_7 \end{array} \begin{array}{c} \begin{array}{ccccccccc} e_1 & e_2 & e_3 & e_4 & e_5 & e_6 & e_7 & e_8 & e_9 \end{array} \\ \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 & 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \end{array}.$$

In a cycle matrix sixth row has single one. The corresponding edge is self loop edge. From this it is clear. Peterson graph has no self loop.

(4) The number of ones in a row is equal to the number of edges in the corresponding cycle. Peterson graph have 5 cycles, 6 cycle, 8 cycles, 9 cycles.

(5) Permeation of any two rows (or) Columns in a cycle matrix corresponds to relabeling the cycles and the edges.

**Theorem 1**

If G is a graph without self-loops and with incidence matrix A and cycle matrix B whose columns are arranged using the same order of edges than every row of B is orthogonal to every row of A that is $AB^T = BA^T \equiv 0 \pmod 2$ where $A^T$ and $B^T$ are the transposes of A and B respectively.

**Theorem 2**

If A and B are matrices of order k x m and n x p respectively, than nullity AB $\leq$ nullity A + nullity B.

**Theorem 3**

If B is a cycle matrix of a connected graph G with n vertices and m edges then the Rank B = m-n+1

**Proof:**

Let A be the incidence matrix of the connected graph G.

B = Cycle matrix of G.

Then $AB^T \equiv 0 \pmod 2$

Using theorem 2,

We have Rank A + Rank $B^T \leq$ m so that

Rank A + Rank B ≤ m

Rank B ≤ m – Rank A

As Rank A = n-1

We get Rank B ≤ m- (n-1)

Rank B ≤ m-n+1 ------- (1)

But Rank B ≥ m-n+1 --------(2)

From (1) & (2)

Rank B = m-n+1

Hence the theorem

If G is a Peterson graph then B is a cycle matrix of the Peterson graph with 10 vertices and 15 edges

Then rank of the cycle matrix is rank B (G) = 15-10+1 By theorem (3)

Rank B (G) =6

**CONCLUSION:**

Peterson graph is a special kind of graph. Cycle matrix is used to solve physical problems. In addition, using the relation of incident matrix and cycle matrix. Peterson cycle matrix Rank can be found. Cycle matrix of the Peterson grph has similarity with the regular graph properties of the cycle matrix.

**References:**


1. H.J. Finck, on the chromatic Number of Graph and its complements “Theory of Graphs proceedings of the colloquium, Thihany Hungaru 1996,99 (113)
2. R. Balakrishnan and K. Renganathan A text Book of Graph theory, Springer 2000.
3. G.Nirmala and D.R Kirubaharan Uses of line graphs International journal of Humanities sciences PMU_Vol 2 -2011.
4. G. Nirmala and S. Priyadharshini – Fuzzy algebra and Groups, International Journal of Computers, Mathematical Sciences and applications – Vol 1- 2011
5. G. Nirmala and S. Priyadharshini – Q- Cut with P- Fuzzy algebra proceedings of International Conference Bishop Heber College Thiruchirappalli – 2011

6. G. Nirmala and S. Priyadharshini – P- Fuzzy algebra with Database to facilitate uncertainity Management – Aryabhatta International journal of Mathematics and informatics – 2012.
7. G. Nirmala and D.R. Kirubaharan Optimal Matching process for the manufacture of slined shaft by using network theoretical approach Aryabhatta, international journal of mathematics and informatics ISSN-0975-7139 May 2012.
8. G. Nirmala and D.R Kirubaharan – Real life problem on mad with RTS, International journal of scientific and Research Publications, Vol-1, May 2012
9. G. Nirmala and D.R. Kirubaharan – Applications of Network on real life problem applied Science periodical, vol.15, no-3, Aug 2013
10. Narasingh Deo, Graph theory with applications to Engineering and computer Science.
11. S. Arumugam and S. Ramachandran " Invitation to graph theory"
12. Douglas B. West, "Introduction to Graph theory"
13. G. Nirmala and M. Sheela (2013), "Domination in Fuzzy Digraphs." Aryabhatta J. of Maths & Info. Vol. 5 (2) pp 275-278.
14. K. Thilakam and A. Sumathi (2013), "Wiener Index of Chain Graphs" Aryabhatta J. of Maths & Info. Vol. 5 (2) pp 347-352.
15. A. Ramesh Kumar, R. Palani Kumar and S. Deepa (2014), "Lapacian matrix in algebraic Graph-Theory." Aryabhatta J. of Maths & Info. Vol. 6 (1) pp 65-76.

# STOCHASTIC MODEL TO FIND THE TESTOSTERONE THERAPY ON FUNCTIONAL CAPACITY IN CONGESTIVE HEART FAILURE PATIENTS USING UNIFORM DISTRIBUTION

**Dr. A. Muthaiyan* & R. J. Ramesh Kumar****
* Assistant Professor, PG Research & Department of Mathematics, Government Arts College, Ariyalur, Tamilnadu, India.
** Associate Professor, Department of Mathematics, Dhanalakshmi Srinivasan Engineering College, Perambalur, Tamilnadu, India.
Email: mathsphd1@gmail.com, rjramesh1980@gmail.com

**ABSTRACT :**
***Heart failure is a serious cardiovascular condition leading to life threatening events, poor prognosis, and degradation of quality of life. According to the present evidences suggesting association between low testosterone level and prediction of reduced exercise capacity as well as poor clinical outcome in patients with heart failure, we sought to determine if testosterone therapy improves clinical and cardiovascular conditions as well as quality of life status in patients with stable chronic heart failure.***
***In the random motion on Poincare half plane, the hyperbolic distance is analyzed and also in the case where returns to the starting point is admitted. The mean hyperbolic distance in all versions of the motion envisaged and it is used to find the role of Testosterone in improvement of functional capacity and quality of life in heart failure patients.***
**Key Words:** ***Testosterone Therapy, Congestive Heart Failure, Poincare Half Plane, Uniform Distribution.***
***2010 Mathematics Subject Classification: 60H99, 60G99***

## 1. INTRODUCTION

A noticeable evolution of therapeutic concepts has taken place with a variety of cardiac and hormonal drugs with the aim of improving patient's survival, preventing sudden death, and improving quality of life [8] & [9]. In a significant proportion of heart failure patients, testosterone deficiency as an anabolic hormonal defect has been proven and identified even in both genders [10]. This metabolic and endocrinological abnormality is frequently associated with impaired exercise tolerance and reduced cardiac function [4]. For this reason, combination therapy with booster cardiovascular drugs and testosterone replacement therapy might be very beneficial in heart failure patients. The physiological pathways involved in these therapeutic processes have been recently examined. First, elevated level of testosterone following replacement therapy is major indicator for increase of peak $VO_2$ in affected men with heart failure explaining improvement of exercise tolerance in these patients [11]. Furthermore, testosterone replacement therapy can reduce circulating levels of inflammatory mediators including tumor necrosis factor $\alpha$ $(TNF-\alpha)$ and interleukin $(IL)-1\beta$, as well as total cholesterol in patients with established simultaneous coronary artery disease and testosterone deficiency.

According to the present evidences suggesting association between low testosterone level and prediction of reduced exercise capacity as well as poor clinical outcome in patients with heart failure, we sought to determine if testosterone therapy improves clinical and cardiovascular conditions as well as quality of life status in patients with stable chronic heart failure.

A random motion on Poincare half plane is studied. The mean hyperbolic distance in all versions especially the motion at finite velocity on the surface of a three dimensional sphere is investigated. In this case we use

$$E(t)=\frac{e^{-\frac{\lambda t}{2}}}{2}\left[\left(e^{\frac{t}{2}\sqrt{\lambda^2-4c^2}}+e^{-\frac{t}{2}\sqrt{\lambda^2-4c^2}}\right)+\frac{\lambda}{\sqrt{\lambda^2-4c^2}}\left(e^{\frac{t}{2}\sqrt{\lambda^2-4c^2}}-e^{-\frac{t}{2}\sqrt{\lambda^2-4c^2}}\right)\right]$$

to find the testosterone therapy (TT) on functional capacity, cardiovascular parameters (CVP), and quality of life in patients with congestive heart failure (CHF).

## 2. NOTATIONS:

| | | |
|---|---|---|
| $TNF-\alpha$ | - | Tumor Necrosis Factor $\alpha$ |
| $IL$ | - | Interleukin |
| $TT$ | - | Testosterone Therapy |
| $CVP$ | - | Cardiovascular Parameters |
| $CHF$ | - | Congestive Heart Failure |
| $6MWD$ | - | 6 Minute Walk Distance |

## 3. INTRODUCTION:

Motion on hyperbolic spaces have been studied since the end of the Fifties and most of papers devoted to them deal with the so called hyperbolic Brownian motion [1] [6] & [7]. More recently also works concerning two dimensional random motions at finite velocity on planar hyperbolic spaces have been introduced and analyzed. While in the corresponds of motion are supposed to be independent, we present here a planar random motion with interacting components. Its counterpart on the unit sphere is also examined and discussed.

The space on which our motion develops is the Poincare upper half plane $H_2^+ = \{(x,y) : y > 0\}$ which is certainly the most popular model of the Lobachevsky hyperbolic space. In the space $H_2^+$ the distance between points is measured by means of the metric $ds^2 = \frac{dx^2+dy^2}{y^2}$ (1)

The propagation of light in a planar non homogeneous medium, according to the Fermat principle, must obey the law $\frac{\sin\alpha(y)}{c(x,y)} = \cos t$, Where $\alpha(y)$ is the angle formed by the tangent to the curve of propagation with the vertical at the point with ordinate $y$. In the case where the velocity $c(x,y) = y$ is independent from the direction, the light propagates on half circles as in $H_2^+$.

It is shown that the light propagates in a non homogeneous half plane $H_2^+$ with refracting index $n(x,y) = 1/y$ with rays having the structure of half circles. Scattered obstacles in the non homogeneous medium cause random deviations in the propagation of light and this lead to the random model analyzed below. The position of points in $H_2^+$ can be given either in terms of Cartesian coordinates $(x,y)$ or by means of the hyperbolic coordinates $(\eta,\alpha)$. In particular, $\eta$ represents the hyperbolic distance of a point of $H_2^+$ from the origin $O$ which has Cartesian coordinates $(0,1)$. We recall than $\eta$ is evaluated by means of (1) on the arc of a circumference with center located on the $x$ axis and joining $(x,y)$ with the origin $O$. The upper half circumference centered on the $x$ axis represents the geodesic lines of the space $H_2^+$ and play the same role of the straight lines in the Euclidean plane [2] & [3].

The angle $\alpha$ represents the slope of the tangent in $O$ to the half circumference passing through $(x,y)$. The formulas which relate the polar hyperbolic coordinates $(\eta,\alpha)$ to the Cartesian coordinates $(x,y)$ are

$$\begin{cases} x = \frac{\sinh\eta\cos\alpha}{\cosh\eta - \sinh\eta\sin\alpha} & \eta > 0 \\ y = \frac{1}{\cosh\eta - \sinh\eta\sin\alpha} & -\frac{\pi}{2} < \alpha < \frac{\pi}{2} \end{cases} \tag{2}$$

for each value of $\alpha$ the relevant geodesic curve is represented by the half circumference with equation

$$(x - \tan\alpha)^2 + y^2 = \frac{1}{cos^2\alpha} \tag{3}$$

for $\alpha = \frac{\pi}{2}$ we get from (3) the positive $y$ axis which also is a geodesic curve of $H_2^+$. From (2) it is easy to obtain the following expression of the hyperbolic distance $\eta$ of $(x,y)$ from the origin $O$:

$$\cosh\eta = \frac{x^2+y^2+1}{2y} \tag{4}$$

from (4) it can be seen that all the points having hyperbolic distance $\eta$ from the origin $O$ from a Euclidean circumference with center at $(0, cosh\,\eta)$ and radius $\sinh\eta$. The expression of the hyperbolic distance between two arbitrary points $(x_1, y_1)$ and $(x_2, y_2)$ is instead given by $\cosh\eta = \frac{(x_1-x_2)^2+y_1^2+y_2^2}{2y_1y_2}$ (5)

In fact, by considering the hyperbolic triangle with vertices at $(0,1), (x_1, y_1)$ and $(x_2, y_2)$, by means of the Carnot hyperbolic formula it is simple to show that the distance $\eta$ between $(x_1, y_1)$ and $(x_2, y_2)$ is given by $\cosh\eta = \cosh\eta_1 \cosh\eta_2 - \sinh\eta_1 \sinh\eta_2 \cos(\alpha_1 - \alpha_2)$ (6)
where $(\eta_1, \alpha_1)$ and $(\eta_2, \alpha_2)$ are the hyperbolic coordinates of $(x_1, y_1)$ and $(x_2, y_2)$ respectively. From (3) we obtain that $\tan\alpha_i = \frac{x_i^2+y_i^2-1}{2x_1}$ for $i = 1, 2, ...$ (7)
and in view of (4) and (7), after some calculations, formula (5) appears. Instead of the elementary arguments of the proof above we can also invoke the group theory which reduces $(x_1, y_1)$ to $(0, 1)$.

If $\alpha_1 - \alpha_2 = \frac{\pi}{2}$ the hyperbolic Carnot formula (6) reduces to the hyperbolic Pythagorean theorem $\cosh\eta = \cosh\eta_1 \cosh\eta_2$, which plays an important role in the present paper.

The motion considered here is the non Euclidean counterpart of the planar motion with orthogonal deviations studied. The main object of the investigation is the hyperbolic distance of the moving point form the origin. We are able to give explicit expressions for its mean value, also under the condition that the number of changes of direction is known. In the case of motion in $H_2^+$ with independent components an explicit expression for the distribution of the hyperbolic distance $\eta$ has been obtained. Here, however, the components of motion are dependent and this excludes any possibility of finding the distribution of the hyperbolic distance $\eta(t)$. We obtain the following explicit formula for the mean value of the hyperbolic distance which reads $E\{\cosh\eta(t)\} = e^{\frac{-\lambda t}{2}}\left\{\cosh\frac{t}{2}\sqrt{\lambda^2+4c^2} + \frac{\lambda}{\sqrt{\lambda^2+4c^2}}\sinh\frac{t}{2}\sqrt{\lambda^2+4c^2}\right\} = Ee^{T(t)}$, where $T(t)$ is a telegraph process with parameters $\frac{\lambda}{2}$ and $c$. The telegraph process represents the random of a particle moving with constant velocity and changing direction at Poisson paced times.

The next section is devoted to motions on the Poincare half plane where the return to the starting point is admitted and occurs at the instants of changes of direction. The mean distance from the origin of these jumping back motions is obtained explicitly by exploiting their relationship with the motion without jumps. In the case where the return to the starting point occurs at the Poisson event $T_1$, the mean value of the hyperbolic distance $\eta_1(t)$ reads $E\{\cosh\eta_1(t) \,|\, N(t) \geq 1\} = \frac{\lambda}{\sqrt{\lambda^2+4c^2}} \frac{\sinh\frac{t}{2}\sqrt{\lambda^2+4c^2}}{\sinh\frac{t}{2}}$. The last section considers the motion at finite velocity, with orthogonal deviations at Poisson times, on the unit radius sphere. The main results concern the mean value $E\{\cos d(P_0P_1)\}$, where $d(P_0P_1)$ is the distance of the current point $P_1$ from the starting position $P_0$. We take profit of the analogy of the spherical motion with its counterpart on the Poincare half plane to discuss the different situations due to the finiteness of the space where the random motion develops.

## 4. MOTIONS WITH JUMPS BACKWARDS TO THE STARTING POINT:

We here examine the planar motion dealt with so far assuming now that, at the instants of changes of direction, the particle can return to the starting point and commence its motion from scratch. The new motion and the original one are governed by the same Poisson process so that changes of direction occur simultaneously in the original as well as in the new motion starting a fresh from the origin. This implies that the arcs of the original sample path and those of the new trajectories have the same hyperbolic length. However, the angles formed by successive segments differ in order to make the hyperbolic Pythagorean Theorem applicable to the trajectories of the new motion.
In order to make our description clearer, we consider the case where, in the interval $(0, t), N(t) = n$ Poisson events $(n \geq 1)$ occur and we assume that the jump to the origin happens at the first change of direction i.e., at the instant

$t_1$. The instants of changes of direction for the new motion are $t_k^{'} = t_{k+1} - t_1$, where $k = 0, 1, \dots, n$ with $t_0^{'} = 0$ and $t_n^{'} = t - t_1$and the hyperbolic lengths of the corresponding arcs are $c\left(t_k^{'} - t_{k-1}^{'}\right) = c(t_{k+1} - t_k)$

Therefore, at the instant $t$, the hyperbolic distance from the origin of the particle performing the motion which has jumped back to $O$ at time $t_1$is

$$\prod_{k=1}^{n} \cosh c\left(t_k^{'} - t_{k-1}^{'}\right) = \prod_{k=1}^{n} \cosh c(t_{k+1} - t_k) = \prod_{k=2}^{n+1} \cosh c\left(t_k - t_{k-1}^{'}\right) \quad (8)$$

where $0 = t_0^{'} < t_1^{'} < \cdots < t_n^{'} = t - t_1$ and $t_{k+1} = t_k^{'} + t_1$. Formula (8) shows the first step has been deleted. However, the distance between the position $P_t$ and the origin $O$ of the moving particle which jumped back to $O$ after having reached the position $P_1$, is different from the distance of $P_t$ from $P_1$ since the angle between successive steps must be readjusted in order to apply the hyperbolic Pythagorean Theorem. If we denote by $T_1$ the random instant of the return to the starting point (occurring at the first Poisson event), we have that

$$\begin{aligned} E\left\{\cosh \eta_1(t) I_{\{N(t)\geq 1\}} \middle| N(t) = n\right\} &= E\left\{\cosh \eta(t - T_1) I_{\{T_1 \leq t\}} \middle| N(t) = n\right\} \\ &= \int_0^t E\left\{\cosh \eta(t - T_1) I_{\{T_1 \in dt_1\}} \middle| N(t) = n\right\} dt_1 \\ &= \int_0^t E\left\{\cosh \eta(t - T_1) \middle| T_1 = t_1, N(t) = n\right\} Pr\left\{T_1 \in dt_1 \middle| N(t) = n\right\} dt_1 \end{aligned} \quad (9)$$

By observing that $E\left\{\cosh \eta(t - T_1) \middle| T_1 = t_1, N(t) = n\right\} = E\left\{\cosh \eta(t - t_1) \middle| N(t) = n - 1\right\}$

$$= \frac{(n-1)!}{(t-t_1)^{n-1}} I_{n-1}(t - t_1)$$

and that $Pr\left\{T_1 \in dt_1 \middle| N(t) = n\right\} = \frac{n!}{t^n} \frac{(t-t_1)^{n-1}}{(n-1)!} dt_1$, with $0 < t_1 < t$, formula (9) becomes

$$E\left\{\cosh \eta_1(t) I_{\{N(t)\geq 1\}} \middle| N(t) = n\right\} = \frac{n!}{t^n} \int_0^t I_{n-1}(t - t_1) dt_1 \quad (10)$$

From (10) we have that the mean hyperbolic distance for the particle which returns to $O$ at time $T_1$, has the form:

$$E\left\{\cosh \eta_1(t) \middle| N(t) \geq 1\right\} = \frac{e^{-\lambda t}}{Pr\{N(t)\geq 1\}} \sum_{n=1}^{\infty} \lambda^n \int_0^t I_{n-1}(t - t_1) dt_1 = \frac{\lambda e^{-\lambda t}}{Pr\{N(t)\geq 1\}} \int_0^t e^{\lambda(t-t_1)} E(t - t_1) dt_1$$

We give here, a general expression for the mean value of the hyperbolic distance of a particle which returns to the origin for the last time at the k^th Poisson event $T_k$. We shall denote the distance by the following equivalent notation $\eta(t - T_k) = \eta_k(t)$ where the first expression underlines that the particle starts from scratch at time $T_k$ and then moves away for the remaining interval of length $t - T_k$. In the general case we have the result stated in the next theorem.

