METHOD FOR SOLVING BRANCH-AND-BOUND TECHNIQUE FOR ASSIGNMENT PROBLEMS USING TRIANGULAR AND TRAPEZOIDALFUZZY NUMBERS

B.Srinivas
Department of Mathematics
ISTS Women's Engineering College, Rajahmundry, Andhra Pradesh, India

G.Ganesan
Department of Mathematics
Adikavi Nannaya University, Rajahmundry, Andhra Pradesh, India

Abstract: Assignment problem is a well-known topic and is used very often in solving Problems of engineering and management science. In this problem $\tilde{C}_{ij}$ denotes the cost for assigning jobs i = 1,2,…., m to the person j = 1,2,…, n . This cost is usually deterministic in nature. In this paper $\tilde{C}_{ij}$ has been considered to be triangular or trapezoidal fuzzy numbers denoted by $\tilde{C}_{ij}$ which are more realistic and general in nature. In this paper first the proposed fuzzy assignment problem has been transformed into crisp assignment problem in the linear programming problem form and solved by using Branch and bound and Robust's ranking method [5] for the fuzzy numbers. A Numerical example is taken to illustrate the solution procedure.

.***Keywords*:** Fuzzy Number, Fuzzy Assignment Problem, Triangular fuzzy number, Trapezoidal fuzzy number, Branch and bound Technique, Robust ranking method

1. Introduction:

Assignment Problem (AP) is used worldwide in solving real world problems. An assignment problem plays an important role in industry and other applications. It is a one of the well-studied optimization problems in Management Science and has been widely applied in both manufacturing and service systems. In an assignment problem, n jobs are to be performed by n persons depending on their efficiency to do the job. In this problem $\tilde{C}_{ij}$ denotes the cost of assigning the i'th job to the j'th person. We assume that one person can be assigned exactly one job; also each person can do at most one job. The problem is to find an optimal assignment so that the total cost of performing all jobs is minimum or the total profit is maximum. In 1965, Lotfi Zadeh [13] has introduced fuzzy sets which provide as a new mathematical tool to deal with uncertainty of information Fortemps and Roubens [5] Ranking and defuzzification methods based area compensation fuzzy sets and systems. The AP introduced by Votaw and Orden [10] can be solved using the linear programming technique, the transportation algorithm or the Hungarian method developed by Kuhn [7]. The Hungarian method is recognized to be the first practical method for solving the standard assignment problem. Balinski and Gomory [2] introduced a labeling algorithm for solving the transportation and assignment problems. Nagarajan and Solairaju [8] presented an algorithm for solving fuzzy assignment problems using Robust ranking technique with fixed fuzzy numbers. Geetha et.al [9] first formulated cost time assignment problem has the multi criterion problem. In existing literature, several researchers developed different metrologies for solving generalized assignment problem. Among this, one may refer to the works of Ross et.al [6]. Bai et.al [1] proposed a method for solving fuzzy generalized assignment problem. Chen [3] proved some theorems and proved a fuzzy assignment model that considers all individuals to have same skills. Wang[11] solved a similar model by graph theory. Kalaiarasi et.al [8] Optimization of fuzzy assignment model with triangular fuzzy numbers, Dubois and Fortemps [4] surveys refinements of the ordering of solutions supplied by the max-min formulation. Chi-Jen et.al [2], Labelling algorithm for the fuzzy assignment problem.

In this paper first the proposed fuzzy assignment problem has been transformed into crisp assignment problem in the linear programming problem form and solved by using Branch and Bound Technique and Robust's ranking method [5] for the fuzzy numbers. The rest of the paper is organized as follows: In section 2, we briefly introduce the basic definitions and arithmetic operations of fuzzy numbers. Section 3 mathematical formulation of fuzzy assignment problem. In Section 4, presents fuzzy assignment algorithms. In section 5, numerical example is presented to show the applications of the proposed algorithms, finally, the conclusion is given in section 6.

2. Preliminaries

Zadeh, first introduced Fuzzy set as a mathematical way of representing impreciseness or vagueness in everyday life in 1965 [13].

