Propagation of Electron-Acoustic Waves In a Relativistic Plasma

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ABSTRACT

The effects of relativistic electrons on the amplitude and width of small amplitude electron acoustic waves are investigated in collisionless, unmagnetized plasma consisting of relativistic cold electrons, hot electrons obeying non maxwellian distributions and stationary ions. The Korteweg De Vries (KdV) equation has been derived. It is found that relativistic factor, density ratio and superthermal parameter effects on the characteristics on EASW.

Keywords*: Electron acoustic wave; Relativistic electrons; KdV equation.*

1. INTRODUCTION

In the last few years, there has been considerable interest in the different types of coherent wave structures in the multispecies plasma. The nonlinear wave structures are beautiful and amazing manifestation of nature, arising out of competition between nonlinearity, dispersion and dissipation. Space environment constitutes a magnificent laboratory for investigation of plasma phenomenon and nonlinear wave structures. EA wave is characterized by two electron populations, referred to as cool and hot electrons [1-6]. The cool electrons provide the inertia necessary to maintain the electrostatic oscillations, while the restoring force comes from hot electron pressure. The propagation of EASWs in a plasma system has been studied by several investigators in Unmagnetized two electon plasmas [11-13] as well as in magnetized plasma [14-18]. Mace and Helberg [9] studies the properties of EASW in a magnetized plasma consisting of cold electron fluids, ion fluids and Maxwellian distributed hot electrons. However, the hot electrons will not have a Maxwellian distribution due to formation of phase space holes caused by the trapping of hot electrons in the potential of EAW. Thus the hot electrons follow a vortex - like distribution in the most cases [10-14].

Most of these investigations on linear and non-linear phenomenon are confined to non relativistic plasmas. But when electron velocity approaches the velocity of light, relativistic effects may significantly modify the soliton behaviour. Relativistic plasmas occur in a variety of situations e.g. in laser-plasma interaction, space plasma phenomenon, plasma sheet boundary layer of earth's magnetosphere[7-8]. As plasma with relativistic velocities are frequently observed in astrophysical and space environments, so there is a need to study electron-acoustic waves in three component plasma with weakly relativistic electrons. In this paper we will derive the KdV equation from the basic set of equations for plasma by reductive perturbation theory and study the effect of relativistic electrons on Unmagnetized Collisionless plasma consisting of cold relativistic electron fluid, non Maxwellian hot electrons and stationary ions. This paper is organised as follow, in section 2, we present the basic set of fluid equations in three component plasma model. In section 3, KdV equation is derived using reductive perturbation method. Finally some conclusions are given in section 4.

2. BASIC EQUATIONS:

We consider Unmagnetized Collisionless plasma consisting of cold relativistic electron fluid, non Maxwellian hot electrons and stationary ions. The non linear dynamics of the electron acoustic solitary waves is governed by the continuity and motion equations for relativistic cold electron and Poisson's equation.

$$\frac{\partial n_c}{\partial t} + \frac{\partial (n_c u_c)}{\partial x} = 0 \quad (1)$$

$$\frac{\partial (\gamma_c u_c)}{\partial t} + u_c \frac{\partial (\gamma_c u_c)}{\partial x} + \frac{3\alpha(1+\alpha)^2 n_c}{\theta} \frac{\partial n_c}{\partial x} - \alpha \frac{\partial \phi}{\partial x} = 0 \quad (2)$$

$$\frac{\partial^2 \phi}{\partial x^2} = \frac{1}{\alpha} n_c + n_h - \left(1 + \frac{1}{\alpha}\right) \quad (3)$$

where

$$n_h = (1 - \beta\phi + \beta\phi^2)e^{\phi}, \gamma_c = 1 + \frac{u_c^2}{2c^2}$$

$\theta = \frac{T_h}{T_c}$ and $\alpha = \frac{n_{h0}}{n_{c0}}$, n_c and n_h are the normalized number densities of cold and hot electrons, respectively. φ is the normalized electrostatic potential, u_c is the normalized velocities of cold electron in the x direction. x and t are also normalized. The normalization is as follows.

The densities of cold and hot electrons are normalized by n_{co} and n_{ho}. The space coordinates x time t, velocity and electrostatic potential ϕ are normalized by the hot electron Debye length$(K_B T_h/$ 4π nhoe2)1/2, inverse of cold electron plasma frequency ωpc−1=(m/4π ncoe2), ce=(KBTh/αm), and KBThe, respectively. Here m is the electron mass, e is the magnitude of the electron charge and K_B is the Boltzmann constant.