**Theorem : 5.1**

If $N(t) \geq k$, then the mean value of the hyperbolic distance $\eta_k$ is equal to

$$\begin{aligned} E\left\{\cosh \eta_k(t) \middle| N(t) \geq k\right\} &= \frac{\lambda^k e^{-\lambda t}}{Pr\{N(t)\geq k\}} \int_0^t e^{\lambda(t-t_k)} E(t - t_k) dt_k \\ &= \frac{\lambda^k e^{-\lambda t}}{Pr\{N(t)\geq k\}(k-1)!} \int_0^t e^{\lambda(t-t_k)} t_k^{k-1} E(t - t_k) dt_k \end{aligned} \quad (11)$$

$$\text{where } E(t) = e^{\frac{-\lambda t}{2}} \left\{\cosh \frac{t\sqrt{\lambda^2+4c^2}}{2} + \frac{\lambda}{\sqrt{\lambda^2+4c^2}} \sinh \frac{t\sqrt{\lambda^2+4c^2}}{2}\right\}$$

**Proof:**

We start by observing that

$$\begin{aligned} E\left\{\cosh \eta_k(t) \middle| N(t) \geq k\right\} &= \sum_{n=k}^{\infty} E\left\{\cosh \eta_k(t) I_{\{N(t)=n\}} \middle| N(t) \geq k\right\} \\ &= \sum_{n=k}^{\infty} E\left\{\cosh \eta_k(t) I_{\{N(t)\geq k\}} \middle| N(t) = n\right\} \frac{Pr(N(t)=n)}{Pr(N(t)\geq k)} \\ &= \sum_{n=k}^{\infty} E\left\{\cosh \eta_k(t) I_{\{N(t)\geq k\}} \middle| N(t) = n\right\} Pr\left\{N(t) = n \middle| N(t) \geq k\right\} \end{aligned} \quad (12)$$

Since $T_k = inf\{t: N(t) = k\}$, the conditional mean value inside the sum can be developed as follows

$$\begin{aligned} E\left\{\cosh \eta_k(t) I_{\{N(t)\geq k\}} \middle| N(t) = n\right\} &= E\left\{\cosh \eta(t - T_k) I_{\{T_k \leq t\}} \middle| N(t) = n\right\} \\ &= \int_0^t E\left\{\cosh \eta(t - T_k) I_{\{T_k \in dt_k\}} \middle| N(t) = n\right\} dt_k \\ &= \int_0^t E\left\{\cosh \eta(t - T_k) \middle| T_k = t_k, N(t) = n\right\} Pr\left\{T_k \in dt_k \middle| N(t) = n\right\} dt_k \end{aligned}$$

Now we consider $E_n(t) = \frac{n!}{t^n} I_n(t)$ (13)

Using the above condition (13), we have that

$$E\{\cosh\eta(t-T_k)\,|\,T_k=t_k, N(t)=n\}=E\{\cosh\eta(t-t_k)\,|\,N(t-t_k)=n-k\}=\frac{(n-k)!}{(t-t_k)^{n-k}}I_{n-k}(t-t_k)$$

and on the base of well known properties of the Poisson process we have that

$$Pr\{T_k\in dt_k\,|\,N(t)=n\}=\frac{n!}{t^n}\frac{(t-t_k)^{n-k}}{(n-k)!}\frac{t_k^{k-1}}{(k-1)!}dt_k,\text{ where } 0<t_k<t.$$

In conclusion we have that $E\{\cosh\eta_k(t)I_{\{N(t)\geq k\}}\,|\,N(t)=n\}=\frac{n!}{t^n}\frac{1}{(k-1)!}\int_0^t t_k^{k-1}I_{n-k}(t-t_k)dt_k$

and, from this and (12), it follows that

$$E\{\cosh\eta_k(t)\,|\,N(t)\geq k\}=\sum_{n=k}^{\infty}\frac{n!}{t^n}\frac{1}{(k-1)!}\int_0^t t_k^{k-1}I_{n-k}(t-t_k)dt_k\frac{e^{-\lambda t}(\lambda t)^n}{n!Pr\{N(t)\geq k\}}$$

$$=\frac{e^{-\lambda t}\lambda^k}{Pr\{N(t)\geq k\}(n-1)!}\int_0^t e^{\lambda(t-t_k)}t_k^{k-1}E(t-t_k)dt_k$$

Finally, in view of Cauchy formula of multiple integrals, we obtain that

$$\frac{e^{-\lambda t}\lambda^k}{Pr\{N(t)\geq k\}(n-1)!}\int_0^t e^{\lambda(t-t_k)}t_k^{k-1}E(t-t_k)dt_k=\frac{e^{-\lambda t}\lambda^k}{Pr\{N(t)\geq k\}}\int_0^t dt_1\ldots\int_{t_{k-1}}^t e^{\lambda(t-t_k)}E(t-t_k)dt_k$$

**Theorem : 5.2**

The mean of the hyperbolic distance of the moving particle returning to the origin at the $k^{th}$ change of direction is

$$E\{\cosh\eta_k(t)\,|\,N(t)\geq k\}=\frac{\lambda^k e^{-\lambda t}}{\sqrt{\lambda^2+4c^2}Pr\{N(t)\geq k\}}\left\{\frac{e^{At}}{A^{k-1}}-\frac{e^{Bt}}{B^{k-1}}+\sum_{i=1}^{k-1}\left(\frac{1}{B^i}-\frac{1}{A^i}\right)\frac{t^{k-i-1}}{(k-i-1)!}\right\} \tag{14}$$

where $A=\frac{1}{2}\left(\lambda+\sqrt{\lambda^2+4c^2}\right)$ and $B=\frac{1}{2}\left(\lambda-\sqrt{\lambda^2+4c^2}\right)$ for $k=1$, the sum in (14) is intended to be zero.

**Proof:**

We can prove (14) by applying both formulas in (11). We start our proof by employing the first one:

$$E\{\cosh\eta_k(t)\,|\,N(t)\geq k\}=\frac{\lambda^k e^{-\lambda t}}{Pr\{N(t)\geq k\}}\int_0^t dt_1\ldots\int_{t_{k-1}}^t e^{\lambda(t-t_k)}E(t-t_k)dt_k \tag{15}$$

Now consider $E(t)=\frac{e^{-\lambda t/2}}{2}\left\{\frac{\lambda+\sqrt{\lambda^2+4c^2}}{\sqrt{\lambda^2+4c^2}}e^{(t/2)\sqrt{\lambda^2+4c^2}}+\frac{\sqrt{\lambda^2+4c^2}-\lambda}{\sqrt{\lambda^2+4c^2}}e^{-(t/2)\sqrt{\lambda^2+4c^2}}\right\}$ (16)

Therefore in view of (16), formula (15) becomes

$$E\{\cosh\eta_k(t)\,|\,N(t)\geq k\}=\frac{\lambda^k e^{-\lambda t}}{Pr\{N(t)\geq k\}}\int_0^t dt_1\ldots\int_{t_{k-1}}^t e^{\lambda(t-t_k)}\left\{\frac{e^{\lambda(t-t_k)/2}}{2}\left[\frac{\lambda+\sqrt{\lambda^2+4c^2}}{\sqrt{\lambda^2+4c^2}}e^{(t-t_k/2)\sqrt{\lambda^2+4c^2}}+\frac{\sqrt{\lambda^2+4c^2}-\lambda}{\sqrt{\lambda^2+4c^2}}e^{-(t-t_k/2)\sqrt{\lambda^2+4c^2}}\right]E(t-t_k)\right\}dt_k$$

By introducing $A$ and $B$ as in (14), we can easily determine the $k$ fold integral

$$E\{\cosh\eta_k(t)\,|\,N(t)\geq k\}=\frac{\lambda^k e^{-\lambda t}}{\sqrt{\lambda^2+4c^2}Pr\{N(t)\geq k\}}\int_0^t dt_1\ldots\int_{t_{k-1}}^t\{Ae^{A(t-t_k)}-Be^{B(t-t_k)}\}dt_k$$

$$=\frac{\lambda^k e^{-\lambda t}}{\sqrt{\lambda^2+4c^2}Pr\{N(t)\geq k\}}\int_0^t dt_1\ldots\int_{t_{k-2}}^t\{e^{A(t-t_{k-1})}-e^{B(t-t_{k-1})}\}dt_{k-1}$$

$$=\frac{\lambda^k e^{-\lambda t}}{\sqrt{\lambda^2+4c^2}Pr\{N(t)\geq k\}}\int_0^t dt_1\ldots\int_{t_{k-3}}^t\left\{\frac{e^{A(t-t_{k-2})}}{A}-\frac{e^{B(t-t_{k-2})}}{B}+\frac{1}{B}-\frac{1}{A}\right\}dt_{k-2}$$

At the $j^{th}$ stage the integral becomes

$$E\{\cosh\eta_k(t)\,|\,N(t)\geq k\}=$$

$$\frac{\lambda^k e^{-\lambda t}}{\sqrt{\lambda^2+4c^2}Pr\{N(t)\geq k\}}\int_0^t dt_1\ldots\int_{t_{k-j-1}}^t\left\{\frac{e^{A(t-t_{k-j})}}{A^{j-1}}-\frac{e^{B(t-t_{k-j})}}{B^{j-1}}+\sum_{i=1}^{j-1}\left(\frac{1}{B^i}-\frac{1}{A^i}\right)\frac{(t-t_{k-j})^{j-i-1}}{(j-i-1)!}\right\}$$

At the $(k-1)^{th}$stage the integral becomes

$$E\{\cosh\eta_k(t)\,|\,N(t)\geq k\}=\frac{\lambda^k e^{-\lambda t}}{\sqrt{\lambda^2+4c^2}Pr\{N(t)\geq k\}}\int_0^t dt_1\left\{\frac{e^{A(t-t_1)}}{A^{k-2}}-\frac{e^{B(t-t_1)}}{B^{k-2}}+\sum_{i=1}^{k-2}\left(\frac{1}{B^i}-\frac{1}{A^i}\right)\frac{(t-t_1)^{k-2}}{(k-i-2)!}\right\}$$

At the $k^{th}$ integration we obtain formula (14). By means of the second formula in (11) and by repeated integrations by parts we can obtain again result (14).

## 5. MOTION AT FINITE VELOCITY ON THE SURFACE OF A THREE DIMENSIONAL SPHERE:

Let $P_0$ be a point on the equator of a three dimensional sphere. Let us assume that the particle starts moves from $P_0$ along the equator in one of the two possible directions (clockwise or counter clockwise) with velocity $c$. At the first Poisson event (occurring at time $T_1$) it starts moving on the meridian joining the north pole $P_N$ with the position reached at time $T_1$(denoted by $P_1$) along one of the two possible directions. At the second Poisson event the particle is located at $P_2$ and its distance from the starting point $P_0$ is the length of the hypotenuse of a right spherical triangle with cathetus $P_0P_1$ and $P_1P_2$; the hypotenuse belongs to the equatorial circumference through $P_0$ and $P_2$.

Now the particle continues its motion (in one of the two possible directions0 along the equatorial circumference orthogonal to the hypotenuse through $P_0$ and $P_2$ until the third Poisson event occurs. In general, the distance $d(P_0P_1)$ of the point $P_1$ from the origin $P_0$ is the length of the shortest arc of the equatorial circumference through $P_0$ and $P_1$and therefore it takes values in the interval $[0,\pi]$. Counter clockwise motions cover the arcs in $\left[-\pi,0\right]$ so that the distance is also defined in $[0,\pi]$ or in $[-\pi/2\,,\pi/2]$ with s shift that avoids negative values for the cosine.

By means of the spherical Pythagorean relationship we have that the Euclidean distance $d(P_0P_2)$ satisfies $\cos d(P_0P_2) = \cos d(P_0P_1)\cos d(P_1P_2)$ and, after three displacements,

$$\cos d(P_0P_3) = \cos d(P_0P_2)\cos d(P_2P_3) = \cos d(P_0P_1)\cos d(P_1P_2)\cos d(P_2P_3)$$

After $n$ displacement the position $P_t$ on the sphere at time $t$ is given by

$$\cos d(P_0P_t) = \prod_{k=1}^{n}\cos d(P_kP_{k-1})\cos d(P_nP_t)$$

Since $d(P_kP_{k-1})$ is represented by the amplitude of the arc run in the interval $(t_k,t_{k-1})$, it results

$$d(P_kP_{k-1}) = c(t_k,t_{k-1})$$

The mean value $E\{\cos d(P_0P_t)\,|\,N(t)=n\}$ is given by

$$E_n(t) = E\{\cos d(P_0P_t)\,|\,N(t)=n\} = \frac{n!}{t^n}\int_0^t dt_1\int_{t_1}^t dt_2 \dots \int_{t_{n-1}}^t dt_n \prod_{k=1}^{n+1}\cos c(\,t_k,t_{k-1}) = \frac{n!}{t^n}H_n(t)$$

Where $t_0 = 0, t_{n+1} = t$ and $H_n(t) = \int_0^t dt_1\int_{t_1}^t dt_2 \dots \int_{t_{n-1}}^t dt_n \prod_{k=1}^{n+1}\cos c(\,t_k,t_{k-1})$

The mean value $E\{\cos d(P_0P_t)\}$ is given by

$$E(t) = E\{\cos d(P_0P_t)\} = \sum_{n=0}^{\infty} E\{\cos d(P_0P_t)\,|\,N(t)=n\}Pr\,\{N(t)=n\} = e^{-\lambda t}\sum_{n=0}^{\infty}\lambda^n H_n(t)$$

By steps similar to those of the hyperbolic case we have that $H_n(t), t\geq 0$, satisfies the difference differential equation $\frac{d^2}{dt^2}H_n = \frac{d}{dt}H_{n-1} - c^2H_n$, where $H_0(t) = \cos ct$ and therefore we can prove the following:

**Theorem: 6.1**

The mean value $E(t) = E\{\cos d(P_0P_t)\}$ satisfies $\frac{d^2}{dt^2}E = -\lambda\frac{d}{dt}E - c^2E$ (17)

with initial conditions $\begin{cases} E(0) = 1 \\ \frac{d}{dt}E(t)\big|_{t=0} = 0 \end{cases}$ (18)

and has the form $E(t) = \begin{cases} e^{-\frac{\lambda t}{2}}\left[\cosh\frac{t}{2}\sqrt{\lambda^2-4c^2} + \frac{\lambda}{\sqrt{\lambda^2-4c^2}}\sinh\frac{t}{2}\sqrt{\lambda^2-4c^2}\right] & 0<2c<\lambda \\ e^{-\frac{\lambda t}{2}}\left[1+\frac{\lambda t}{2}\right] & \lambda = 2c>0 \\ e^{-\frac{\lambda t}{2}}\left[\cosh\frac{t}{2}\sqrt{4c^2-\lambda^2} + \frac{\lambda}{\sqrt{4c^2-\lambda^2}}\sinh\frac{t}{2}\sqrt{4c^2-\lambda^2}\right] & 2c>\lambda>0 \end{cases}$ (19)

**Proof:**

The solution to the problem (17) and (18) is given by

$$E(t) = \frac{e^{-\frac{\lambda t}{2}}}{2}\left[\left(e^{\frac{t}{2}\sqrt{\lambda^2-4c^2}} + e^{-\frac{t}{2}\sqrt{\lambda^2-4c^2}}\right) + \frac{\lambda}{\sqrt{\lambda^2-4c^2}}\left(e^{\frac{t}{2}\sqrt{\lambda^2-4c^2}} - e^{-\frac{t}{2}\sqrt{\lambda^2-4c^2}}\right)\right] \quad (20)$$

so that (19) emerges.

For large values of $\lambda$, the first expression furnishes $E(t)\sim 1$ and therefore the particle hardly leaves the starting point. If $\frac{\lambda}{2} < c$, the mean value exhibits an oscillating behavior; in particular , the oscillations decrease as time goes on, and this means that the particle moves further and further reaching in the limit the poles of the sphere.