2.1 Definition: A fuzzy set is characterized by a membership function mapping elements of a domain, space, or universe of discourse X to the unit interval [0, 1].

(i,e) A = {(x, μ_A(x) ; x ∈ X}, Here μ_A(x): X →[0,1] is a mapping called the degree of membership function of the fuzzy set A and μ_A(x) is called the membership value of x ε X in the fuzzy set A. These membership grades are often represented by real numbers range from [0,1].

2.2 Definition: A fuzzy number A is defined to be a trapezoidal fuzzy number if its membership functions $\mu_{\tilde{A}}(x)$:ℝ→ [0, 1] is equal to.

$$\mu_{\tilde{A}}(x) = \begin{cases} \frac{x-a_1}{a_2-a_1} & if \quad x \in [a_1, a_2] \\ \frac{a_3-x}{a_3-a_2} & if \quad x \in [a_2, a_3] \\ 0 & otherwise \end{cases}$$

where $a_1 \le a_2 \le a_3$. This fuzzy number is denoted by (a_1, a_2, a_3).

2.3 Definition: A fuzzy number A is defined to be a triangular fuzzy number if its membership functions $\mu_{\tilde{A}}(x)$:ℝ→ [0, 1] is equal to

$$\mu_{\tilde{A}}(x) = \begin{cases} \frac{x-a_1}{a_2-a_1} & , a_1 \le x \le a_2 \\ 1 & , a_2 \le x \le a_3 \\ \frac{x-a_4}{a_3-a_4} & , a_3 \le x \le a_4 \end{cases}$$

2.4 Definition: (α-cut of a Triangular fuzzy number):

The α-cut of a fuzzy number A(x) is defined as A(α) = { x : μ(x) ≥ α, α Є [0,1]}

2.5 Definition: Addition of two trapezoidal fuzzy numbers can be performed as

$(a_1,b_1,c_1,d_1) + (a_2,b_2,c_2,d_2) = (a_1+a_2, b_1+b_2, c_1+c_2, d_1+d_2)$.

2.6 Robust's Ranking Techniques

Robust's ranking technique [5] which satisfies costs, linearity, and additives properties and provides results which are consistent with hu-man intuition. Give a convex fuzzy number ã, the Robust's Ranking Index is defined by

R (ã) = $\int_0^1 \boldsymbol{o.5}(a_L^{\alpha}, a_U^{\alpha})$, where $(, a_L^{\alpha}, a_U^{\alpha})$ is the α – level cut of the fuzzy number ã.

Where $(a^L{}_{\alpha}, a^U{}_{\alpha})$ ={ [(b-aα) +a] , [d-(d- cα]}]}

In this paper we use this method for ranking the objective values. The Robust's ranking index R (ã) gives the representative value of the fuzzy number ã. It satisfies the linearity and additive property:

3. The Assignment Problem can be stated in the form of n×n cost matrix $[a_{ij}]$ of real numbers as given in the following:

Mathematically assignment problem can be stated as

Minimize Z= $\sum_{i=1}^{n} \sum_{j=1}^{n} a_{ij} x_{ij}$

Subjet to $\sum_{i=1}^{n} x_{ij}$ =1

$\sum_{j=1}^{n} x_{ij}$ =1

$x_{ij} = \begin{cases} 1 & \text{if ith pension is assigned to j th job} \\ 0 & \text{other wise} \end{cases}$

is the decision variable denoting the assignment of the person i to job j. $\tilde{a}_{ij}$ is the cost of assigning the j^{th} job to the i^{th} person.