3. KdV equation:

According to RPM, the independent variables are scaled as

$$\xi = \epsilon^{1/2}(x - v_g t), \quad (4)$$

$$\tau = \epsilon^{3/2} t, \quad (5)$$

Where ϵ is small (0< ϵ<1) expansion parameter charcterizing the strength of the non-linearity and v_g is the phase velocity of the electron-acoustic wave.The dependent variables are expanded around the equilibrium values in the power of ϵ in the following forms:

$$n_c = 1 + \epsilon n_1 + \epsilon^2 n_2 + \epsilon^3 n_3 + \ \ldots\ldots$$

$u_c = u_0 + \epsilon u_1 + \epsilon^2 u_2 + \epsilon^3 u_3 + \ \dots\dots\dots$

$\phi_c = \epsilon\phi_1 + \epsilon^2\phi_2 + \epsilon^3\phi_3 + \ \dots\dots\dots$ (6)

Using stretching coordinates Eqs.(4-5) and Eq.(6) in Eqs.(1-3) and collecting the terms of ϵ . We obtain the following equations in lowest power of :

$$-v_g \frac{\partial n_1}{\partial \xi} + \frac{\partial u_1}{\partial \xi} + u_0 \frac{\partial n_1}{\partial \xi} = 0 \quad (7)$$

$$-v_g \gamma_1 \frac{\partial u_1}{\partial \xi} + u_0 \gamma_1 \frac{\partial u_1}{\partial \xi} + \frac{3\alpha(1+\alpha)^2}{\theta} \frac{\partial n_1}{\partial \xi} - \alpha \frac{\partial \phi_1}{\partial \xi} = 0$$

(8)

$$(1-\beta)\phi_1 + \frac{1}{\alpha} n_1 = 0 \quad (9)$$

From (7) to (9), we can express the first order quantities in terms of ϕ_1 as

$$n_1 = -\alpha(1-\beta)\phi_1 \quad (10)$$

$$u_1 = -\alpha(1-\beta)(v_g - u_0)\phi_1 \quad (11)$$

where

$\gamma_1 = 1 + \frac{3}{2}U^2$, U = $\frac{u_0}{c}$

Algebraic manipulations of these equations lead to the following dispersion relation

$$V = v_g - u_0 = \sqrt{\frac{1}{\gamma_1}\left[\frac{3\alpha(1+\alpha)^2}{\theta} + \frac{1}{1-\beta}\right]} \quad (12)$$

The phase velocity given by Eq.(12) depends on relativistic factor U(u_0/c), cold to hot electron density ratio(α), hot to cold electron temperature ratio(θ) and nonthermal electrons distribution parameter β. To see the effect of α on the phase velocity with relativistic factor (U) Fig. 1 is drawn. V is plotted vs. U for different value of α, viz. α= 0.5 (solid line), 1.0 (dashed line), 5 (dotted line). The other parameters are β = 0.1, θ=20. From Fig. it is seen that phase velocity decreases with increase in the value of the relativistic factor. As the value of α is increases, phase velocity also increases.

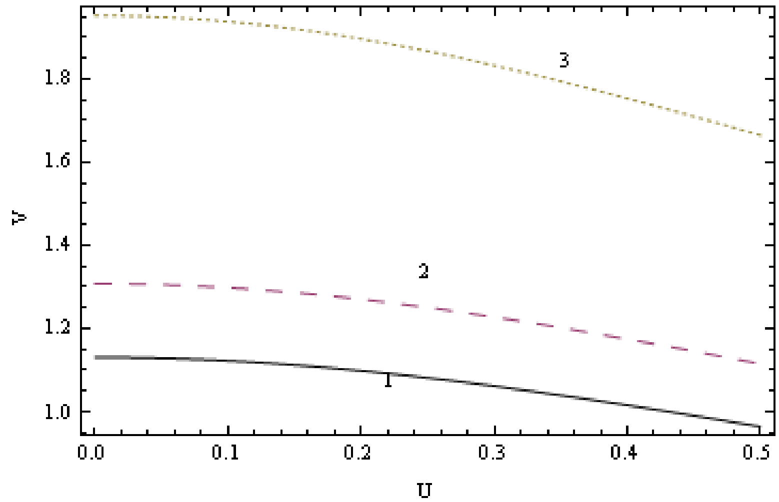

Figure 1: V is plotted against U for different values of α. The other parameters are β = 0.1, θ=20. Solid curve correspond to α= 0.5, dashed curve to α=1.0 and dotted curve to α=5.0.