## 6. EXAMPLE:

A total of 50 male patients who suffered from congestive heart failure were recruited in a double blind, placebo controlled trial and randomized to receive an intramuscular (gluteal) long acting androgen injection ($1ml$ of testosterone enanthate $250mg/ml$) once every four weeks for 12 weeks or receive intramuscular injections of saline ($1ml$ of $0.9\%$ $wt/vol\ NaCl$) with the same protocol. Comparing baseline variables and clinical parameters across the two groups who received testosterone or placebo did not show any significant difference, except for $6MWD$ that was higher in the testosterone group. During the 12 week study period, no significant differences were revealed in the trend of the changes in hemodynamic parameters including systolic and diastolic blood pressures as well as heart rate between the two groups. Also, the changes in body weight were comparable between the groups, while, unlike the group received placebo, those who received testosterone had a significant increasing trend in $6MWD$ parameter within the study period ($6MWD$ at baseline was $407.44 \pm 100.23m$ and after 12 weeks of follow up reached $491.65 \pm 112.88m$ following testosterone therapy, $P = 0.019$). According to post hoc analysis, the mean 6 walk distance parameter was improved at three time points of 4 weeks, 8 weeks, and 12 weeks after intervention compared with baseline; however no differences were found in this parameter at three post intervention time points. The discrepancy in the trends of changes in $6MWD$ between study groups remained significant after adjusting baseline variables ($mean\ square = 243.262, F - index = 4.402$ and $P = 0.045$) [8-10] & [12-13].

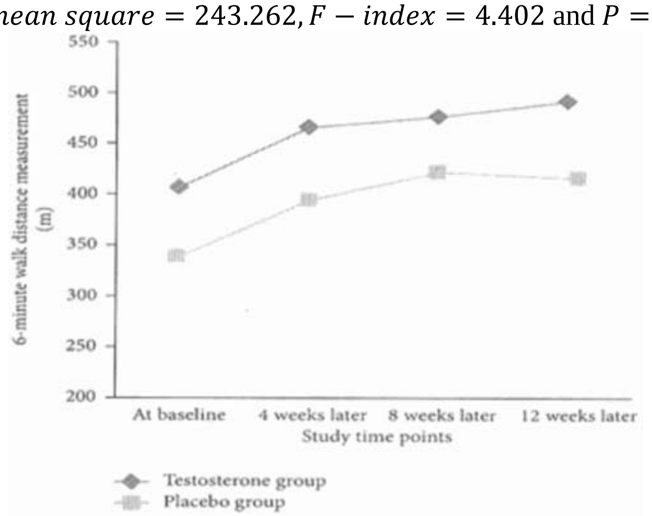
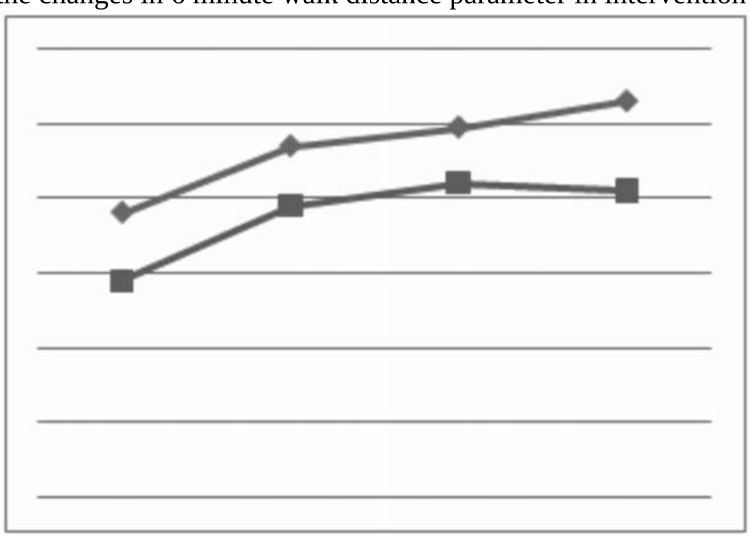



**Figure 1:** Trend of the changes in 6 minute walk distance parameter in intervention and placebo groups



**Blue Line:** Testosterone Group **Red Line:** Placebo Group

**Figure 2:** Trend of the changes in 6 minute walk distance parameter in intervention and placebo groups using Uniform Distribution

## 7. CONCLUSION:

The changes in body weight, hemodynamic parameters, and left ventricular dimensional echocardiographic indices were all comparable between the two groups. Regarding changes in diastolic functional state and using Tei index, this parameter was significantly improved. Unlike the group received placebo, those who received testosterone had a significant increasing trend in 6 walk mean distance ($6MWD$) parameter within the study period ($P = 0.019$). The discrepancy in the trends of changes in $6MWD$ between study groups remained significant after adjusting baseline variables ($mean\ square = 243.262, F\ index = 4.402$ and $P = 0.045$). Our study strengthens insights into the beneficial role of testosterone in improvement of functional capacity and quality of life in heart failure patients. This results while using motion on Poincare half plane also gives the same result by using uniform distribution. The medical reports {Figure (1)} are beautifully fitted with the mathematical model {Figure (2)}; ($i.e$) the results coincide with the mathematical and medical report.

## REFERENCES:


1. Gertsenshtein M E & Vasiliev V B, “Waveguides with random in homogeneities and Brownian motion in the Lobachevsky plane” Theory of Applied Probability, Volume 3, Page Number 391–398, 1959.
2. Kulczycki S, “Non Euclidean Geometry”, Pergamon, Oxford, 1961.
3. S. Lakshmi & M Anusurya (2012) “Stochastic model representation of BGTLNB model for the effect of liglocaine on Arginine Vasopressin level.” Aryabhatta J. of Maths & Info. Vol. 4 (2) pp 253-258.
4. Tappler B & Katz M, “Pituitary Gonadal Dysfunction in Lowoutput Cardiac Failure,” Clinical Endocrinology, Volume 10, Page Number 219–226, 1979.
5. Rogers L C G & Williams D, “Diffusions Markov Processes and Martingales”, Wiley, Chichester, 1987.
6. Comtet A & Monthus C, “Diffusion in a one dimensional random medium and hyperbolic Brownian motion”, Journal of Physics and Applied Mathematics, Volume 29, Page Number 1331–1345, 1996.
7. Monthus C & Texier C, “Random walk on the Bethe lattice and hyperbolic Brownian motion”, Journal of Physics and Applied Mathematics, Volume 29, Page Number 2399–2409 1996.
8. Nieminen M S, Ohm M B & Cowie M R, “Executive summary of the guidelines on the diagnosis and treatment of Acute Heart Failure: the Task Force on Acute Heart Failure of the European Society of Cardiology”, European Heart Journal, Volume 26, Page Number 384–416, 2005.
9. Malkin C J, Pugh P J, West J N & Van Beek E J R, “Testosterone Therapy in men with Moderate Severity Heart Failure: a double blind Randomized Placebo Controlled Trial,” European Heart Journal, Volume 27, Page Number 57–64, 2006.
10. Jankowska E A, Biel B & Majda J, “Anabolic Deficiency in men With Chronic Heart Failure: Prevalence and Detrimental Impact on Survival,” Circulation, Volume 114, Page Number 1829–1837, 2006.
11. Jankowska E A, Filippatos G & Ponikowska B, “Reduction in Circulating Testosterone Relates to Exercise Capacity in men with Chronic Heart Failure,” Journal of Cardiac Failure, Volume 15, P.N 442–450, 2009.
12. Muthaiyan A & Ramesh Kumar R J, “Stochastic Model to Find the Prognostic Ability of NT Pro-BNP in Advanced Heart Failure Patients Using Gamma Distribution” International Journal Emerging Engineering Research and Technology (IJEERT), Volume 2, Issue 5, August 2014, Page Number 40-50.
13. Ramesh Kumar R J & Muthaiyan A “Stochastic Model to Find the Triiodothyronine Repletion in Infants during Cardiopulmonary Bypass for Congenital Heart Disease Using Normal Distribution” International Journal of Research in Advent Technology (IJRAT), Volume 2, Issue 9, September 2014.

Aryabhatta Journal of Mathematics & Informatics Vol. 6, No. 2, July-Dec, 2014 ISSN : 0975-7139
Journal Impact Factor (2013) : 0.489

# A NOVEL APPROACH OF VISUAL CRYPTOGRAPHY(T,N) SCHEME WITH DYNAMIC GROUP

**Hemlata*, Lalit Himral****
*Research Scholar, YIET Gadholi Yamuna Nagar, India
**Assistant Professor, Yamuna Instt. of Engg. & Technology, Gadholi, Yamuna Nagar
E-mail: goldygsohal@gmail.com*

**ABSTRACT :**
***Visual cryptography (VC) Scheme is a secret sharing scheme where a secret image is encoded into transparencies, and the stacking of any out of transparencies reveals the secret image. The stacking of or fewer transparencies is unable to extract any information about the secret data. We discuss the additions and deletions of users in a dynamic group. To reduce overhead of generating and distributing transparencies in user change, this paper proposes a VC scheme with unlimited based on the probabilistic model. The proposed model allows to change dynamically in order to include new transparencies without regenerating and redistributing the original transparencies. Specifically, an extended VC scheme based on basis matrices and a probabilistic model is proposed in the system. An equation is derived from the fundamental definitions of the VC scheme, and then the VC scheme achieving maximal contrast can be designed by using the derived equation. The maximal contrasts are explicitly solved in this developed paper.***

**Index Terms –** *VC Scheme, Halftone Technique,*

## 1. INTRODUCTION

VISUAL cryptography (VC) scheme is a branch of secret sharing. In the VC scheme, a secret image is encoded into transparencies, and the content of each transparency is noise-like so that the secret information cannot be retrieved from any one transparency via human visual observation or signal analysis techniques. In general, a -threshold VC scheme has the following properties: The stacking of any out of those VC generated transparencies can reveal the secret by visual perception, but the stacking of any or fewer number of transparencies cannot retrieve any information other than the size of the secret image. Naor and Shamir [1] proposed a –threshold VC scheme based on basis matrices, and the model had been further studied and extended. The related works include the VC schemes based on probabilistic models [2]–[4], general access structures [5], [6], VC over halftone images [7], [8], VC for color images [9], cheating in VC [10], [11], the general formula of VC schemes [12], and region incrementing VC . Contrast is one of the important performance metrics for VC schemes. Generally, the stacking revelation of the secret with higher contrast represents the better visual quality, and therefore the stacking secret with high contrast is the goal of pursuit in VC designs. Naor and Shamir [1] defined a contrast formula which has been widely used in many studies. Based on the definition of contrast, there are studies attempting to achieve the contrast bound of VC scheme [4], [14]–[20]. For instance, Blundo et al. [17] gave the optimal contrast of VC schemes. Hofmeister et al. [19] provide a linear program which is able to compute exactly the optimal contrast for VC schemes. Krause and Simon [20] provides the upper bound and lower bound of the optimal contrast for VC schemes. Moreover, there exist VC related researches using differential definitions of contrast [21]–[23]. Another important metric is
the pixel expansion denoting the number of sub pixels in transparency used to encode a secret pixel. The minimization of pixel expansions has been investigated in previous studies [25]. The probabilistic model of the VC

scheme was first introduced by Ito et al. [2], where the scheme is based on the basis matrices, but only one column of the matrices is chosen to encode a binary secret pixel, rather than the traditional VC scheme utilizing the whole basis matrices. The size of the generated transparencies is identical to the secret image. Yang [31] also proposed a probabilistic model of VC scheme, and the two cases and are explicitly constructed to achieve the optimal contrast. Based on Yang [31], Cimato et al. [32] proposed a generalized VC scheme in which the pixel expansion is between the probabilistic model of VC scheme and the traditional VC scheme. Encrypting an image by random grids (RGs) was first introduced by Kafri and Keren [26] in 1987. A binary secret image is encoded into two noise-like transparencies with the same size of the original secret image, and stacking of the two transparencies reveals the content of the secret. Comparing RGs with basis matrices, one of the major advantages is that the size of generated transparencies is unexpanded. The RG scheme is similar to the probabilistic model of the VC scheme, but the RG scheme is not based on the basis matrices. The recent studies include the RG for color image [27], RG, and RG schemes [28]-[29]. We also compare the proposed method with RG and gray scale.

**(A) Our system implementation process**

1. Input Image modules.
2. Matrices (Black and White) Method.
3. VC Scheme Method.
4. Encoding Algorithm Method.

**(B) Visual Cryptography (VC) Scheme**

Visual Cryptography technique is a special encryption technique to hide information in images in such a way that it can be decrypted by the human vision if the correct key image is used. The technique was first proposed by Naor and Shamir in 1994. Visual Cryptography uses two transparent images. One image contains random pixels and the other image contains the secret information to secure data. It is impossible to retrieve the secret information from one of the images. Both transparent images and layers are required to reveal the information. The easiest way to implement Visual Cryptography is to print the two layers onto a transparent sheet to show secrete encrypted image. Proposed method is based on the basis matrices and the idea of probabilistic model. For a (t, n) VC scheme, the “totally symmetric” form of (B0)and(B1) are both constructed and described as H0 and H1, respectively.

VC scheme with flexible value of (n). From the practical perspective, the proposed scheme accommodates the dynamic changes of users without regenerating and redistributing the transparencies, which reduces computation and communication resources required in managing the dynamically changing user group.

## 2. PROPOSED SYSTEM

We have proposed a (t, n) VC scheme with flexible value of (n). From the practical perspective, the proposed scheme accommodates the dynamic changes of users without regenerating and redistributing the transparencies, which reduces computation and communication resources required in managing the dynamically changing user group. From the theoretical perspective, the scheme can be considered as the probabilistic model of (t, n) VC with unlimited. Initially, the proposed scheme is based on basis matrices, but the basis matrices with infinite size cannot be constructed practically. Therefore, the probabilistic model is adopted in the scheme.

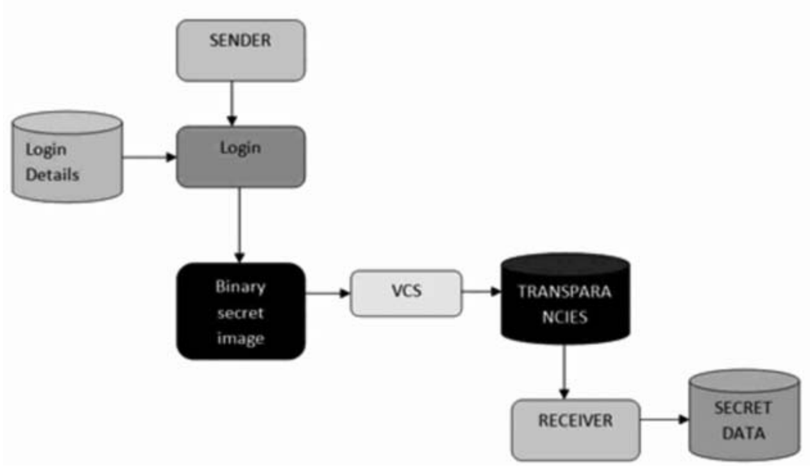



**Fig:-System Design architecture**

## 3. Result

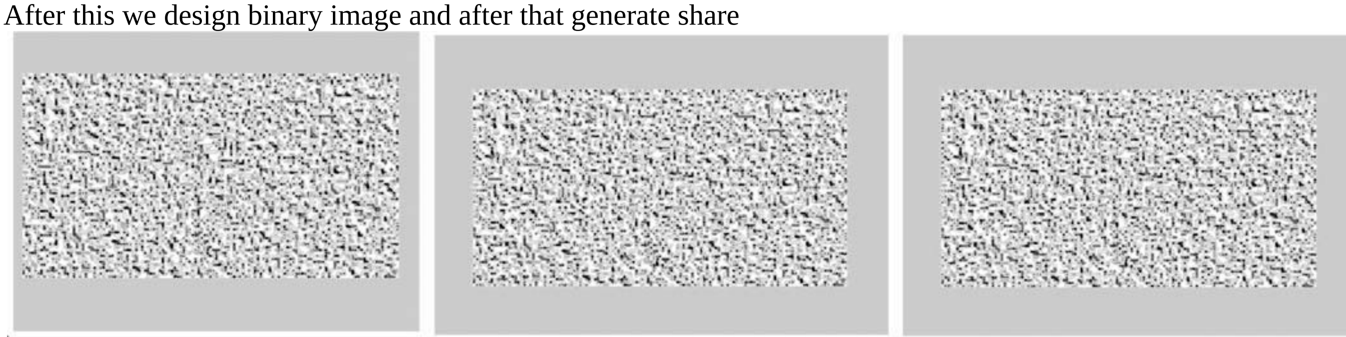
First we take RGB image and then convert to CMY


Fig:-RGB image

After RGB we convert CMY


Fig:-Converted CMY image

After this we design binary image and after that generate share



(a) Share 1 (b) Share 2 (c) Share 3

**Fig : Share**

Result

**Fig: Final Result**

## 4. CONCLUSION

We have proposed a VC scheme with flexible value of . From the practical perspective, the proposed scheme accommodates the dynamic changes of users without regenerating and redistributing the transparencies, which reduces computation and communication resources required in managing the dynamically changing user group. From the theoretical perspective, the scheme can be considered as the probabilistic model of VC with unlimited . Initially, the proposed scheme is based on basis matrices, but the basis matrices with infinite size cannot be constructed practically. Therefore, the probabilistic model is adopted in the scheme. As the results listed in Table I, the proposed scheme also provides the alternate verification for the lower bound proved by Krause and Simon [20]. For, the contrast is very low so that the secret is visually insignificant. Therefore, in practical applications, the values of 2 or 3 for are empirically suggested for the proposed scheme.

## REFERENCES


1. M. Naor and A. Shamir, "Visual cryptography," in *Proc. Advances in Cryptography (EUROCRYPT'94)*, 1995, vol. 950, LNCS, pp. 1–12.
2. R. Ito, H. Kuwakado, and H. Tanaka, "Image size invariant visual cryptography," *IEICE Trans. Fundam. Electron., Commun., Comput. Sci.*, vol. 82, pp. 2172–2177, Oct. 1999.
3. C. N. Yang, "New visual secret sharing schemes using probabilistic method," *Pattern Recognit. Lett.*, vol. 25, pp. 481–494, Mar. 2004.
4. S. J. Lin, S. K. Chen, and J. C. Lin, "Flip visual cryptography (FVC) with perfect security, conditionally-optimal contrast, and no expansion," *J. Vis. Commun. Image Represent.*, vol. 21, pp. 900–916, Nov. 2010.
5. G. Ateniese, C. Blundo, A. De Santis, and D. R. Stinson, "Visual cryptography for general access structures," *Inf. Computat.*, vol. 129, no. 2, pp. 86–106, Sep. 1996.
6. F. Liu, C. Wu, and X. Lin, "Step construction of visual cryptography schemes," *IEEE Trans. Inf. Forensics Security*, vol. 5, no. 1, pp. 27–38, Mar. 2010.
7. Z. Zhou, G. R. Arce, and G. Di Crescenzo, "Halftone visual cryptography," *IEEE Trans. Image Process.*, vol. 15, no. 8, pp. 2441–2453, Aug. 2006.
8. Z. Wang, G. R. Arce, and G. Di Crescenzo, "Halftone visual cryptography via error diffusion," *IEEE Trans. Inf. Forensics Security*, vol. 4, no. 3, pp. 383–396, Sep. 2009.