	Job1	Job2	Job3	.Jobj	JobN
Person1	$\tilde{a}_{11}$	$\tilde{a}_{12}$	$\tilde{a}_{13}$	.. $\tilde{a}_{ij}$	$\tilde{a}_{1n}$
Person 2	$\tilde{a}_{21}$	$\tilde{a}_{22}$	$\tilde{a}_{23}$	... $\tilde{a}_{ij}$	$\tilde{a}_{2n}$
. . Person i .	- - $\tilde{a}_{i1}$	- - $\tilde{a}_{i2}$	- - $\tilde{a}_{i3}$	- - .. $\tilde{a}_{ij}$. -	- - -
Person N	$\tilde{a}_{n1}$	$\tilde{a}_{n2}$	$\tilde{a}_{n3}$	.. $\tilde{a}_{nj}$ ..	$\tilde{a}_{nn}$

The objective is to minimize the total cost of assigning all the jobs to the available persons. (one job to one person When the costs or time ãij are fuzzy numbers, then the total cost becomes a fuzzy number. Robust's ranking technique [5] which satisfies compensation, linearity, and additive properties and provides results which are consistent with human intuition.

Hence it cannot be minimized directly. For solving the problem we de-fuzzify the fuzzy cost coeffients into crisp ones by a fuzzy number ranking method.

4. BRANCH BOUND TECHNIQE AND BOUND TECHNIQUE FOR ASSIGNMENT PROBLEM;

The assignment problem can also be solved using a branch and bound algorithm:

It is a curtailed enumeration technique. The terminologies of branch and bound technique applied to the assignment problem are presented below.

1. Let k be the level number in the branch tree (for root node it is 0) σ be an assignment in the current node of a branching tree.

2. P_σ^k be an assignment at level k of the branching tree. A is the set of assigned cells (partial assignment) up to the node P_σ^k from the root node (set of i and j values with respect to the assigned cells up to the node P_σ^k from the root node).

3. V_σ be the lower bound of the partial assignment up to P_σ^k such that,

$$V_\sigma = \sum_{i,j \varepsilon A} C_{i,j} + \sum_{i \varepsilon X} \left(\sum_{j \varepsilon Y} \min C_{ij} \right)$$

Where C_{ij} ithe cell entry of the cost matrix with respect to the i^{th} row and j^{th} column. X be the set of rows which are not deleted up to the node P_σ^k from the root node in the branching node.

Branching guidelines:

1. At Level k, the row marked as k of the assignment assigned problem, will be assigned with the best column of the assignment problem.

2. If there is tie on the lower bound then the terminal node at the lower –most is to be considered for the further branching.

3. Stooping rule: If the minimum lower bound happiness to be at any of the terminal node at the $(n-1)^{th}$ level, the optimality is reached. Then the assignments on the path of the node to that node along with the missing pair of row-column combination from the optimum solution.

5. Numerical Example: (Triangular Fuzzy Number)

Four different jobs can be done on four different machines and take down time costs are

Prohibitively high for change over. The matrix below gives the cost in rupees of producing jobs i on machine j. A company has four sources S1, S2, S3, S4 and destinations D1, D2, D3, D4. The fuzzy transportation cost for unit quantity of product from i^{th} sources j^{th} destinations is C_{ij}

(1,5,9)	(3,7,11)	(7,11,15)	(2,6,10)
(4,8,12)	(1,5,9)	(4,9,13)	(2,6,10)
(0,4,8)	(3,7,11)	(6,10,14)	(3,7,11)
(6,10,14)	(0,4,8)	(4,8,12)	(-1,3,7)

Solution: In conformation to model the fuzzy assignment problem can be formulated in the following

Min R(1,5,9) x_{11} + R(3,7,11)x_{12} + R(7,11,15)x_{13} +R(2,6,10)x_{14} + R(4,8,12)x_{21}+ R(1,5,9)x_{22}+R(4,9,13)x_{23}+ R(2,6,10)x_{24} + R(0,4,8)x_{31}+ R(3,7,11)x_{32} + R(6,10,14)x_{33} + R(3,7,11)x_{34}+R(6,10,14)x_{41}+ R(0,4,8)x_{42}+ R(4,8,12)x_{43}+ R(-1,3,7)x_{44}

Subject to
$x_{11}+x_{12}+x_{13}+x_{14}=1$ $x_{11}+x_{21}+x_{31}+x_{41}=1$
$x_{21}+x_{22}+x_{23}+x_{24}=1$ $x_{12}+x_{22}+x_{32}+x_{42}=1$
$x_{31}+x_{32}+x_{33}+x_{34}=1$ $x_{13}+x_{23}+x_{33}+x_{43}=1$
$x_{41}+x_{42}+x_{43}+x_{44}=1$ $x_{14}+x_{24}+x_{34}+x_{44}=1$ where $x_{ij} \in [0,1]$

now we calculate R(1,5,9) by applying Robust ranking method.