To next order of ε, we get

$$\frac{\partial n_1}{\partial \tau} - (v_g - u_0)\frac{\partial n_2}{\partial \xi} + \frac{\partial u_2}{\partial \xi} + \frac{\partial (n_1 u_1)}{\partial \xi} = 0 \quad (13)$$

$$\gamma_1 \frac{\partial u_1}{\partial \tau} - (v_g - u_0)\gamma_1 \frac{\partial u_2}{\partial \xi} + [\gamma_1 - 2\gamma_2\left(\frac{v_g - u_0}{u_0}\right) u_1 \frac{\partial u_1}{\partial \xi} + \frac{3\alpha(1+\alpha)^2}{\theta} n_1 \frac{\partial n_1}{\partial \xi} + \frac{3\alpha(1+\alpha)^2}{\theta}\frac{\partial n_2}{\partial \xi} - \alpha\,\frac{\partial \phi_2}{\partial \xi} = 0 \quad (14)$$

$$\frac{\partial^2 \phi_1}{\partial \xi^2} = (1-\beta)\phi_2 + \frac{1}{\alpha} n_2 + \frac{1}{2}\phi_1^2 \quad (15)$$

Where

$\gamma_1 = 1 + \frac{3}{2}U^2$, $\gamma_2 = \frac{3}{2}U^2$, U = $\frac{u_0}{c}$

After some algebraic manipulations, second order quantities are eliminated and ϕ_1 is found to satisfy the following KdV equation

$$\frac{\partial \phi_1}{\partial \tau} + A\phi_1 \frac{\partial \phi_1}{\partial \xi} + B\frac{\partial^3 \phi_1}{\partial \xi^3} = 0 \quad (16)$$

where A is the coefficient of non - linear term and B is the coefficient of dispersion term.

The non - linearity coefficient A and dispersive coefficient B is defined as follow:

$$A = \frac{(v_g - u_0)}{2(1-\beta)} - \frac{3}{2}(v_g - u_0)(1-\beta)\alpha + \frac{\gamma_2}{u_0}\frac{\alpha(1-\beta)(v_g - u_0)^2}{\gamma_1} - \frac{3\alpha(1+\alpha)^2}{2\theta\gamma_1(v_g - u_0)(1-\beta)} - \frac{3\alpha^2(1+\alpha)^2(1-\beta)}{2\theta\gamma_1(v_g - u_0)} \quad (17)$$

and

$$B = \frac{1}{2\gamma_1(v_g - u_0)(1-\beta)^2} \quad (18)$$

On introducing the new variable $\eta = \xi - v\tau$, where v is a constant velocity, the solution of Eq.(16) is given as

$$\phi = \phi_1(\eta) = \phi_m \, sech^2\left(\frac{\eta}{L}\right) \quad (19)$$

where peak amplitude ϕ_m and width L of soliton are respectively given by

$$\phi_m = \frac{3V}{A} \quad (20)$$

and

$$L = \sqrt{\frac{4B}{v}} \quad (21)$$

4. CONCLUSION AND REMARKS

A study of small but finite amplitude electron-acoustic radiation in unmagnetized plasma containing cold relativistic electron fluid, Maxwellian hot electrons, a relativistic electron beam and stationary ions has been carried out using reductive perturbation theory. The corresponding KdV equation describing the nonlinear evolution of the solitons has been derived. To examine the effect of the relativistic electrons and electron temperatures on the nature of the solitary waves, we numerically analyze both of the amplitude and the width of the solitary waves. We have displayed their variation graphically in Figs. 2-4. it is seen that the amplitude and width of the solitons increases with relativistic factor U while decreases with θ and α . The results presented here may be applicable to some plasma environments, such as terrestrial magnetosphere.