9. F. Liu, C. K.Wu, and X. J. Lin, “Colour visual cryptography schemes,” *IET Inf. Security*, vol. 2, no. 4, pp. 151–165, Dec. 2008.
10. G. Horng, T. Chen, and D. S. Tsai, “Cheating in visual cryptography,” *Designs, Codes, Cryptography*, vol. 38, no. 2, pp. 219–236, Feb. 2006.
11. C. M. Hu and W. G. Tzeng, “Cheating prevention in visual cryptography,” *IEEE Trans. Image Process.*, vol. 16, no. 1, pp. 36–45, Jan. 2007.
12. H. Koga, “A general formula of the -threshold visual secret sharing scheme,” in *Proc. 8th Int. Conf. Theory and Application of Cryptology and Information Security: Advances in Cryptology*, Dec. 2002, pp. 328–345.
13. R. Z. Wang, “Region incrementing visual cryptography,” *IEEE Signal Process. Lett.*, vol. 16, no. 8, pp. 659–662, Aug. 2009.
14. M. Bose and R. Mukerjee, “Optimal visual cryptographic schemes for general ,” *Designs, Codes, Cryptography*, vol. 55, no. 1, pp. 19–35, Apr. 2010.
15. C. Blundo, P. D’Arco, A. De Santis, and D. R. Stinson, “Contrast optimal threshold visual cryptography schemes,” *SIAM J. DiscreteMath.*, vol. 16, no. 2, pp. 224–261, Feb. 2003.
16. M. Bose and R. Mukerjee, “Optimal visual cryptographic schemes,” *Designs, Codes, Cryptography*, vol. 40, no. 3, pp. 255–267, Sep. 2006.
17. C. Blundo, A. De Santis, and D. R. Stinson, “On the contrast in visual cryptography schemes,” *J. Cryptology*, vol. 12, no. 4, pp. 261–289, 1999.
18. S. Cimato, R. De Prisco, and A. De Santis, “Optimal colored threshold visual cryptography schemes,” *Designs, Codes, Cryptography*, vol. 35, no. 3, pp. 311–335, Jun. 2005.
19. T.Hofmeister,M. Krause, andH. U. Simon, “Contrast-optimal out of secret sharing schemes in visual cryptography,” *Theoretical Comput. Sci.*, vol. 240, no. 2, pp. 471–485, Jun. 2000.
20. M. Krause and H. U. Simon, “Determining the optimal contrast for secret sharing schemes in visual cryptography,” *Combinatorics, Probability, Comput.*, vol. 12, no. 3, pp. 285–299, May 2003.
21. E. R. Verheul and H. C. A. Van Tilborg, “Constructions and properties of out of visual secret sharing schemes,” *Designs, Codes, Cryptography*, vol. 11, no. 2, pp. 179–196, May 1997.
22. P. A. Eisen and D. R. Stinson, “Threshold visual cryptography schemes with specified whiteness levels of reconstructed pixels,” *Designs, Codes, Cryptography*, vol. 25, no. 1, pp. 15–61, 2002.
23. F. Liu, C. K. Wu, and X. J. Lin, “A new definition of the contrast of visual cryptography scheme,” *Inf. Process. Lett.*, vol. 110, no. 7, pp. 241–246, Mar. 2010.
24. C. Blundo, S. Cimato, and A. De Santis, “Visual cryptography schemes with optimal pixel expansion,” *Theoretical Comput. Sci.*, vol. 369, no. 1, pp. 169–182, Dec. 2006.
25. H. Hajiabolhassan and A. Cheraghi, “Bounds for visual cryptography schemes,” *Discrete Appl. Math.*, vol. 158, no. 6, pp. 659–665, Mar. 2010.
26. O. Kafri and E. Keren, “Encryption of pictures and shapes by random grids,” *Opt. Lett.*, vol. 12, no. 6, pp. 377–379, Jun. 1987.
27. S. J. Shyu, “Image encryption by random grids,” *Pattern Recognit.*, vol. 40, no. 3, pp. 1014–1031, Mar. 2007.
28. S. J. Shyu, “Image encryption by multiple random grids,” *Pattern Recognit.*, vol. 42, no. 7, pp. 1582–1596, Jul. 2009.

29. T. H. Chen and K. H. Tsao, “Visual secret sharing by random grids revisited,” *Pattern Recognit.*, vol. 42, no. 9, pp. 2203–2217, Sep. 2009.
30. N. Macon and A. Spitzbart, “Inverses of Vandermonde matrices,” *Amer. Math. Monthly*, vol. 65, no. 2, pp. 95–100, Feb. 1958.
31. C. N. Yang, “New visual secret sharing schemes using probabilistic method,” *Pattern Recognit. Lett.*, vol. 25, no. 4, pp. 481–494, Mar. 2004.
32. S. Cimato, R. De Prisco, and A. De Santis, “Probabilistic visual cryptography schemes,” *Computer J.*, vol. 49, no. 1, pp. 97–107, Jan. 2006.
33. G. B. Horng, T. G. Chen, and D. S. Tsai, “Cheating in visual cryptography,” *Designs, Codes, Cryptography*, vol. 38, no. 2, pp. 219–236, Feb. 2006.

Aryabhatta Journal of Mathematics & Informatics Vol. 6, No. 2, July-Dec, 2014 ISSN : 0975-7139
Journal Impact Factor (2013) : 0.489

# EFFECT ON TOTAL DISK ACCESS TIME USING ZONED-BIT RECORDING TECHNOLOGY BY PLACING MORE FREQUENTLY USED DATA AT OUTER ZONE

**Er. Sachin Sharma*, Er. Silky Miglani**, T.P Singh*****
*Asstt. Prof., CSE, Hindustan Institute of Tech. & Management, Dheen, Ambala
**Asstt. Prof., CSE, Yamuna Institute of Engg. & Technology, Gadholi, Yamunanagar, Haryana
***Professor Mathematics & Computing, Yamuna Institute of Engg. & Tech., Gadholi, Yamunanagar, Haryana
Email: sachinsharma.one@gmail.com*,silky.cse@gmail.com**,tpsingh78@yahoo.com***

**ABSTRACT :**
***In the present communication, we have explored new mechanism to place most frequently used data at outer zone of tracks over hard disk. The effect on transfer time has been observed using Zoned-Bit Recording Technology An attempt has been made to minimize the seek time through scheduling techniques on track requests. Our efforts have shown that transfer time gets reduced with reasonable difference on applying this technique.***

**Keywords:** ***Zoned-Bit recording, fragmentation, transfer time***

## 1. INTRODUCTION

It has been observed that total disk access time depends upon seek time, controller over head, internal transfer time and rotational delay. In this paper, an attempt has been made to minimize the seek time by applying scheduling techniques on track requests. Internal data transfer rate must be minimized so that total disk access time gets reduced. Earlier, disk controller could not work with complicated arrangements of sectors in tracks. Thus every track contains same number of sectors. The first hard disk contains 17 sectors per track. According to disk geometry the outer concentric circles i.e. outer tracks contain more sectors as compared to inner tracks. Circumference area increases as we move from inner track to outer track. Practically, according to state technology, we place same number of sectors by reducing their bit density towards outer tracks, which creates the wastage of space. To overcome this situation , zone bit recording or multiple zone bit recording plays an important role in which tracks are grouped into zones. Each zone contains different number of tacks and sectors per track as well as different data transfer rate because platter of hard disk spins at constant speed. Packing of sectors (bits in sector) is more on outer zones due to bits per inch and larger area as compared to inner zone.

Little work has been done related to Zoned-Bit Recording over the disk. **Chen (1995)** discussed the Zoned-Bit Recording enhanced Video Data Layout Strategies. They studied the video data layout issues in the video server design. They presented a family of novel video data layout strategies, called Zone-Bit-Recording-Enhanced (ZBRE) layout schemes, which take into account the multiple zone-recording features of modern disk drives. **Zhai et al (1999)** developed a scheme to produce a maximal storage utilization and bandwidth utilization. They fully utilized the storage and width of each physical zone of ZBR disks. **Wang et al (2001)** presented a novel performance-oriented data reorganizing scheme, called PROFS, which boosts the I/O performance of LFS (Log-structured File System). Their scheme organize data on the disk during LFS garbage collection and system idle periods.

A general **PC** has three type of storage related to hard disk. Permanent storage is on hard disk platters while temporary storage is on buffer and in system memory. Therefore data move in these three storage devices. Transfer of data from platters to buffer and vice versa is called Internal Data Transfer Rate whereas transfer of data from buffer to system memory (array) and vice versa is called External Data Transfer Rate. However, Internal Data Transfer Rate is always greater than External Data Transfer Rate.

In modern hard disk drive, manufacturers mention range of either minimum or maximum Internal Data Transfer Rate. For example, IBM Desk star 34GXP (model DPTA-373420) has a media transfer rate that varies between 171Mb/s and 284 Mb/s (approximately) depending on the disk that is meant for reading whose drive has 12 different zones.

In the present communication, we have explored new mechanism to place most frequently used data at outer zone of tracks over hard disk. The effect on disk transfer time has been seen using Zoned-Bit Recording Technology. Our efforts have shown that transfer time gets reduced with reasonable difference on applying this technique.

The rest of the paper is organized as follows:

Model description has been provided in Section 2.Section 3 deals with problem formulation while Section 4 is devoted to the proposed work. In Section 5, the result and analysis has been facilitated through tables and graphs. Finally conclusions are drawn in Section 6.

## 2. ZONED-BIT RECORDING TECHNOLOGY

The capacity and performance of the disks have been improved through the special arrangement made using **areal density** of the disk. Areal density is defined as the number of bits that can be packed into each unit of area on the disk. The areal density of the disk provides such an arrangement through which different number of sectors per track can be packed into different number of cylinders on the disk so the tracks in the outer cylinder will have more number of sectors per track than the inner one.

When the track possesses more than one set of sectors it is known as Zoned Bit Recording. With this approach, tracks are grouped into zones that eliminate the wasted space on the disk.

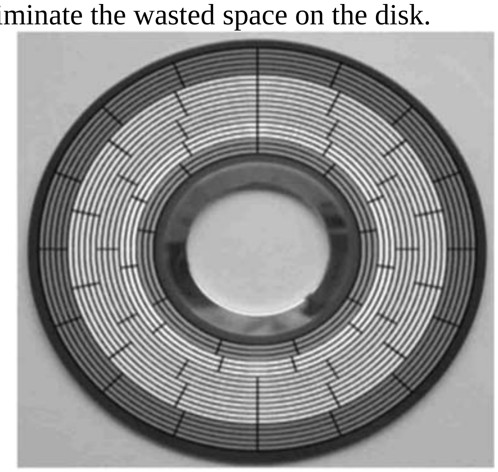


**Figure 1: Tracks grouped into Zones**

## 3. PROBLEM FORMULATION

Recently, **Riska et al (2006)** presented a characterization of disk drive workloads measured in systems representing the Enterprises, Desktop and Consumer Electronic instruments and showed the general analysis in Table 1 which lists all Traces and their main characteristics and the number of disks in the system, trace length in homes, number of requests in the trace, read/write ratio etc.

Table 4.1 shows the characterization of disk drive workloads with respect to different computing environments, categorized based on their complexity, applications and performance requirements. It is observed through analysis made by Alma Riska et al that **E-Mail** is an activity that is requested maximum number of times nearly 1,606,434 times by the user in 25 hrs. This implies that our hard disk must be configured in such a way that the data related to E-Mail activity must be on the outermost tracks of the disk so that it must easily and quickly Read or Write. If the data related to most frequently accessed trace is on inner tracks of the disk, it is very expensive to read/write that data as the disk arm will have to move non-uniformly time and again. This is a time consuming process.

**Table 1: Different Traces and their respective number of requests**

| Trace | Length | R/W % | No. of Requests |
|---|---|---|---|
| Web | 7.3hrs | 44/56 | 114,814 |
| Email | 25hrs | 99/1 | 1,606,434 |
| S/W Development | 12hrs | 88/12 | 483,563 |
| User Accounts | 12hrs | 87/13 | 168,148 |
| Desktop 1 | 21hrs | 52/48 | 146,248 |
| Desktop 2 | 18hrs | 15/85 | 159,405 |
| Desktop 3 | 24hrs | 44/56 | 29,779 |
| PVR A | 20hrs | 95/5 | 880,672 |
| PVR B | 2.8hrs | 54/46 | 138,155 |
| MP 3 | 2.2hrs | 69/31 | 40,451 |
| Game Console | 1.4hrs | 83/17 | 33,076 |

To overcome the above mentioned problem, we have adopted the mechanism called Zoned-Bit Recording that divides the disk into zones such that the more frequently used data is placed at outer zones. There arises a severe requirement to execute all the requests over disk as fast as possible. This objective is achieved by varying the Internal Data Transfer Rate among individual zones in order to transfer the data at the higher data rate between cache (buffer) and permanent storage of platters. Therefore, in this chapter, we **compute the impact on Total Disk Access Time by placing more frequent data in outer zones.**

## 4. PROPOSED WORK

In today's era, every person has its own field of interest. According to choice of their work, they frequently demand for particular data whenever needed. For example a person working on a project since 5 months, tries to access related data whenever required. Similarly, a person playing a game daily sends frequent requests to access desired data.

**Zoned-Bit Recording is a mechanism that is implemented to solve such problems for accessing data related to frequently accessed items.** This technique reduces the Total Disk Access Time of most frequently used data to much extent .We noticed that outer zones have faster data transfer rate as compared to inner zones. Thus we can store the frequently used data at outer zone so that it can be transferred at high speed. But generally we debut storage from outer zones in hard disk. During defragmentation process, we can place the frequently used data at outer zones so as to minimize the transfer time.

We have two methods to store files at outer zone. **First**, we can reserve space for most frequent data (as per results of analysis made by Riska et al [2006]) at outer zones. **Second**, we can monitor track requests and then calculate the high frequency used blocks of data. After detecting frequent used data we can place or swap data among different zones of hard disk during defragmentation process. But second method is time consuming and sometimes many files cannot be moved. There would be increase in over head of disk controller.

We considered Hitachi Travel Star 5k500B 2 ½ inch HDD. The Data Transfer Rate of different Zones that comprises of varying number of Sectors per rack is shown in Table 4.2.This information for our specific model of Hard Disk is gathered through the book namely "**Upgrading and Repairing PCs 21st edition**" authored by Scott Mueller.

**Table 2: Arrangement of different zones with their respective Transfer Rates**

| Zone | Sectors per track | Transfer rate (MBps) |
|---|---|---|
| 0 | 1920 | 117.96 |
| 1 | 1840 | 113.05 |
| 2 | 1800 | 110.59 |
| 3 | 1760 | 108.13 |
| 4 | 1720 | 105.68 |
| 5 | 1680 | 103.22 |
| 6 | 1632 | 100.27 |
| 7 | 1600 | 98.3 |
| 8 | 1560 | 95.85 |
| 9 | 1520 | 93.39 |
| 10 | 1480 | 90.93 |
| 11 | 1440 | 88.47 |
| 12 | 1360 | 83.56 |
| 13 | 1320 | 81.1 |
| 14 | 1296 | 79.63 |
| 15 | 1280 | 78.64 |
| 16 | 1240 | 76.19 |
| 17 | 1200 | 73.73 |
| 18 | 1140 | 70.04 |
| 19 | 1104 | 67.83 |
| 20 | 1080 | 66.36 |
| 21 | 1020 | 62.67 |
| 22 | 960 | 58.98 |
| 23 | 912 | 56.03 |

From the above table, it is observed that there are 24 zones in our Hitachi Disk and each zone possesses their respective Data Transfer Rates with variation in number of Sectors per Track ranging from 1,920 Sectors per Track over Ist Zone to 912 Sectors per Track over 24th Zone. Hence, the Data Transfer Rate varies zone to zone.

Graphically, the Data Transfer Rate of different zones is depicted in the following Figure 2 showing 24 Zones their respective Data Transfer rates in Mega Bytes per Second (MBps).

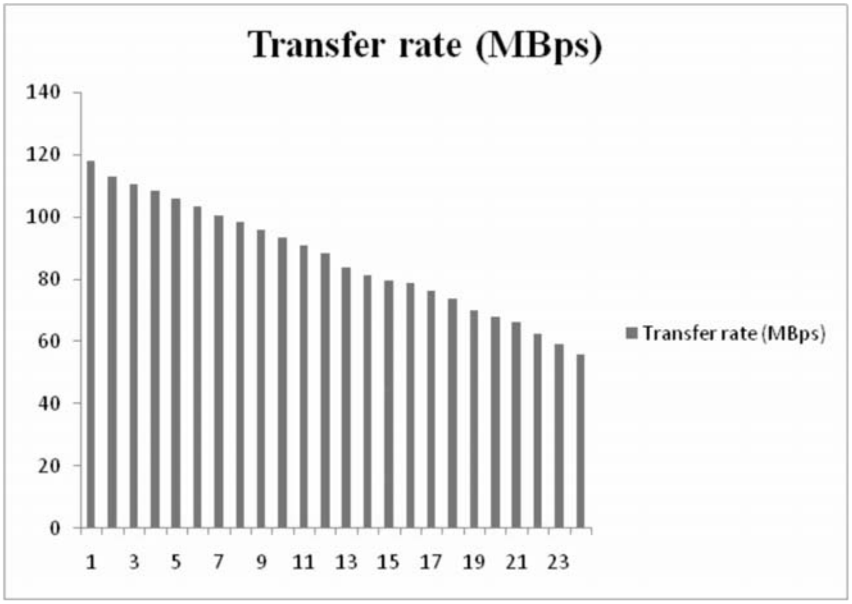



**Figure 2: Data Transfer Rates of different Zones**

## 5. RESULTS & ANALYSIS

Presently, the data files are stored on to the disk in the same manner as they arrives mainly in sequential order as shown in table 4.3.From Table 3, we observe that the file of 442MB is placed in zone number 11which has been requested 65 times. The time spend to transfer this file is 324.7428507 seconds. Total requests generated by CPU are 654 and the average time taken to fulfill these requests is 292.7479547 seconds as shown below. But this time can further be reduced to much extent if we follow the technique of Zoned-Bit Recording as discussed in section 2. Table 3 shows the arrangement of data files in accordance to the requests made.