For which $(a^L_{\alpha}, a^U_{\alpha}) = \{ (b-a)\alpha + a, c-(c-b)\alpha\}$

$R(\tilde{C}_{11}) = \int_0^1 0.5(1 + 4\alpha + 9 - 4\alpha)\, d\alpha = 5$

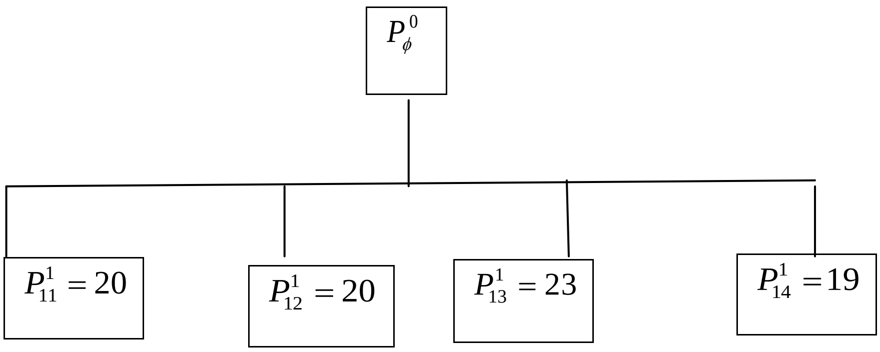
Similarly, $R(\tilde{C}_{12}) = 7$ $R(\tilde{C}_{13}) = 11$ $R(\tilde{C}_{14}) = 6$
$R(\tilde{C}_{21}) = 8$ $R(\tilde{C}_{22}) = 5$ $R(\tilde{C}_{23}) = 9$ $R(\tilde{C}_{24}) = 6$
$R(\tilde{C}_{31}) = 4$ $R(\tilde{C}_{32}) = 7$ $R(\tilde{C}_{33}) = 10$ $R(\tilde{C}_{34}) = 7$
$R(\tilde{C}_{41}) = 10$ $R(\tilde{C}_{42}) = 4$ $R(\tilde{C}_{43}) = 8$ $R(\tilde{C}_{44}) = 3$

We replace these values for their corresponding to given problem which results in a conventional assignment problem in the LPP form. Solving it, we get the solution a

Crisp form of the problem:

5	7	11	6
8	5	9	6
4	7	10	7
10	4	8	3

Initially, no job is assigned to any operator, so the assignment(σ) at the root (level o) of the branching tree is a null set and the corresponding lower bound is also 0 for each. **Further branching**: the four different sub problem under the root nodes are shown as in Fig lower bound the solution problems shown on its right hand side.



Compute the lower bound for P_{11}^{1}:

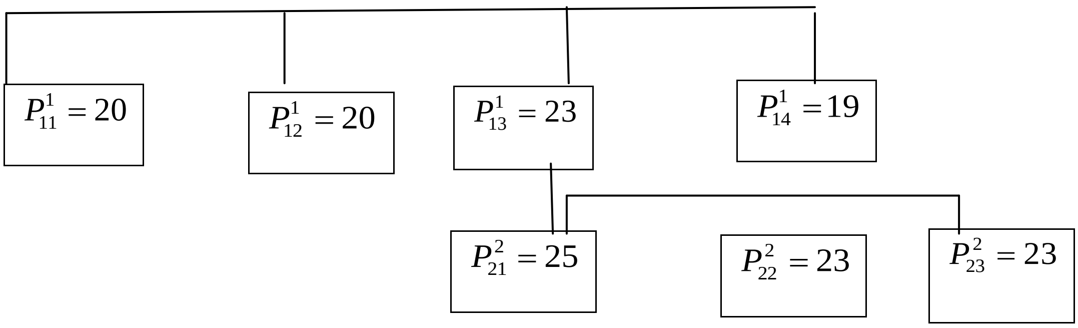
$$V_{\sigma} = \sum_{i,j\varepsilon A} C_{i,j} + \sum_{i\varepsilon X} \left(\sum_{j\varepsilon Y} \min C_{ij} \right)$$