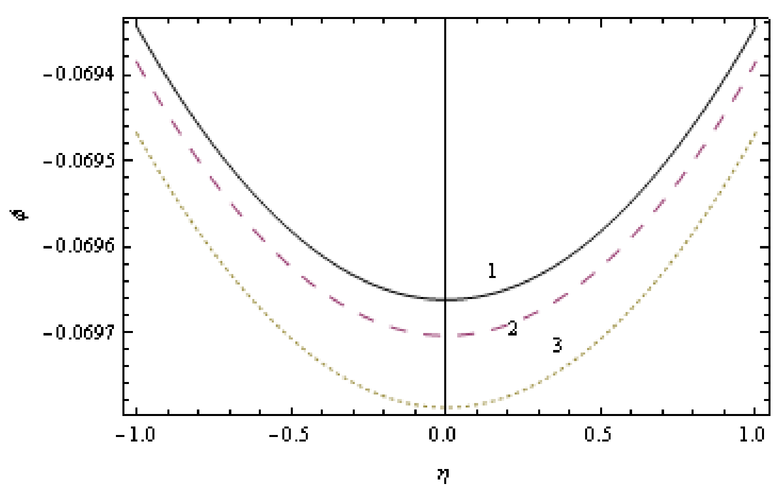

Figure. 2: Variation of soliton amplitude and width against η for three different values of U = 0.01 (curve 1), 0.04 (curve 2), 0.08 (curve 3) where α=0.5, β=0.1, θ=20 and v=0.01.

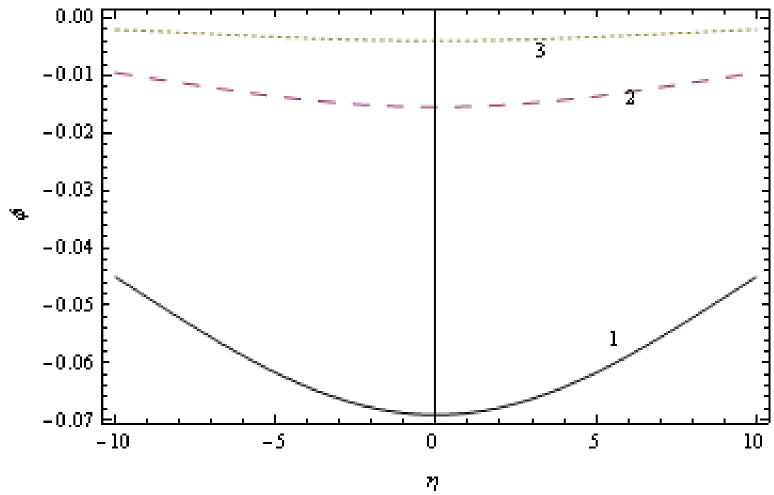

Figure. 3: Variation of soliton amplitude and width against η for three different values of θ = 20 (curve 1), 40 (curve 2), 60 (curve 3) where α=0.5, β=0.1, U=0.01 and v=0.01.

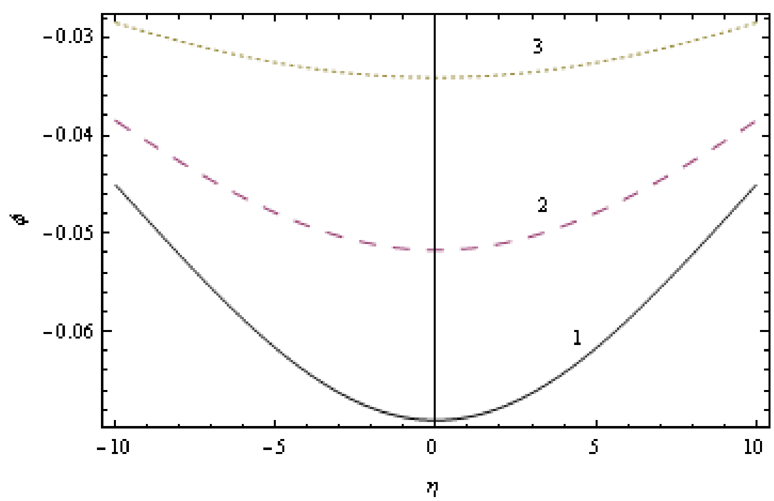

Figure. 4: Variation of soliton amplitude and width against η for three different values of α = 0.5 (curve 1), 1.0 (curve 2), 2.0 (curve 3) where U=0.01, β=0.1, θ=20 and v=0.01.

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