On the basis of above research done by Riska et al, we extended and modified their work by considering different files sizes that may be requested by the user. The files are arranged within the zones as discussed by Scott Mueller. The requests were generated randomly using rand function in C for a particular file. Following table shows the various files of different size that are placed on different zones according to the number of times that file has been requested.

**Table 3: Time taken by different files in different zones for frequent requests**

| Zone | Sectors per track | Transfer Rate (MBps) | Requested File Size (MB) | Time (Second) | Frequent requests | Total Time (seconds) |
|---|---|---|---|---|---|---|
| 0 | 1920 | 117.96 | 300 | 2.543234995 | 20 | 50.8646999 |
| 1 | 1840 | 113.05 | 455 | 4.024767802 | 15 | 60.37151703 |
| 2 | 1800 | 110.59 | 222 | 2.007414775 | 7 | 14.05190343 |
| 3 | 1760 | 108.13 | 455 | 4.207897901 | 23 | 96.78165172 |
| 4 | 1720 | 105.68 | 43 | 0.406888721 | 12 | 4.882664648 |
| 5 | 1680 | 103.22 | 222 | 2.150745979 | 28 | 60.22088742 |
| 6 | 1632 | 100.27 | 456 | 4.547721153 | 23 | 104.5975865 |
| 7 | 1600 | 98.3 | 654 | 6.653102747 | 45 | 299.3896236 |
| 8 | 1560 | 95.85 | 344 | 3.588941054 | 12 | 43.06729264 |
| 9 | 1520 | 93.39 | 666 | 7.131384517 | 34 | 242.4670736 |
| 10 | 1480 | 90.93 | 33 | 0.362916529 | 4 | 1.451666117 |
| **11** | **1440** | **88.47** | **442** | **4.996043857** | **65** | **324.7428507** |
| 12 | 1360 | 83.56 | 562 | 6.725706079 | 43 | 289.2053614 |
| 13 | 1320 | 81.1 | 545 | 6.720098644 | 55 | 369.6054254 |
| 14 | 1296 | 79.63 | 6554 | 82.30566369 | 44 | 3621.449203 |
| 15 | 1280 | 78.64 | 343 | 4.361648016 | 14 | 61.06307223 |
| 16 | 1240 | 76.19 | 367 | 4.816905106 | 34 | 163.7747736 |
| 17 | 1200 | 73.73 | 555 | 7.527465075 | 23 | 173.1316967 |
| 18 | 1140 | 70.04 | 4888 | 69.78869218 | 5 | 348.9434609 |
| 19 | 1104 | 67.83 | 443 | 6.531033466 | 13 | 84.90343506 |
| 20 | 1080 | 66.36 | 667 | 10.05123568 | 22 | 221.1271851 |
| 21 | 1020 | 62.67 | 111 | 1.771182384 | 14 | 24.79655337 |
| 22 | 960 | 58.98 | 122 | 2.068497796 | 55 | 113.7673788 |
| 23 | 912 | 56.03 | 320 | 5.711226129 | 44 | 251.2939497 |
| | | | | | **654** | **292.7479547** |

The time (seek time) taken by the disk to locate that file on the particular sector was calculated in seconds as shown below in the Table 4.The total time in seconds denote the total amount of time taken to locate the file over a disk as many times the contents of the file has been requested or retrieved.

**Table 4: Time taken by different files in different zones for frequent requests according to Zoned-Bit Recording**

| Zone | Sectors per Track | Transfer rate (MBps) | Request file size (MB) | Time (Second) | Frequent requests | Total Time |
|---|---|---|---|---|---|---|
| 0 | 1920 | 117.96 | 442 | 3.747032893 | 65 | 243.557138 |
| 1 | 1840 | 113.05 | 545 | 4.820875719 | 55 | 265.1481645 |
| 2 | 1800 | 110.59 | 122 | 1.103173886 | 55 | 60.6745637 |
| 3 | 1760 | 108.13 | 654 | 6.048275224 | 45 | 272.1723851 |
| 4 | 1720 | 105.68 | 6554 | 62.01741105 | 44 | 2728.766086 |
| 5 | 1680 | 103.22 | 320 | 3.100174385 | 44 | 136.4076729 |
| 6 | 1632 | 100.27 | 562 | 5.604866859 | 43 | 241.009275 |
| 7 | 1600 | 98.3 | 666 | 6.775178026 | 34 | 230.3560529 |
| 8 | 1560 | 95.85 | 367 | 3.828899322 | 34 | 130.1825769 |
| 9 | 1520 | 93.39 | 222 | 2.377128172 | 28 | 66.55958882 |
| 10 | 1480 | 90.93 | 455 | 5.003849115 | 23 | 115.0885296 |
| 11 | 1440 | 88.47 | 456 | 5.15428959 | 23 | 118.5486606 |
| 12 | 1360 | 83.56 | 555 | 6.64193394 | 23 | 152.7644806 |
| 13 | 1320 | 81.1 | 667 | 8.224414303 | 22 | 180.9371147 |
| 14 | 1296 | 79.63 | 300 | 3.767424338 | 20 | 75.34848675 |
| 15 | 1280 | 78.64 | 455 | 5.785859613 | 15 | 86.7878942 |
| 16 | 1240 | 76.19 | 343 | 4.501903137 | 14 | 63.02664392 |
| 17 | 1200 | 73.73 | 111 | 1.505493015 | 14 | 21.07690221 |
| 18 | 1140 | 70.04 | 443 | 6.324957167 | 13 | 82.22444318 |
| 19 | 1104 | 67.83 | 43 | 0.633937786 | 12 | 7.607253428 |
| 20 | 1080 | 66.36 | 344 | 5.18384569 | 12 | 62.20614828 |
| 21 | 1020 | 62.67 | 222 | 3.542364768 | 7 | 24.79655337 |
| 22 | 960 | 58.98 | 4888 | 82.87555103 | 5 | 414.3777552 |
| 23 | 912 | 56.03 | 33 | 0.588970195 | 4 | 2.355880778 |
| | | | | | **654** | **240.9158438** |

The above table shows the arrangement for frequently used data at outer zone by sorting the requests larger to smaller.

## 6. CONCLUDING REMARKS & FUTURE SCOPE

In our study it has been observed that when we store most frequent data at outer zones, this reduces the Transfer Time to just **240.91 seconds** and further reduces the Internal data Transfer rate whereas if we store data sequentially as it comes the amount of transfer time leads to greater value of **292.7479547 Seconds.** Thus our method of organizing the data in proper order proves to be one of the better methods that reduce the transfer time as well the transfer rate to much extent. The results are more significant and relevant.

This technique can be applied in future in large databases of multimedia for broadcasting where most of the data is frequently utilized.

## REFERENCES

1. **[1965]** Zadeh, L.A "*Fuzzy sets and systems*", System Theory, Brooklyn N.Y., pp 29-37.
2. **[1981]** Yager, R.R, "*A procedure for ordering fuzzy subset of the unit interval*", Information Science VOL. 24, pp 143-161
3. **[1993]** Kai Hwang, "*Computer Organisation and Architecture*", Tata McGraw Hills
4. **[1994]** B. L. Worthington, G. R. Ganger and Y. N. Patt, "*Scheduling Algorithms for Modern Disk Drives*", Appeared in the Proceedings of the ACM Sigmetrics Conference, May 1994, pp. 241-251.
5. **[1996]** S. Chen and M. Thapar, "*Zone-bit-recording enhanced video data layout strategies*", in Proceedings of the 4th Int'l Workshop on Modelling, Analysis and Simulation of Computer and Telecommunication Systems(MASCOTS'96).
6. **[1999]** Chen S.H & Hseih, C.J "*Graded Mean Integration Representation of Generalized Fuzzy Numbers*", Journal of Chinese Fuzzy System, Vol.5 (2), pp 1-7.
7. **[1999]** Zhai, Shunnian and Dr. Chan,Tony K.Y, "*Continuous Media Servers with Zone-Bit-Recording (ZBR) Disks*", CG Topics 6/99, pp 20-21.
8. **[2001]** Zacker Craig and Rourke John, "*The Complete Reference-PC Hardware*", Osborne/Tata McGraw Hill Publications.
9. **[2001]** J. Wang and Y. Hu, "*Improving performance of log structured file systems on multi-zone disks*," Tech. Rep. TR256/02/01/ECECS, Department of Electrical & Computer Engineering and Computer Science, University of Cincinnati, Feb. 2001.
10. **[2003]** Tannenbaum Andrew S. And Woodhull Albert S., "*Operating Systems: Design and Implementation*", 2nd Edition, Prentice Hall, India.
11. **[2004]** Mueller Scott, "*Upgrading and Repairing PC's*", 21st Edition, Que Publishing.
12. **[2006]** Riska Alma and Riedel Erik, "*Disk Drive Level Workload Characterization*", Seagate Research, USENIX Annual Technical Conference 2006.
13. **[2006]** Al Mamum Abdullah, GuoXiao Guo and Chao Bi, "*Hard Disk Drive: Mechatronics and Control*", CRC Press, Nov. 2006.
14. **[2010]** Sunita Gupta, T.P.Singh "*Bi-objective in fuzzy scheduling on parallel machines*" Aryabhatta Journal of Mathematics & Informatics Vol. 2 No. 1, pp 149-152.
15. **[2011]** Operating Systems Achyut S. Godbole Tata McGraw Hill 2nd edition.
16. **[2013]** Singh T.P, Meenu Mittal and D.Gupta, "*A Heuristic algorithm for general weightage job scheduling under uncertain environment including transportation time*", Aryabhatta Journal of Mathematics & Informatics Vol. 5 No. 2, pp 381-388.
17. **[March 2014]** Sachin & Silky, Singh T. P. "*Performance Measure Of CPU Scheduling For Real-Time Operating System In Fuzzy Environment*" Yamuna J. Of Tech. & Business Research Vol.4 (1-2) pp 54.1-54.9.
18. **[June 2014]** Sachin & Silky, Singh T. P. "*Comparative study of CPU Scheduling Algorithms for Real-Time Operating System in Uncertainty*", Aryabhatta Journal of Mathematics & Informatics Vol.6 No.1, pp 149-158.

Aryabhatta Journal of Mathematics & Informatics Vol. 6, No. 2, July-Dec, 2014 ISSN : 0975-7139
Journal Impact Factor (2013) : 0.489

# TRANSFORMING 3SAT TO STEINER PROBLEM IN PLANAR GRAPH WHICH IS NP-COMPLETE

**Dr. G. Nirmala*, C. Sujatha****

*Head & Associate Professor, PG & Research, Dept. of Mathematics,
K.N. Govt. Arts College for women (Autonomous), Thanjavur-613007. (Tamilnadu)
**Research Scholar, Dept. of Mathematics, K.N. Govt. Arts College for women (Autonomous), Thanjavur-613007.

**ABSTRACT :**
***Complexity theory has many facts. In this work, we propose an NP-completeness result for the Steiner problem in planar graphs.***
**Keywords:** ***Planar graph, NP-complete, Steiner problem in planar graph, 3-Satisfiability.***

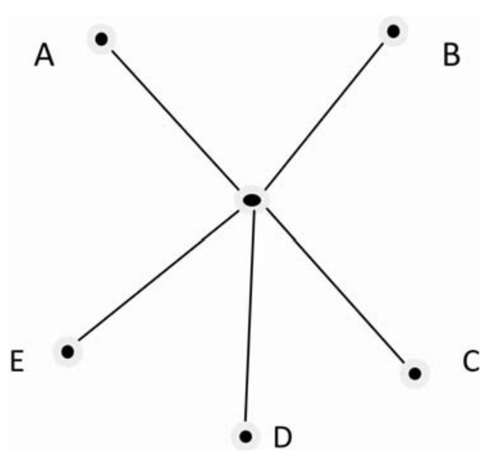
## 1. INTRODUCTION

More precisely in the abstract notions of complexity theory which we are about to define now, the inputs, also called the instances wills always be graphs, networks or finite sequence of integers, etc. For many decisions problem, Steiner problem in graphs and satisfiability, no polynomial time algorithm is known. Nevertheless some of these problems have a property which is not inherent to every decision problem, there exists algorithm which, if presented with the instance of the problem. [i.e. a graph G terminal set k, and a bound B, respectively a Boolean formula F] and in addition with a potential solution x [i.e. a sub graph T of G respectively a truth assignment τ for the variables in F] these algorithm verify in polynomial time whether x is a valid solution. [Whether T is a Steiner tree for K terminals contains almost B edges respectively whether τ satisfies F] The decision problems with this property form the NP. This abbreviation comes from non deterministic polynomial time. A very important concept in complexity theory is the concept of reducibility.

## 2. PRELIMINARIES

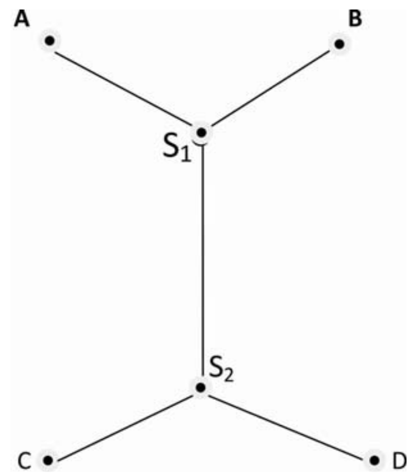
A Steiner tree is a tree in a distance graph which spans a given subset of vertices (Steiner point) with the minimal total distance on its edges.

**Examples 2.1:**




Steiner tree for five points A, B, C, D, E [note that there is direct connection between A, B, C, D, E]

**Examples 2.2:-**




Solution for four points (note that there are two Steiner points' $S_1$ and $S_2$)

**Definition 2.2:-**

A literal is a Boolean variable or its negation. A clause is a disjunction (or) if literals. A formula is in the conjunctive normal for if it is conjunction and of clauses

**Definition 2.3:** A decision problem is a question with yes or no answer. The Boolean satisfiability problem (SAT) is a decision problem of determining whether the given Boolean formula is satisfiable.

**Definition 2.4:-** The class of problem solvable by non deterministic polynomial time algorithm is called NP.

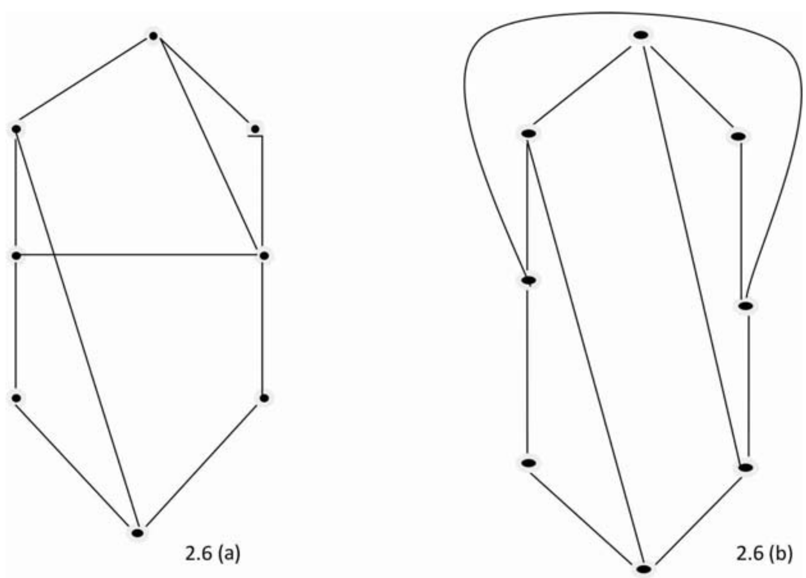
**Definition 2.5:-**

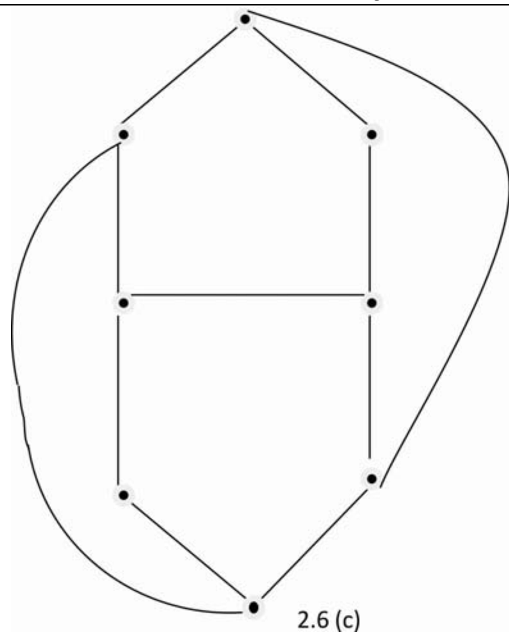
**A problem is NP-complete if**

1. It is an element of the class NP
2. Another NP-complete problem is polynomial times reducible to it.

**Definition 2.6:-** A graph is said to be planar or embeddable in the plane if it can be drawn in the plane so that its edges intersect only at end vertices. Such a drawing of a planar graph G is called a planar embedding of G or plane graph. A graph that is not planar is called non-planar graph.

**Examples 2.6:-**





2.6 (c)

The graph G in fig 2.6(a) is planar since it can be drawn in such a way that none of their edges cross and figure 2.6 (b) and 2.6 (c) gives the plane embedding of G.

**Definition 2.7:**

A planar embedding of a planar graph divides the plane into union of disjoint regions called faces of G. The numbers of regions or faces in a planar graph is denoted by P. Each plane graph has exactly one unbounded face and it is called the exterior face.