Where σ ={(11)} A= {(11)} X= {2,3,4} Y= {2,3,4} then

$$V_{(11)} = C_{11} + \sum_{i\varepsilon\{2,3,4\}} \left(\sum_{j\varepsilon\{2,3,4\}} \min C_{ij} \right)$$

P_{11}^{1} =5+ (5+7+3)=20 P_{12}^{1} =7+(6+3+3)=20 P_{13}^{1}=11+(5+4+3)=23 P_{14}^{1} =6+(5+4+4)=19

Further branching: further branching is done from the terminal node which has the least lower bound at this stage, the nodes P_{11}^{1}, P_{12}^{1} P_{13}^{1} P_{14}^{1} are the terminal nodes. The node P_{14}^{1} has the least lower bound. Hence, further branching from this node is shown as in Fig.

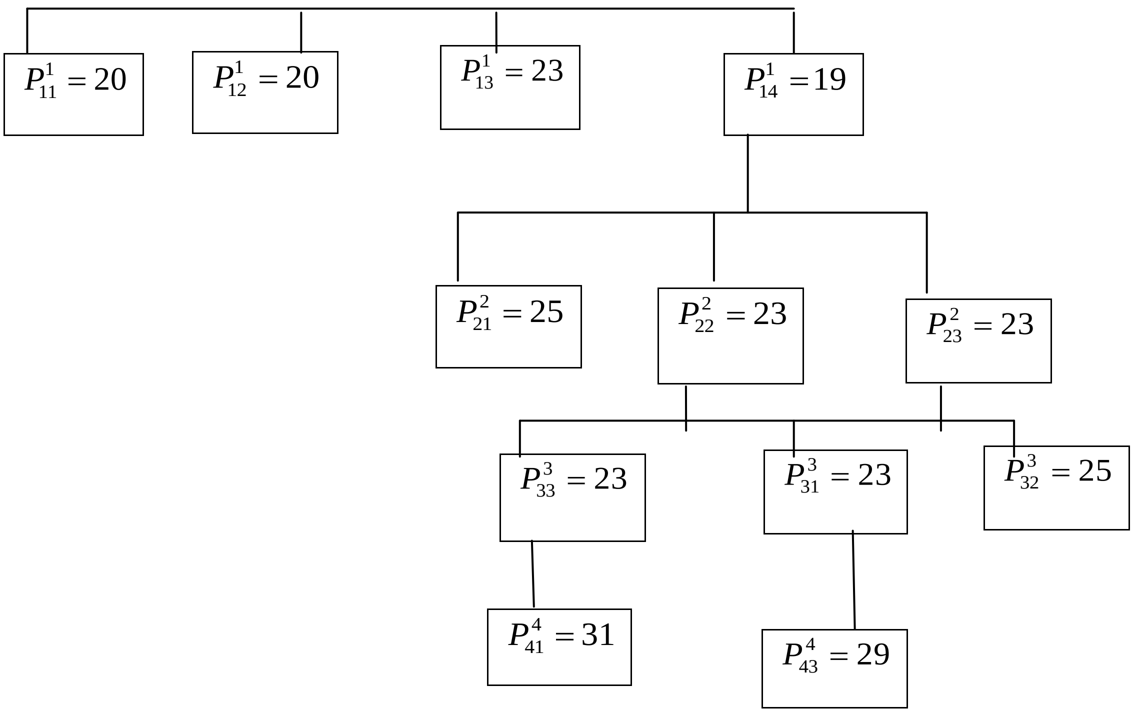


$V_{(21)} = C_{11} + C_{21} + \sum_{i \varepsilon \{,3,4\}} \left(\sum_{j \varepsilon \{2,4\}} \min C_{ij} \right)$

P_{21}^2 =6+8+(7+4)=25 P_{22}^2 =6+5+(4+8)=2 3 P_{23}^2 =6+9+(4+4)=23

Further branching: At this stage the nodes P_{14}^1, P_{21}^2, P_{22}^2, P_{23}^2 are the terminal node. Among these nodes P_{22}^2, P_{23}^2 are tie on the lower bound. Then these terminal nodes at lower-most are to be considering further branching.