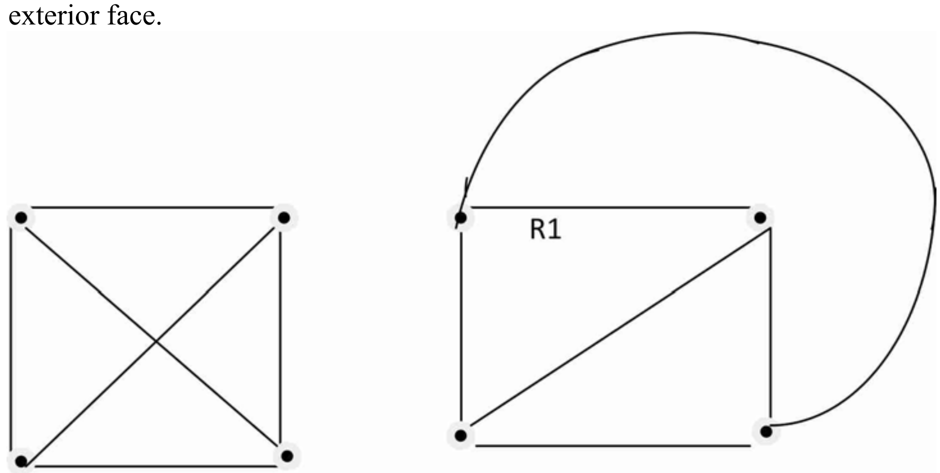



Fig 2.7

Consider the graph G and its planar embedding in figure 2.7 in which r1, r2, r3, r4 are faces with r4 as the exterior face.

**Properties of planar graphs.**

**Theorem: 2.8:**

If G is a connected planar (p,q) graph having r faces then p - q + r=2

**Corollary 2.9:**

If G is a plane (p, q) graph with r faces and K components then p-q + r=k+1

**Proof:-**

Consider a plane embedding of G such that the exterior face of each component contains all other components. Let the i th component be a (p,q) graph with $r_i$ faces for each i .Then by theorem 2.8

$$P_i - q_i + r_i = 2$$

**Hence** $\Sigma p_i - \Sigma q_i + \Sigma r_i = 2k \longrightarrow$ **(1)**

**But** $\Sigma p_i = p$, $\Sigma q_i = q$ **and** $\Sigma r_i = r + (k-1)$

Since the exterior face is counted k times in $\Sigma r_i$.

Hence (1) gives

$$p-q+r+k-1=2k$$

$$p-q+r=2k-k+1=k+1$$

**hence the result.**

**Corollary 2.10:**

If G is a (p,q) plane graph in which every face is an n cycle then $q = \frac{n(p-2)}{n-2}$

**Proof:**

Given that every face is an n-cycle. Hence every edge of G lies on the boundary of two faces of G.

Let $f_1.f_2$,----------$f_r$ be the faces of G.

Then $2q = \Sigma_{i=1}^{r}$(number of edges in the boundary of face $f_i$ )=nr

$$r = \frac{2q}{n}$$

By theorem 2.8

**p-q+r=2**

$p - q + \frac{2q}{n} = 2$

$p-2 = q(1 - 2/n)$

$p-2 = q(\frac{n-2}{n})$

$q = \frac{n(p-2)}{n-2}$

**Definition 2.11:**

A plane graph G is called maximal planar if for every pair of non-adjacent vertices x and y, the graph G + xy is non planar.

**Example**:-

In the following fig: 2.11(a) graph G is planar but it is not maximal planar. Graph in fig 2.11(b) is maximal planar.

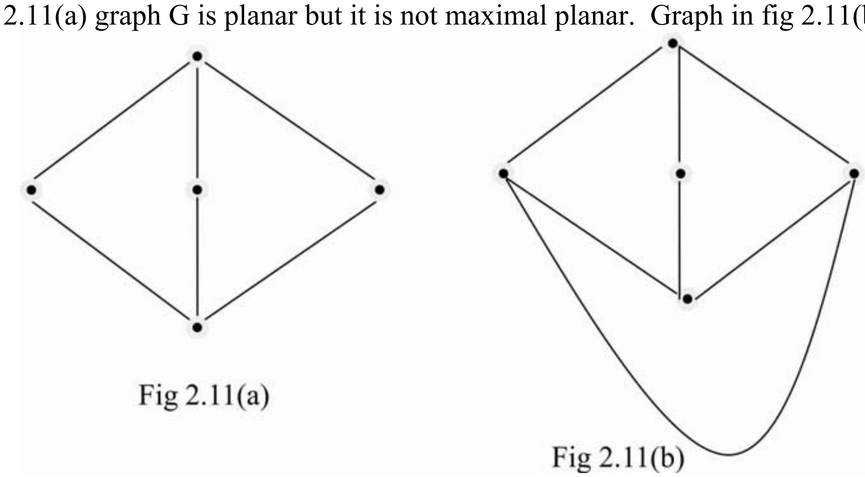


Fig 2.11(a) Fig 2.11(b)

**Corollary 2.12:**

If G is a maximal planar (p,q) graph with p≥3 then $q$ =3p-6

**Proof:**

Let r be the number of faces of G. since G is maximal, the boundary of every face is a triangle. Hence the number of edges bounding the r faces in 3r. But each edges of G lies on boundaries counted twice in the sum 3r

**(ie) $2q$=3r**

**(ie) r=$\frac{2q}{3}$**

By theorem 2.8:

$$p- q+r+=2$$

$$p-q+\frac{2q}{3}=2$$

$$3p- 3q+ 2q=6$$

$$3p- q = 6$$

$$q = 3p - 6$$

**Corollary 2.13:**

If G is planar (p,q) graph with p≥3 then q≤3p-6

**Proof:**

Let G′ be the maximal planar graph obtained from G after inserting some edges between non-adjacent vertices of G. If q′ is the number of edges of G′ then q≤ q′. But P=P′. For the maximal planar graph G′. By corollary 2.12

q′ =3p′ -6=3p-6

but q≤q′

q≤3p-6

**Corollary 2.14:**

Every planar graph G contains a vertex of degree at most 5.

**Proof:**

Let G be a plane (p,q) graph with $x_1$,$x_2$,-------$x_p$) as its vertices.

If p≤6 then deg $v_i$≤5 for each i and hence the result is true.

Suppose that p≥7 since G is planar, by corollary 2.13

$$q \leq 3p + 6$$

But $\boldsymbol{\Sigma}_{i=1}^{p}$ deg $x_i$=2g≤ 6p-12 ⇢ (1)

If deg $x_i \geq 6$ for every vertex x, then $\boldsymbol{\Sigma}_{i=1}^{i}$ deg $x_i$ ≥6p ⇢ (2)

From (1&2) we get a contradiction. Hence there must be at least one vertex for which the degree $\leq 5$

**Corollary 2.15**:

If G is a connected plane (p, q) graph without triangles and p ≥ 3, then q ≤ $2p - 4$

**Proof:**

Since G has no triangles, the boundary of each face has at least four edges. Hence the member of edges bounding the r faces ≥ 4 r. But each edge of G lies on boundaries of two faces, and hence 2q≥ $4r$

By theorem 2.8:

p-q+r=2

r=2-p+q

Hence 2q ≥ $4(2 - p + q)$

2q≥ $8 - 4p + 4q$

4p-8 $\geq 2q$

q$\leq 2p-4$

**Corollary 2.16:**

The graph $k_{3,3}$ is non planar

**Proof:**

$k_{3,3}$ is a bipartite graph and hence it has no triangles. Suppose that $\boldsymbol{k_{3,3}}$ is planar and G be a plane embedding of $\boldsymbol{k_{3,3}}$ **.** Then G has 6 vertices and 9 edges.

By corollary 2.15

q$\leq 2p-4$

q$\leq 2(6)-4$

q$\leq$8

Which is a contradiction. Hence $\boldsymbol{k_{3,3}}$ **is non planar.**

**Corollary 2.17:**

The graph $k_5$ is non planar

**Proof**:

If $k_5$ is planar then by corollary 2.13, q $\leq 3p-6$

(ie) 10$\leq 3(5)-6$

=15-6=9 10$\leq$ 9

Which is a contradiction. Hence $k_5$ is non-planar

## 3. STEINER PROBLEM IN PLANAR GRAPH IS NP-COMPLETE.

**Theorem:** Steiner problem in planar graph is NP-complete

**Proof**:

Let the Steiner problem in planar graph is $\in$ NP, it sufficient to show that Steiner problem in planar graph is in fact NP-complete.

To see this, we reduce 3SAT to Steiner problem in planar graph. Let $x_1$,$x_2$-------$x_n$ be the variables and $c_1$,$c_{2,}$________$c_m$ the clauses in an arbitrary instance of 3 SAT.

Our aim is to construct a graph G = (V,E) a terminal set k, and a bound B such that t contains steiner tree T for K of size B if and only if the given 3 SAT instance is satisfiable.

**Planar 3 SAT**

A 3SAT formula F=$c_1$ Λ-----Λcm using variables $x_1$,$x_2$-------$x_n$such that the following graph G = (V, E) is planar.

V= { $x_{i,}$ $x_{i,}$ $y_{i,}$ $\bar{z_i}$/1≤i≤ $n$} Ụ{$c_j$ /l≤ $j \leq m$}

E= { $y_i$. $x_i$ },{ $y_i$. $x_i$ },{ $x_i$, $z_i$ }$\bar{}${ $x_i$, $z_i$ }/l≤ $i \leq n$}

Ụ {$z_i$, $y_i$+1}/≤ $i \leq n$ } Ụ {$z_n$,$y_1$} Ụ {λ, cj}/ l≤ $j \leq m$, λ a literal of cj}Satisfying assignment for F.

A graph G is constructed as follows. First we connect two vertices $z_n$ and $y_1$ by variable path as shown in the figure.

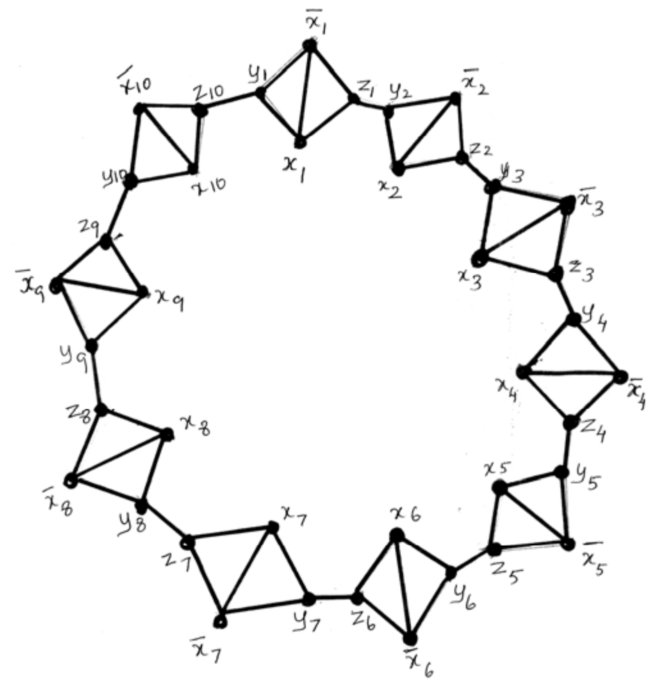


**Figure 3.1. Transforming 3SAT to Steiner problem in planar graph; the variable path.**

Clause gadget consisting of a $c_i$vertex connected to the literals contained in the clause $c_i$ by path of length t=2n+1.

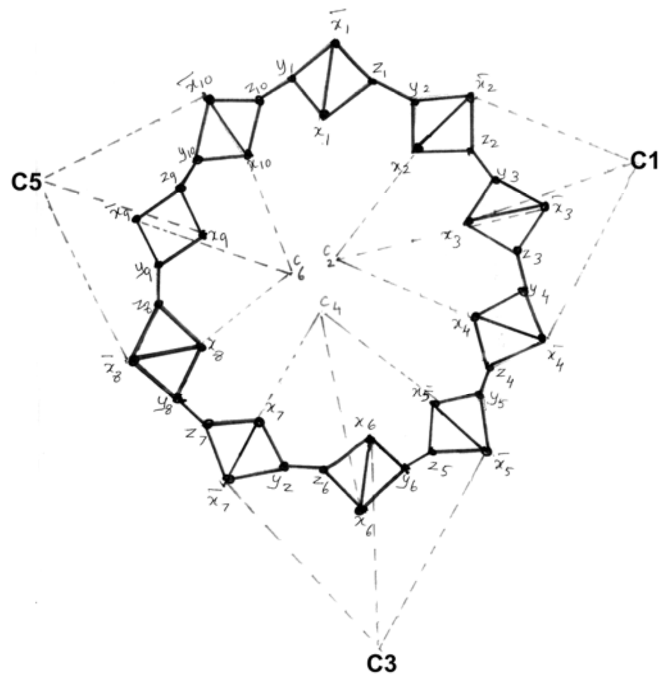



**Figure 3.2. The clause gadgēt for the clause $c_i$=$x_1$V $x_2$ V$x_3$.**

The dashed line indicate the path of length t=2n+1 from $c_i$ to the appropriate variable path.
Now we take n=10. To form the clauses {$c_1$, $c_2$, $c_3$—$c_6$} and the terminals set k = {y1z10, v}Ụ{$c_1$,$c_2$,$c_3$—$c_6$} and set B=2n+t.m
B=2n+t.m
t=2n+1
t=2(10)+1
t=21

B=2(10)+21(6)

=20+126=146

Assume first that the 3SAT instance is satisfiable. To construct a Steiner tree for K we start with $y_1$-$z_{10}$ path p reflecting a satisfying assignment. That is let $x_i \in p$ if $x_i$, is set to true in this assignment and $x_i \in p$ otherwise.

Next observe that for every clause the vertex $c_i$ can be connected to P by a path of length t. In this way we obtained a Steiner tree for k of length 2n + t. m=b.

To see other direction assume now that T is a Steiner tree for K of length at most B. Trivially for each clause the vertex $C_i$ has connected to the variable path.

Then $|E\ (T)| \geq (m+1).t>B$

$|E\ (T)| \geq (6+1).21>B$

$\geq 7(21)>B$

$\geq 147>146$

This shows that $y_1$-$z_{10}$ can only be connected along variable path, which requires $2_n$ edges.

**Result: 1**

Steiner problem in planar graph is NP – Complete.

**Result : 2**

Transforming 3SAT to Steiner problem in planar graph which is NP – Complete.

## CONCLUSION:

By using Planar 3 satisfiability to a NP – Complete Steiner Planar graph, it is obtained that Steiner problem in planar graph is NP – Complete.

## REFERENCES:


1. G.Nirmala and C.Sujatha, Every u-v Path of NP – Complete Steiner graphs contains Exactly 2n – edges, International journal of scientific and research publication, Volume – 3, Issue – 4, September 2014.
2. G.Nirmala and D.R. Kirubaharan, uses of Line graph, International Journal of Humanities and sciences, PMU – VOL 2, 2011.
3. G.Nirmala and M.Sheela, Fuzzy effective shortest spanning tree algorithm, International Journal of scientific transaction in environment and techno vision, Volume I – 2012.
4. Han Jurgen promel Angelika steger ‘The steiner tree problem’ Page 42-58
5. Alessandra Santurari, steiner Tree. NP – Completeness proof May 7, 2003
6. Michael R Gamey and David S.Johnson computers and Intractability. A Guide to the theory of NP – Completeness. W.H.Freeman and company 1979.
7. G. Nirmala & M. Sheela (2013) “Domination in Fuzzy Di-Graphs” Aryabhatta J. of Maths & Info. Vol. 5 (2) pp 275-278.
8. Professor Luca Trevisan Notes on NP – completeness – August 2009
9. Piete, Oloff de wet, Geometric Steiner Minimal Tiees, University of South Africa,January 2008.
10. Frank Harary, Graph Theory 1988, Page 32-40
11. K. Thilakam & A. Sumathi (2013) “Wiener Index of Chain Graphs” Vol. 5 (2) pp 347-352.
12. R.Balakrishnan, K.Ranganathan A Text Book of Graphs theory Page 152 – 180
13. G. Nirmala & K. Dharzabal (2013) “Algorithm for construction of self complementary $Kv_1v_2$ Fuzzy Graph” Aryabhatta J. of Maths & Info. Vol. 5 (1) pp 93-100.

Aryabhatta Journal of Mathematics & Informatics Vol. 6, No. 2, July-Dec, 2014 ISSN : 0975-7139
Journal Impact Factor (2013) : 0.489

# CYCLIC CODES OF LENGTH $4p^n$ OVER $GF(q)$, WHERE $q$ IS PRIME POWER OF THE FORM $4k+1$

**Sheetal Chawla*, Jagbir Singh****
*Department of Mathematics, IIT, Delhi-110016 (India)
**Department of Mathematics, M.D. University, Rohtak-124001 (India)
*chawlaasheetal@gmail.com, **ahlawatjagbir@yahoo.com

**ABSTRACT :**
***The explicit expression for the $4(n+1)$ primitive idempotents in $FG$ (the group algebra of the cyclic group $G$ of order $4p^n$, where $p$ is an odd prime, $n \geq 1$) over the finite field $F$ of prime power order $q$, where $q$ is of the form $4k+1$ and is a primitive root modulo $p^n$ are obtained. The minimum distances, dimensions and the generating polynomials of the minimal cyclic codes generated by these primitive idempotents are also obtained.***
***AMS Mathematical Subject Classification (2000): 11T71, 11G25, 22D20.***
**Keywords.** ***Group algebra, primitive idempotents, cyclotomiccosets.***

## 1. INTRODUCTION

Let $F = GF(q)$ be a field of prime power order $q$ and $G = \langle g \rangle = C_{4p^n}$ be a cyclic group of order $4p^n$, where $p$ is an odd prime and $n$ is a positive integer such that $g.c.d.(p,q) = 1$. Then $FC_{4p^n}$, the group algebra of the cyclic group $C_{4p^n}$ over $F$, is semi-simple. Let $t$ be the multiplicative order of $q$ modulo $p^n$, then $1 \leq t \leq \phi\left(p^n\right)$ [4]. Pruthi and Arora in [1, 7] computed the primitive idempotents of the minimal cyclic codes of length $m$ in case, when $t = \phi(m)$ and $m = 2, 4, p^n, 2p^n$. For $t = \frac{\phi\left(p^n\right)}{2}$, the minimal quadratic residue cyclic codes of length $p^n$ were studied by Batra and Arora[3]. The minimal cyclic codes of length $p^n q$ ($p$ and $q$ are odd primes) were studied by Bakshi and Raka[2], of length $2^m$ by Singh and Arora[8,9] and of length $4p^n$ for the field of prime power order of the form $4k+3$ by Pankaj and Arora[5].