$V_{(3,1)} = C_{14} + C_{22} + C_{31} + \sum_{i \varepsilon \{4\}} \left(\sum_{j \varepsilon \{3\}} \min C_{ij} \right)$ P_{31}^3 =6+5+4+8=23 P_{33}^3 =6+5+10+10=31



We replace these values for their corresponding the above table which results in Conventional assignment problem in the LPP form. Solving it, we get the solution as with the optimal objective value 23 which represents the optimal total cost. In other words the optimal assignment is **(1,4), (2,2), (3,1), (4,3)** The fuzzy optimal cost is calculated

(2,6,10)+(1,5,9)+(0,4,8)+(4,8,12)=(7,23,39)

Also 6+5+4+8=23

5. Numerical Example

Example5.1. Let us consider a Fuzzy assignment problem with rows representing 4 persons A,B,C,D and columns representing the4 jobs Job1, Job2,Job3 and Job4. The cost matrix $\tilde{C}_{ij}$ is given whose elements are trapezoidal fuzzy numbers. The problem is to find the assignment of persons to jobs that will minimize the total fuzzy cost

Find the optimal assignment so that the total cost of job assignment becomes minimum.

Persons	Jobs			
	1	2	3	4
A	(3,5,6,7)	(5, 8, 11, 12)	(9, 10, 11, 15)	(5, 8, 10, 11)
B	(7, 8, 10, 11)	(3, 5, 6,7)	(6, 8, 10, 12)	(5, 8, 9, 10)
C	(2, 4, 5,6)	(5, 7, 10, 11)	(8, 11, 13, 15)	(4, 6, 7, 10)
D	(6, 8, 10, 12)	(2, 5, 6, 7)	(5, 7, 10, 11)	(2, 4, 5, 7)

Solution:

In Conformation to model the fuzzy assignment problem can be formulated in the following

Min{R(3,5,6,7) +R(5,8,11,12)+R(9,10,11,15)+R(5,8,10,11) +R(7,8,10,11) +R(3,5,6,7) +R(6,8,10,12) +R(5,8,9,10)+R(2,4,5,6) +(5,7,10,11)+R(8,11,13,15)+(4,6,7,10)+(6,8,10,12)+(2,5,6,7)+ (5,7,10,11)+(2,4,5,7)}

Subject to
$x_{11}+x_{12}+x_{13}+x_{14}=1$ $x_{11}+x_{21}+x_{31}+x_{41}=1$
$x_{21}+x_{22}+x_{23}+x_{24}=1$ $x_{12}+x_{22}+x_{32}+x_{42}=1$
$x_{31}+x_{32}+x_{33}+x_{34}=1$ $x_{13}+x_{23}+x_{33}+x_{43}=1$
$x_{41}+x_{42}+x_{43}+x_{44}=1$ $x_{14}+x_{24}+x_{34}+x_{44}=1$ where $x_{ij} \in [0,1]$

Where $R(\tilde{C}_{11}) = \int_0^1 0.5(3 + 2 \propto +7-\propto)d \propto = 2.096$

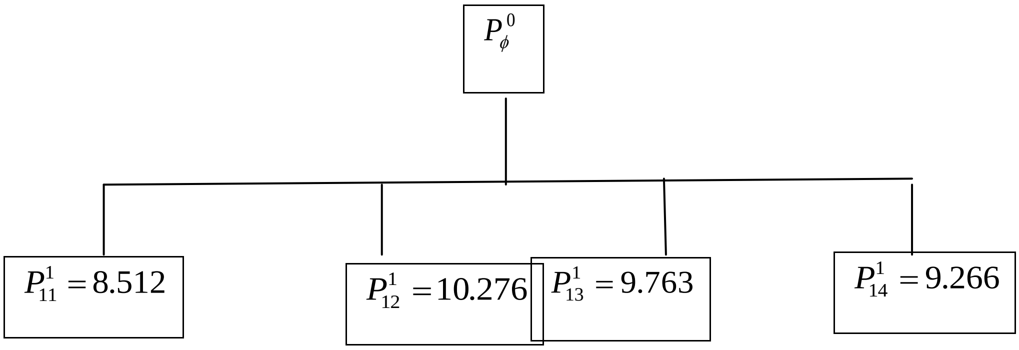
$R(\tilde{C}_{12}) = 3.608$ $R(\tilde{C}_{13}) = 4.213$ $R(\tilde{C}_{14}) = 3.414$

$R(\tilde{C}_{21}) = 3.5$ $R(\tilde{C}_{22}) = 2.096$ $R(\tilde{C}_{23}) = 3.5$ $R(\tilde{C}_{24}) = 3.219$

$R(\tilde{C}_{31}) = 1.704$ $R(\tilde{C}_{32}) = 3.262$ $R(\tilde{C}_{33}) = 4.023$ $R(\tilde{C}_{34}) = 2.571$

$R(\tilde{C}_{41}) = 3.5$ $R(\tilde{C}_{42}) = 2.052$ $R(\tilde{C}_{43}) = 3.262$ $R(\tilde{C}_{44}) = 1.75$

2.096	3.608	4.213	3.414
3.5	2.096	3.5	3.219
1.704	3.262	4.023	2.571
3,5	2.052	3.262	1.75



Compute the lower bound for P_{11}^{1}:

$$V_{\sigma} = \sum_{i,j\varepsilon A} C_{i,j} + \sum_{i\varepsilon X} \left(\sum_{j\varepsilon Y} \min C_{ij} \right)$$

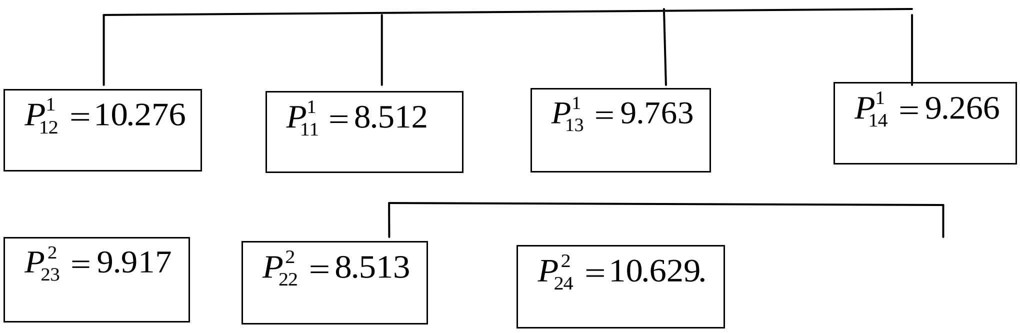
Where σ ={(11)} A= {(11)} X= {2,3,4} Y= {2,3,4} then

$$V_{(11)} = C_{11} + \sum_{i\varepsilon\{2,3,4\}} \left(\sum_{j\varepsilon\{2,3,4\}} \min C_{ij} \right)$$ similarly

P_{11}^{1} =2.096+ (2.096+2.571+1.75)=8.513 P_{12}^{1} =3.608+(2.096+1.704+1.75)=10.276

P_{13}^{1}=4.213+(2.096+1.704+1.75)=9.736 P_{14}^{1}=3.414+(2.096+1.704+2.052)=9.266

here P_{11}^{1} has the least lower bound. Hence, further branching from this node is



$V_{(21)}=C_{11}+C_{21}+\sum_{i\varepsilon\{,3,4\}} (\sum_{j\varepsilon\{2,4\}} \min C_{ij})$