Let $S = \{0,1,2,\ldots,8p^n - 1\}$. For $a, b \in S$, say that $a \sim b$ iff $a \equiv bq^i \pmod{4p^n}$ for some integer $i \geq 0$. This is an equivalence relation on the set $S$ and partitions $S$ into $4(n+1)$ $q$-cyclotomic cosets given by

$$\Omega_0 = \{0\},\ \Omega_{p^n} = \{p^n\},\ \Omega_{2p^n} = \{2p^n\},\ \Omega_{\lambda p^n} = \{\lambda p^n\},$$

and for $0 \leq j \leq n-1$,

$$\Omega_{p^j} = \left\{p^j, p^j q, \ldots, p^j q^{\phi\left(p^{n-j}\right)-1}\right\}, \Omega_{2p^j} = \left\{2p^j, 2p^j q, \ldots, 2p^j q^{\phi\left(p^{n-j}\right)-1}\right\},$$

$$\Omega_{4p^j} = \left\{4p^j, 4p^j q, \ldots, 4p^j q^{\phi\left(p^{n-j}\right)-1}\right\}, \Omega_{\lambda p^j} = \left\{\lambda p^j, \lambda p^j q, \ldots, \lambda p^j q^{\phi\left(p^{n-j}\right)-1}\right\}, \text{where } \lambda = 1 + 2p^n.$$

The 4(n+1)primitive idempotents are obtained in Theorem 2.14.In Section 3, we discuss minimal polynomials, generating polynomials, dimensions and minimum distances of the corresponding minimal cyclic codes of length $4p^n$ over $F$. To illustrate the paremeters an example is discussed in Section 4.

## 2. PRIMITIVE IDEMPOTENTS

**2.1 Notation.** Throughout this paper, $\alpha$ is a $4p^n$th root of unity in some extension field of $F$. Let $M_s$ be the minimal ideal in $R_{4p^n}$ generated by $\frac{\left(x^{4p^n}-1\right)}{m_s(x)}$. We denote $\theta_s(x)$, the primitive idempotent in $R_{4p^n}$, corresponding to the minimal ideal $M_s$, given by $\theta_s(x)=\frac{1}{4p^n}\sum_{i=0}^{4p^n-1}\varepsilon_i^s x^i$ where $\varepsilon_i^s=\sum_{j\in\Omega_s}\alpha^{-ij}$ [2]. Also, denote $\overline{C_s}=\sum_{s\in\Omega_s}x^s$.

**2.2 Lemma.** For any odd prime $p$ and positive integer $k$, if $\delta$ is a primitive $p^k$th root of unity in some extension field of $F$ and $q$ is a primitive root modulo $p^k$, then

$$\sum_{s=0}^{\phi\left(p^k\right)-1}\delta^{q^s}=\begin{cases}-1 & \text{if } k=1\\ 0 & \text{if } k\ge 2.\end{cases}$$

**2.3 Lemma.** For any odd prime $p$ and positive integer $k$, if $\delta$ is a primitive $2p^k$th root of unity in some extension field of $F$ and $q$ is a primitive root modulo $p^k$, then

$$\sum_{s=0}^{\phi\left(2p^k\right)-1}\delta^{q^s}=\begin{cases}1 & \text{if } k=1\\ 0 & \text{if } k\ge 2.\end{cases}$$

**2.4Lemma.** For $0\le i\le n$, $\left(1+2p^n\right)q^{\frac{\phi\left(p^n\right)}{2}}\equiv -1 \left(\text{mod } 4p^n\right)$.

**Proof.**Since $q^{\frac{\phi\left(p^n\right)}{2}}\equiv -1\left(\text{mod } p^n\right)$.

Also$\left(1+2p^n\right)\equiv 1 \left(\text{mod } p^n\right)$.Therefore, $\left(1+2p^n\right)q^{\frac{\phi\left(p^n\right)}{2}}\equiv -1\left(\text{mod } p^n\right)$.

Further, since $p\equiv 1 \ (\text{mod } 4)$, so $\frac{\phi\left(p^n\right)}{2}$ is even. Also, $q$ is of the form $4k+1$, therefore,$q^{\frac{\phi\left(p^n\right)}{2}}$ is of the form $4k_1+1$, so $\left(1+2p^n\right)q^{\frac{\phi\left(p^n\right)}{2}}\equiv -1 \left(\text{mod } 4\right)$.

But $g.c.d.\left(4, p^n\right)=1$, thus$\left(1+2p^n\right)q^{\frac{\phi\left(p^n\right)}{2}}\equiv -1\left(\text{mod } 4p^n\right)$.

**2.5 Remark.**By above lemma, we obtain that$-\Omega_{p^j}=\Omega_{\lambda p^j}$.

**2.6 Lemma.** For cyclotomiccosets $\Omega_{p^j}$, $0\le j\le n$,

$$\lambda^2\Omega_{p^j}=\Omega_{p^j}=\lambda\Omega_{\lambda p^j}$$

**2.7 Lemma.**If $r=1$ or $\lambda$ then for $0\le j\le n-1$,

$$r\Omega_{2p^j}=\Omega_{2p^j}=2\Omega_{\lambda p^j}=2\Omega_{p^j}.$$

**2.8 Lemma.**If $r=1,2,4$ or $\lambda$ then for $0\le j\le n-1$,

$$r\Omega_{4p^j}=\Omega_{4p^j}=4\Omega_{\lambda p^j}=2\Omega_{2p^j}.$$

**2.9Notation.** Let $\alpha$ be a fixed primitive $4p^n$th root of unity in some extension field of $F$. For $0\le j\le n-1$, we define,

$$T_j=p^j\sum_{s\in\Omega_{p^j}}\alpha^s,\ S_j=p^j\sum_{s\in\Omega_{\lambda p^j}}\alpha^s.$$

Then, $T_j,\ S_j\in F$.

**2.10 Lemma.** For $0 \le j \le n-1$, $T_j + S_j = 0$.

**Proof.** Since $S_j = p^j \sum_{s \in \Omega_{\lambda p^j}} \alpha^s = p^j \left( \alpha^{\lambda p^j} + \alpha^{\lambda p^j q} + \ldots + \alpha^{\lambda p^j q^{\phi\left(p^{n-j}\right)-1}} \right)$

and $\alpha^{\lambda p^j} = \alpha^{\left(1+2p^n\right)p^j} = \alpha^{p^j} \alpha^{2p^n p^j} = -\alpha^{p^j}$.

Therefore, $S_j = -p^j \left( \alpha^{p^j} + \alpha^{p^j q} + \ldots + \alpha^{p^j q^{\phi\left(p^{n-j}\right)-1}} \right) = -p^j \sum_{s \in \Omega_{p^j}} \alpha^s = -T_j$

or, $S_j + T_j = 0$.

**2.11 Lemma.** For $0 \le j \le n-1,\ 0 \le i \le n$

$$\sum_{s\in\Omega_{p^j}} \alpha^{p^i s} = \sum_{s\in\Omega_{\lambda p^j}} \alpha^{\lambda p^i s} = -\sum_{s\in\Omega_{p^j}} \alpha^{\lambda p^i s} = -\sum_{s\in\Omega_{\lambda p^j}} \alpha^{p^i s} = \begin{cases} \phi(p^{n-j})\alpha^{p^{i+j}} & \text{if } i+j \ge n \\ \dfrac{1}{p^j} T_{i+j} & \text{if } i+j \le n\text{-1.} \end{cases}$$

**Proof.** Let $\delta = \alpha^{p^{i+j}}$. Then

$$\sum_{s\in\Omega_{p^j}} \alpha^{p^i s} = \sum_{t=0}^{\phi\left(p^{n-j}\right)-1} \alpha^{p^{i+j} q^t} = \sum_{t=0}^{\phi\left(p^{n-j}\right)-1} \delta^{q^t}.$$

Now consider the following cases:

**Case 1.**If $i + j \ge n$, then $\delta$ is a4th root of unity and

$\delta^{q^l} = \delta^{q^r}$ iff $q^l \equiv q^r \left(\bmod 4\right)$ iff $l \equiv r \left(\bmod \phi\left(4\right)\right)$.

Hence $\sum_{s\in\Omega_{p^j}} \alpha^{p^i s} = \phi(p^{n-j})\alpha^{p^{i+j}}$.

**Case 2.**If $i + j \le n-1$, then $\delta$ is a $4p^{n-i-j}$ th root of unity and

$\delta^{q^l} = \delta^{q^r}$ iff $q^l \equiv q^r \left(\bmod 4p^{n-i-j}\right)$ iff $l \equiv r \left(\bmod \phi\left(p^{n-i-j}\right)\right)$.

Therefore, $\sum_{s\in\Omega_{p^j}} \alpha^{p^i s} = \dfrac{\phi\left(p^{n-j}\right)}{\phi\left(p^{n-i-j}\right)} \sum_{t=0}^{\phi\left(p^{n-i-j}\right)-1} \delta^{q^t} = \dfrac{1}{p^j} p^{i+j} \sum_{t=0}^{\phi\left(p^{n-i-j}\right)-1} \alpha^{p^{i+j}q^t} = \dfrac{1}{p^j} p^{i+j} \sum_{s\in\Omega_{p^{i+j}}} \alpha^s$.

Since $p^{i+j} \sum_{s\in\Omega_{p^{i+j}}} \alpha^s = T_{i+j}$, therefore, $\sum_{s\in\Omega_{p^j}} \alpha^{p^i s} = \dfrac{1}{p^j} T_{i+j}$.

The proof of the following lemmas will go on similar lines as of the above lemma.

**2.12 Lemma.**For $0 \le j \le n-1,\ 0 \le i \le n$,

$$\sum_{s\in\Omega_{p^j}} \alpha^{2p^i s} = \sum_{s\in\Omega_{\lambda p^j}} \alpha^{2p^i s} = \sum_{s\in\Omega_{2p^j}} \alpha^{p^i s} = \sum_{s\in\Omega_{2p^j}} \alpha^{\lambda p^i s} = \begin{cases} -\phi\left(p^{n-j}\right) & \text{if } i+j \ge n \\ p^{n-j-1} & \text{if } i+j = n-1 \\ 0 & \text{if } i+j < n\text{-1.} \end{cases}$$

**2.13 Lemma.**For $0 \le i \le n$, $0 \le j \le n-1$,

$$\sum_{s\in\Omega_{2p^j}} \alpha^{2p^i s} = \sum_{s\in\Omega_{4p^j}} \alpha^{p^i s} = \sum_{s\in\Omega_{\lambda p^j}} \alpha^{4p^i s} = \sum_{s\in\Omega_{4p^j}} \alpha^{\lambda p^i s} = \sum_{s\in\Omega_{p^j}} \alpha^{4p^i s} = \sum_{s\in\Omega_{4p^j}} \alpha^{2p^i s}$$

$$= \sum_{s\in\Omega_{4p^j}} \alpha^{4p^i s} = \sum_{s\in\Omega_{2p^j}} \alpha^{4p^i s} = \begin{cases} \phi\left(p^{n-j}\right) & \text{if } i+j \ge n \\ -\mathrm{p}^{n-j-1} & \text{if } i+j = n\text{-1} \\ 0 & \text{if } i+j < n\text{-1.} \end{cases}$$

**2.14 Theorem.** The explicit expressions for the $4(n+1)$ primitive idempotents in $R_{4p^n}$ are given by

$$\theta_0(x)=\frac{1}{4p^n}\left[\overline{C}_0+\overline{C}_{p^n}+\overline{C}_{2p^n}+\overline{C}_{\lambda p^n}+\sum_{i=0}^{n-1}\left\{\overline{C}_{p^i}+\overline{C}_{2p^i}+\overline{C}_{4p^i}+\overline{C}_{\lambda p^i}\right\}\right]$$

$$\theta_{p^n}(x)=\frac{1}{4p^n}\left[\overline{C}_0-\alpha^{p^{2n}}\overline{C}_{p^n}-\overline{C}_{2p^n}+\alpha^{p^{2n}}\overline{C}_{\lambda p^n}-\sum_{i=0}^{n-1}\left\{\alpha^{p^{n+i}}\overline{C}_{p^i}+\overline{C}_{2p^i}-\overline{C}_{4p^i}-\alpha^{p^{n+i}}\overline{C}_{\lambda p^i}\right\}\right]$$

$$\theta_{2p^n}(x)=\frac{1}{4p^n}\left[\overline{C}_0-\overline{C}_{p^n}+\overline{C}_{2p^n}-\overline{C}_{\lambda p^n}-\sum_{i=0}^{n-1}\left\{\overline{C}_{p^i}-\overline{C}_{2p^i}-\overline{C}_{4p^i}+\overline{C}_{\lambda p^i}\right\}\right]$$

$$\theta_{\lambda p^n}(x)=\frac{1}{4p^n}\left[\overline{C}_0+\alpha^{p^{2n}}\overline{C}_{p^n}-\overline{C}_{2p^n}-\alpha^{p^{2n}}\overline{C}_{\lambda p^n}+\sum_{i=0}^{n-1}\left\{\alpha^{p^{n+i}}\overline{C}_{p^i}-\overline{C}_{2p^i}+\overline{C}_{4p^i}-\alpha^{p^{n+i}}\overline{C}_{\lambda p^i}\right\}\right]$$

and for $0\le j\le n-1$,

$$\theta_{p^j}(x)=\frac{1}{4p^n}\Big[\phi\left(p^{n-j}\right)\left\{\overline{C}_0-\alpha^{p^{n+j}}\overline{C}_{p^n}-\overline{C}_{2p^n}+\alpha^{p^{n+j}}\overline{C}_{\lambda p^n}\right\}+p^{n-j-1}\left\{\overline{C}_{2p^{n-j-1}}-\overline{C}_{4p^{n-j-1}}\right\}$$
$$-\phi\left(p^{n-j}\right)\sum_{i=n-j}^{n-1}\left\{\alpha^{p^{i+j}}\overline{C}_{p^i}+\overline{C}_{2p^i}-\overline{C}_{4p^i}-\alpha^{p^{i+j}}\overline{C}_{\lambda p^i}\right\}-\frac{1}{p^j}\sum_{i=0}^{n-j-1}\left\{T_{i+j}\overline{C}_{p^i}-T_{i+j}\overline{C}_{\lambda p^i}\right\}\Big]$$

$$\theta_{2p^j}(x)=\frac{1}{4p^n}\Big[\phi\left(p^{n-j}\right)\left\{\overline{C}_0-\overline{C}_{p^n}+\overline{C}_{2p^n}-\overline{C}_{\lambda p^n}\right\}-\phi\left(p^{n-j}\right)\sum_{i=n-j}^{n-1}\left\{\overline{C}_{p^i}-\overline{C}_{2p^i}-\overline{C}_{4p^i}+\overline{C}_{\lambda p^i}\right\}$$
$$+p^{n-j-1}\left\{\overline{C}_{p^{n-j-1}}-\overline{C}_{2p^{n-j-1}}-\overline{C}_{4p^{n-j-1}}+\overline{C}_{\lambda p^{n-j-1}}\right\}\Big]$$

$$\theta_{4p^j}(x)=\frac{1}{4p^n}\Big[\phi\left(p^{n-j}\right)\left\{\overline{C}_0+\overline{C}_{p^n}+\overline{C}_{2p^n}+\overline{C}_{\lambda p^n}\right\}+\phi\left(p^{n-j}\right)\sum_{i=n-j}^{n-1}\left\{\overline{C}_{p^i}+\overline{C}_{2p^i}+\overline{C}_{4p^i}+\overline{C}_{\lambda p^i}\right\}$$
$$-p^{n-j-1}\left\{\overline{C}_{p^{n-j-1}}+\overline{C}_{2p^{n-j-1}}+\overline{C}_{4p^{n-j-1}}+\overline{C}_{\lambda p^{n-j-1}}\right\}\Big]$$

$$\theta_{\lambda p^j}(x)=\frac{1}{4p^n}\Big[\phi\left(p^{n-j}\right)\left\{\overline{C}_0+\alpha^{p^{n+j}}\overline{C}_{p^n}-\overline{C}_{2p^n}-\alpha^{p^{n+j}}\overline{C}_{\lambda p^n}\right\}+p^{n-j-1}\left\{\overline{C}_{2p^{n-j-1}}-\overline{C}_{4p^{n-j-1}}\right\}$$
$$+\phi\left(p^{n-j}\right)\sum_{i=n-j}^{n-1}\left\{\alpha^{p^{i+j}}\overline{C}_{p^i}-\overline{C}_{2p^i}+\overline{C}_{4p^i}-\alpha^{p^{i+j}}\overline{C}_{\lambda p^i}\right\}+\frac{1}{p^j}\sum_{i=0}^{n-j-1}\left\{T_{i+j}\overline{C}_{p^i}-T_{i+j}\overline{C}_{\lambda p^i}\right\}\Big]$$

where $T_{n-1}=\sqrt{p^{2n-1}-2p^{2n-2}}$, andfor $j\le n-2$, $T_j=0$.

## 3. DIMENSION, GENERATING POLYNOMIAL AND MINIMUM DISTANCE

If $\alpha$ is a primitive $4p^n$ th root of unity, then $m_s(x)=\prod_{s\in\Omega_s}\left(x-\alpha^s\right)$ denotes the minimal polynomial for $\alpha^s$.

Assume that $\beta$ is 4th root of unity in F.

**3.1 Remark.[6]**If $m_s(x)$ denotes the minimal polynomial for $\alpha^s$, $s\in\Omega_s$, then generating polynomial for cyclic code of length $4p^n$ corresponding to the cyclotomiccoset $\Omega_s$ is $\dfrac{x^{4p^n}-1}{m_s(x)}$ and the dimension of the minimal cyclic code $M_s$ is equal to the cardinality of the class $\Omega_s$. Thus, the dimensions of the codes $M_0, M_{p^n}, M_{2p^n}, M_{\lambda p^n}, M_{2p^j}$ and $M_{4p^j}$ are1, 1, 1, 1, $\phi\left(p^{n-j}\right)$ and $\phi\left(p^{n-j}\right)$ respectively.

**3.2 Lemma.[2]** If $l$ is a cyclic code of length $m$ generated by $g(x)$ and its minimum distance is $d$, then the code $\widehat{l}$ of length $mk$ generated by $g(x)\left(1+x^m+x^{2m}+\ldots+x^{(k-1)m}\right)$ is a repetition code of $l$ repeated $k$ times and its minimum distance is $dk$.

**3.3Theorem.(i)** The generating polynomials of the codes $M_0, M_{p^n}, M_{2p^n}$ and $M_{\lambda p^n}$ are $\left(1+x+x^2+\ldots+x^{4p^n-1}\right)$, $(x^2-1)(x+\beta)\left(1+x^4+\ldots+x^{4\left(p^n-1\right)}\right)$, $(x^3-x^2+x-1)\left(1+x^4+\ldots+x^{4\left(p^n-1\right)}\right)$ and $(x^2-1)(x-\beta)\left(1+x^4+\ldots+x^{4\left(p^n-1\right)}\right)$ respectively.

**(ii)** The generating polynomials of the codes $M_{2p^j}$ and $M_{4p^j}$, for $1\le j\le n-1$, are $\left(x^{p^{n-j-1}}+1\right)\left(x^{p^{n-j}}-1\right)\left(x^{2p^{n-j}}+1\right)\left(1+x^{4p^{n-j}}+\ldots+x^{4p^{n-j}\left(p^j-1\right)}\right)$ and $\left(x^{p^{n-j-1}}-1\right)\left(x^{p^{n-j}}+1\right)\left(x^{2p^{n-j}}+1\right)\left(1+x^{4p^{n-j}}+\ldots+x^{4p^{n-j}\left(p^j-1\right)}\right)$ respectively.

**(iii)** The generating polynomial for $M_{p^j}\oplus M_{\lambda p^j}$, for $1\le j\le n-1$, is $\left(x^{2p^{n-j-1}}+1\right)\left(x^{4p^{n-j-1}}+1\right)\left(x^{2p^{n-j}}-1\right)\left(1+x^{8p^{n-j}}+\ldots+x^{8p^{n-j}\left(p^j-1\right)}\right)$.

**Proof. (i)** The minimal polynomial for $\alpha^0, \alpha^{p^n}, \alpha^{2p^n}$ and $\alpha^{\lambda p^n}$ are $(x-1)$, $(x-\beta)$, $(x+1)$ and $(x+\beta)$ respectively. By remark 3.1, the corresponding generating polynomials are $\left(1+x+x^2+\ldots+x^{4p^n-1}\right)$, $(x^2-1)(x+\beta)\left(1+x^4+\ldots+x^{4\left(p^n-1\right)}\right)$, $(x^3-x^2+x-1)\left(1+x^4+\ldots+x^{4\left(p^n-1\right)}\right)$ and $(x^2-1)(x-\beta)\left(1+x^4+\ldots+x^{4\left(p^n-1\right)}\right)$ respectively.

**(ii)** Consider the polynomial $x^{p^{n-j-1}\phi(p)}-x^{p^{n-j-1}(\phi(p)-1)}+\ldots-x^{p^{n-j-1}}+1=\dfrac{x^{p^{n-j}}+1}{x^{p^{n-j-1}}+1}$ with root $\alpha^{2p^j}$. Since $2p^j\in\Omega_{2p^j}$ and $|\Omega_{2p^j}|=\phi\left(p^{n-j}\right)$, therefore, $\dfrac{x^{p^{n-j}}+1}{x^{p^{n-j-1}}+1}$ is the minimal polynomial of $\alpha^{2p^j}$. By remark 3.1, the corresponding generating polynomial for $M_{2p^j}$ is $\left(x^{p^{n-j-1}}+1\right)\left(x^{p^{n-j}}-1\right)\left(x^{2p^{n-j}}+1\right)\left(1+x^{4p^{n-j}}+\ldots+x^{4p^{n-j}\left(p^j-1\right)}\right)$.

Similarly, $\left(x^{p^{n-j-1}}-1\right)\left(x^{p^{n-j}}+1\right)\left(x^{2p^{n-j}}+1\right)\left(1+x^{4p^{n-j}}+\ldots+x^{4p^{n-j}\left(p^j-1\right)}\right)$ is the generating polynomial for $M_{4p^j}$.

**(iii)** The product of the minimal polynomials satisfied by $\alpha^{p^j}$ and $\alpha^{\lambda p^j}$ is $\dfrac{\left(x^{2p^{n-j}}+1\right)}{\left(x^{2p^{n-j-1}}+1\right)}$. Therefore, the generating polynomial for $M_{p^j}\oplus M_{\lambda p^j}$ is $\left(x^{2p^{n-j-1}}+1\right)\left(x^{2p^{n-j}}-1\right)\left(1+x^{4p^{n-j}}+\ldots+x^{4p^{n-j}\left(p^j-1\right)}\right)$.

**3.4Theorem.** The minimum distance of the codes $M_0$, $M_{p^n}$, $M_{2p^n}$ and $M_{\lambda p^n}$ are $4p^n$, $4p^n$, $4p^n$ and $4p^n$ respectively.

**Proof.** Since the generating polynomial for the code $M_0$ is $\left(1+x+\ldots+x^{8p^n-1}\right)$, which is itself a polynomial of length $8p^n$, hence its minimum distance is $8p^n$.

Also, the generating polynomial for the cyclic code $M_{p^n}$ is $(x^2-1)(x+\beta)\left(1+x^4+\ldots+x^{4\left(p^n-1\right)}\right)$. If we take a cyclic code of length 4 generated by the polynomial $(x^2-1)(x+\beta)$, then the minimum distance of this code is 4. Since the cyclic code of length $4p^n$ with generating polynomial $(x^2-1)(x+\beta)\left(1+x^4+\ldots+x^{4\left(p^n-1\right)}\right)$, is a repetition of the cyclic code of length 4 with generating polynomial $(x^2-1)(x+\beta)$, repeated $p^n$ times, therefore, its minimum distance is $4p^n$.

Similarly, the minimum distance of the cyclic code $M_{2p^n}$ with generating polynomial $(x^3-x^2+x-1)\left(1+x^4+\ldots+x^{4\left(p^n-1\right)}\right)$ is $4p^n$ as it is repetition of the cyclic code of length 4 with generating polynomial $(x^3-x^2+x-1)$ whose minimum distance is 4, repeated $p^n$ times.

In a similar way, the minimum distance of the cyclic code $M_{\lambda p^n}$ with generating polynomial $(x^2-1)(x-\beta)\left(1+x^4+\ldots+x^{4\left(p^n-1\right)}\right)$ is $4p^n$ as it is repetition of the cyclic code of length 4 with generating polynomial $(x^2-1)(x-\beta)$ whose minimum distance is 4, repeated $p^n$ times.

**3.5 Theorem.** For $0\le j\le n-1$, the minimum distance of the cyclic codes $M_{2p^j}$ and $M_{4p^j}$ are $8p^j$ and for the codes $M_{p^j}$ and $M_{\lambda p^j}$ are greater than or equal to $4p^j$.

**Proof.** Consider the cyclic code $M_{2p^j}$. Since the generating polynomial of the cyclic code of length $4p^n$ is $\left(x^{p^{n-j-1}}+1\right)\left(x^{p^{n-j}}-1\right)\left(x^{2p^{n-j}}+1\right)\left(1+x^{4p^{n-j}}+\ldots+x^{4p^{n-j}\left(p^j-1\right)}\right)$, therefore, if we take a cyclic code $C$ of length $p^{n-j}$ generated by the polynomial $\left(x^{p^{n-j-1}}+1\right)$, then the minimum distance of this code is 2. Now consider the cyclic code $C_1$ of length $2p^{n-j}$ generated by the polynomial $\left(x^{p^{n-j-1}}+1\right)\left(x^{p^{n-j}}-1\right)$, and then the minimum distance of this code is 4, as it is 2 time repetition of the code $C$. Further, the minimum distance of the code $C_2$ of length $4p^{n-j}$ generated by the polynomial $\left(x^{p^{n-j-1}}+1\right)\left(x^{p^{n-j}}-1\right)\left(x^{2p^{n-j}}+1\right)$ is 8, as it is 2 time repetition of the code $C_1$. Hence the minimum distance of the code $C_3$ of length $4p^n$ generated by the polynomial $\left(x^{p^{n-j-1}}+1\right)\left(x^{p^{n-j}}-1\right)\left(x^{2p^{n-j}}+1\right)\left(1+x^{4p^{n-j}}+\ldots+x^{4p^{n-j}\left(p^j-1\right)}\right)$ is a repetition code of the cyclic code $C_2$, repeated $p^j$ times, therefore, its minimum distance is $8p^j$.

Similarly, the minimum distance of the cyclic code $M_{4p^j}$ of length $4p^n$ with generating polynomial $\left(x^{p^{n-j-1}}-1\right)\left(x^{p^{n-j}}+1\right)\left(x^{2p^{n-j}}+1\right)\left(1+x^{4p^{n-j}}+\ldots+x^{4p^{n-j}\left(p^j-1\right)}\right)$, is also $8p^j$. Since the product of generating polynomial for the cyclic codes $M_{p^j}$ and $M_{\lambda p^j}$ is $\left(x^{2p^{n-j-1}}+1\right)\left(x^{2p^{n-j}}-1\right)\left(1+x^{4p^{n-j}}+\ldots+x^{4p^{n-j}\left(p^j-1\right)}\right)$, therefore, if we take a code $C$ of length $4p^{n-j}$ generated by the polynomial $\left(x^{2p^{n-j-1}}+1\right)\left(x^{2p^{n-j}}-1\right)$, then the minimum distance of this code is 4. Since the cycliccode $C_1$ of length $4p^n$ generated by the polynomial $\left(x^{2p^{n-j-1}}+1\right)\left(x^{2p^{n-j}}-1\right)\left(1+x^{4p^{n-j}}+...+x^{4p^{n-j}\left(p^j-1\right)}\right)$ is a repetition code of the code $C$, repeated $p^j$ times. Hence its minimum distance is $4p^j$.

The codes corresponding to $\Omega_{p^j}$ and $\Omega_{\lambda p^j}$ are the sub codes of above code sotheir minimum distance is greater than or equal to $4p^j$.

**Example 4**

**4.1 Example.** Cyclic code of length 12

Take $p=3, n=1, q=5$. Then, the $q$-cyclotomiccosets modulo 12 are

$\Omega_0=\{0\}$, $\Omega_1=\{1,5\}$, $\Omega_2=\{2,10\}$, $\Omega_3=\{3\}$,

$\Omega_4=\{4,8\}$, $\Omega_6=\{6\}$, $\Omega_7=\{7,11\}$, $\Omega_9=\{9\}$

Also, $\beta=\alpha^3=2,\ T_0=1,\ S_0=4$, and the corresponding primitiveidempotents in $\frac{GF(17)[x]}{<x^{24}-1>}$ are

$$\theta_0(x)=\frac{1}{12}\left[\overline{C}_0+\overline{C}_3+\overline{C}_6+\overline{C}_9+\overline{C}_1+\overline{C}_2+\overline{C}_4+\overline{C}_7\right]$$
$$=\frac{1}{12}\left[1+x+x^2+x^3+x^4+x^5+x^6+x^7+x^8+x^9+x^{10}+x^{11}\right]$$
$$\theta_1(x)=\frac{1}{12}\left[2\overline{C}_0+\overline{C}_3+3\overline{C}_6+4\overline{C}_9+4\overline{C}_1+\overline{C}_2+4\overline{C}_4+\overline{C}_7\right]$$
$$=\frac{1}{12}\left[2+4x+x^2+x^3+4x^4+4x^5+3x^6+x^7+4x^8+4x^9+x^{10}+x^{11}\right]$$
$$\theta_2(x)=\frac{1}{12}\left[2\overline{C}_0+3\overline{C}_3+2\overline{C}_6+3\overline{C}_9+\overline{C}_1+4\overline{C}_2+4\overline{C}_4+\overline{C}_7\right]$$
$$=\frac{1}{12}\left[2+x+4x^2+3x^3+4x^4+x^5+2x^6+x^7+4x^8+3x^9+4x^{10}+x^{11}\right]$$
$$\theta_3(x)=\frac{1}{12}\left[\overline{C}_0+2\overline{C}_3+4\overline{C}_6+3\overline{C}_9+3\overline{C}_1+4\overline{C}_2+\overline{C}_4+2\overline{C}_7\right]$$
$$=\frac{1}{12}\left[1+3x+4x^2+2x^3+x^4+3x^5+4x^6+2x^7+x^8+3x^9+4x^{10}+2x^{11}\right]$$
$$\theta_4(x)=\frac{1}{12}\left[2\overline{C}_0+2\overline{C}_3+2\overline{C}_6+2\overline{C}_9+4\overline{C}_1+4\overline{C}_2+4\overline{C}_4+4\overline{C}_7\right]$$
$$=\frac{1}{12}\left[2+4x+4x^2+2x^3+4x^4+4x^5+2x^6+4x^7+4x^8+2x^9+4x^{10}+4x^{11}\right]$$
$$\theta_6(x)=\frac{1}{12}\left[\overline{C}_0+4\overline{C}_3+\overline{C}_6+4\overline{C}_9+4\overline{C}_1+\overline{C}_2+\overline{C}_4+4\overline{C}_7\right]$$
$$=\frac{1}{12}\left[1+4x+x^2+4x^3+x^4+4x^5+x^6+4x^7+x^8+4x^9+x^{10}+4x^{11}\right]$$
$$\theta_7(x)=\frac{1}{12}\left[2\overline{C}_0+4\overline{C}_3+3\overline{C}_6+\overline{C}_9+\overline{C}_1+\overline{C}_2+4\overline{C}_4+4\overline{C}_7\right]$$
$$=\frac{1}{12}\left[2+x+x^2+4x^3+4x^4+x^5+4x^6+4x^7+4x^8+x^9+x^{10}+4x^{11}\right]$$

$$\theta_9(x) = \frac{1}{12}\left[\overline{C}_0 + 3\overline{C}_3 + 4\overline{C}_6 + 2\overline{C}_9 + 2\overline{C}_1 + 4\overline{C}_2 + \overline{C}_4 + 3\overline{C}_7\right]$$
$$= \frac{1}{12}\left[1 + 2x + 4x^2 + 3x^3 + x^4 + 2x^5 + 4x^6 + 3x^7 + x^8 + 2x^9 + 4x^{10} + 3x^{11}\right]$$

Minimal polynomials of $\alpha^0, \alpha^1, \alpha^2, \alpha^3, \alpha^4, \alpha^6, \alpha^7$ and $\alpha^9$ are $x-1$, $x^2-2x-1$, $x^2+4x+1$, $x-2$, $x^2+x+1$, $x+1$, $x^2+2x-1$ and $x+2$ respectively.

The minimal codes, $M_0, M_1, M_2, M_3, M_4, M_6, M_7$ and $M_9$ of length 12 are as follows:

| Code | Dimension | Minimum Distance Bound | Generating Polynomial |
|---|---|---|---|
| $M_0$ | 1 | 12 | $1+x+x^2+x^3+x^4+x^5+x^6+x^7+x^8+x^9+x^{10}+x^{11}$ |
| $M_1$ | 2 | $4 \le d \le 8$ | $1+3x+3x^3+4x^4+4x^6+2x^7+2x^9+x^{10}$ |
| $M_2$ | 2 | 8 | $4+4x+x^3+x^4+4x^6+4x^7+x^9+x^{10}$ |
| $M_3$ | 1 | 12 | $3+4x+2x^2+x^3+3x^4+4x^5+2x^6+x^7+3x^8+4x^9+2x^{10}+x^{11}$ |
| $M_4$ | 2 | 8 | $4+x+4x^3+x^4+4x^6+x^7+4x^9+x^{10}$ |
| $M_6$ | 1 | 12 | $4+x+4x^2+x^3+4x^4+x^5+4x^6+x^7+4x^8+x^9+4x^{10}+x^{11}$ |
| $M_7$ | 2 | $4 \le d \le 8$ | $1+2x+2x^3+4x^4+4x^6+3x^7+3x^9+x^{10}$ |
| $M_9$ | 1 | 12 | $2+4x+3x^2+x^3+2x^4+4x^5+3x^6+x^7+2x^8+4x^9+3x^{10}+x^{11}$ |

## References


[1] Arora, S.K., Pruthi, M.: Minimal Cyclic Codes of Length $2p^n$, Finite Fields Appl., 5, 177-187 (1999)

[2] Bakshi, G.K., Raka, M.: Minimal Cyclic codes of length $p^n q$, Finite Fields Appl., 9, 432-448 (2003)

[3] Batra, S., Arora, S.K.: Minimal Quadratic Residue Cyclic Codes of Length $p^n$(p odd prime), J. Appl. Math. Comput. (Old: KJCAM), 8, 531-547 (2001)

[4] Hardy, G.H., Wright, E.M.: An Introduction to the Theory of Numbers, Oxford University Press (1959)

[5] Kumar, P., Arora, S.K.: Primitive idempotents in $\frac{GF(l)[x]}{<x^m-1>}$, (m - odd) and minimal cyclic codes of length $p^r q^t$ over GF($l$), Doctoral Thesis, Maharshi Dayanand University, India (2007)

[6] Pless, V.: Introduction of the Theory of Error Correcting Codes, Wiley, New York (1981)

[7] Pruthi, M., Arora, S.K., Minimal Codes of Prime-Power Length, Finite Fields Appl., 3, 99-113 (1997)

[8] Singh, K., Arora, S.K.: Primitive Idempotents in $FC_{2^n}$ -I, Int. J. Algebra, 4, 1231-1241 (2010)

[9] Singh, K., Arora, S.K.: Primitive Idempotents in $FC_{2^n}$ -II, Int. J. Algebra, 4, 1243-1254 (2010)

[10] Vinocha O.P, K. Binni etal. “Cyclotomic cosets of even mode for RS Codes” Aryabhatta J. of Maths & Info. Vol. 3 (1) pp 119-122 (2011)