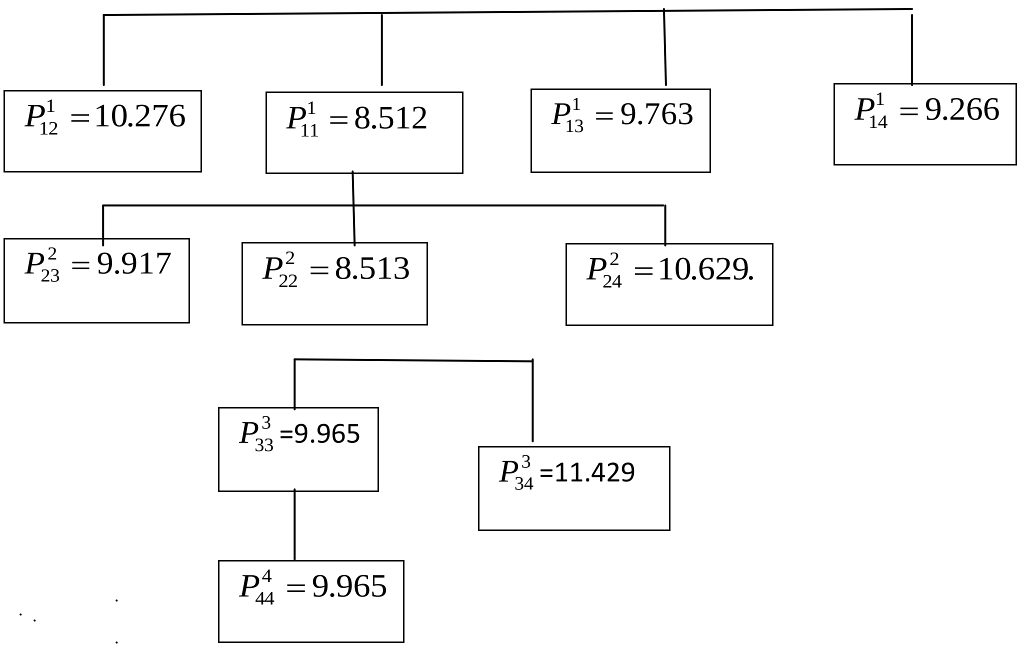
P_{21}^{2} =3.414+3.5+(3.262+2.052)=12.228 P_{22}^{2} =2.096+2.096+(2.571+1.75)=8.513

P_{23}^{2} =2.096+3.5+(2.571+1.75)=9.917 P_{24}^{2} =2.096+3.219+(3.262+2.052)=10.629

P_{22}^{2} having lowest bound further

P_{33}^{3} =2.096+2.096+4.023+(1.75)=9.965 P_{34}^{3} =2.096+3.5+2.571+(3.262)=11.429

node P_{33}^{3} which is at the bottom-most level is considered for further branching. Since this node lies at (n-1) th level(k=3) of the branching tree, where n is the size of assignment problem, optimality is reached. The corresponding solution is traced from the root node P_{33}^{3} along with the missing pair of the job and operator combination,(4,4) as shown in the table



We replace these values for their corresponding table which results in conventional assignment problem in the LPP form. Solving it, we get the solution as with the optimal objective value 23which represents the optimal total cost. In other words the optimal assignment is (1,1), (2,2), (3,3), (4,4) The fuzzy optimal cost is calculated (3,5,6,7)+(3,5,6,7)+(8,11,13,15)+(5,7,10,11)=(19,28,35,40)

Also 2.096+2.096+3.5+1.75=9.442

6. Conclusion:

In this paper, the assignment cost has been considered as imprecise numbers described by fuzzy numbers which are more realistic and general in nature. Here, the fuzzy assignment problem has been converted into crisp assignment problem using Robust ranking indices [10] and Branch bounded method has been applied to find an optimal solution. Numerical example has been shown that the total cost obtained is optimal. This method is systematic procedure, easy to apply and can be utilized for all type of assignment problem whether maximize or minimize objective function.

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