ENVIRONMENTALLY RESPONSIBLE REPAIR AND WASTE DISPOSAL INVENTORY MODELS

W.Ritha* and I.Antonitte Vinoline**

*,** Department of Mathematics, Holy Cross College (Autonomous), Tiruchirapalli – 620 002 (Tamilnadu) • Email : ritha_prakash@yahoo.com

ABSTRACT :

Traditional inventory models involve different decisions that attempt to optimize material lot sizes by minimizing total annual switching costs. However, the increasing concern on environmental issues stresses the need to treat inventory management decisions as a whole, by integrating economic and environmental objectives. Recent studies have underlined the need to incorporate additional criteria in traditional inventory models in order to design "responsible inventory systems". This paper explores the problem of determining the optimal batch sizes for production and recovery in an EOQ (economic order/production quantity) repair and waste disposal model context. This paper assumes that a first shop is manufacturing new products as well as repairing products used by a second shop. The used products can either be stored in the second shop and then be brought back to the first shop in an approach used to reduce inventory costs, or be disposed outside the system. The works available in the literature assumed a general time interval and ignored the very first time interval where no repair runs are performed. This assumption resulted in an over estimation of the average inventory level and subsequently the holding cost. These works also have not accounted for switching costs when alternating between production and recovery runs, which are common when switching among products or jobs in a manufacturing facility. This paper addresses these two limitations. Mathematical models are developed with numerical examples presented and results discussed.

Keywords : EOQ model, Production/recovery, Reuse, Waste disposal. Switching cost.

1. INTRODUCTION

In competitive markets products are pushed faster and faster to customers along supply chains resulting in faster generation of waste and depletion of natural resources. These environmental issues that many governments around the world are confronted with led their legislators to devise laws that require manufacturers to initiate product recovery programs that collect used items of products from their customers once these products reach their economic or useful lives. This gave rise to reverse logistics (RL) as a business term (e.g., [1,2]). Carter and Ellarm (1998) have collected a number of definitions of reverse logistics. We shall cite one of the definitions. The more general definition is "Reverse logistic is such activity which helps to continue an environmental effective policy of firms with reuse of necessary materials, remanufacturing, and with reduction of amount of necessary materials." This efficiency touches the personal in production supply and consumption process. Carter and Ellarm (1998) approach reverse logistics from point of view of environmental protection. Environmental consciousness occurs at three level of activity of firms : governmental regulation, social pressure and voluntary self restriction. Reverse Logistic Executive Council (RLEC) has given a more general definition of reverse logistics which summarizes the above definitions : Reverse logistics is a movement of materials from a typical final consumption in an opposite direction in order to regain value or to dispose of wastes. This reverse activity includes take back of damaged products, renewal and enlargement of inventories through product take back remanufacturing of packaging materials, reuse of containers, and renovation of products, and handling of obsolete appliances.

Reverse logistics is an extension of logistic, which deals with handling and reuse of reusable used products withdrawn from production and consumption process. Such a reuse is eg recycling or repair of spare parts. It has an advantage from economic point of view, as reduction of environmental load through return of used items in the manufacturing process, but the exploitation of natural resources can be decreased with this reuse that saves the resources from extreme consumption for the future generation.

Reverse logistics (RL) extends the traditional forward flow of raw materials, components, finished products to account for activities such as reusing, recycling, refurbishing or recovering of these products, components and raw materials. Rogers and Tibben-Lembke (2001) defined RL as the process of planning, implementing, and controlling the efficient and cost effective flow of raw materials, in-process inventory, finished goods, and related information from the point of consumption to the point of origin for the purpose of recapturing value or proper disposal. Fleischmann et al. (1997) provided a survey addressing the logistics of industrial reuse of products and materials from an Operational Research perspective. They subdivided RL into three main areas, namely distribution planning, inventory control, and production planning. Inventory management of returned items (including repaired/recovered items) will be the focus of this paper.

Richter (2000) assumed the production and repair system to consist of two shops. The first shop manufactures new items of a product and repairs used items of the same product collected by the second shop. The used items are collected at a repair rate β , $0 \le \beta < 1$. Richter (2000, 2003) also assumed that not all the collected items are repairable, and therefore some are disposed as waste outside the system at a rate $\alpha = 1 - \beta$, $0 \le \beta < 1$, which may display the ecological behavior of the system. This assumption is different from that adopted in Schrady (1967) who assumed a continuous flow of used items to the manufacturer. Richter extended his work cited above in several directions. He has done so either individually or in collaboration with Dobos. Richter investigated the cost analysis for extreme waste disposal rates. He showed that the pure (bang-bang) policy of either no waste disposal (total repair) or no repair (total waste disposal) dominates a mixed waste disposal and repair strategy. Richter and Dobos (1999) further examined the bang-bang policy where they showed that the properties of the minimal cost function and the optimal solution known for the continuous EOQ repair and waste disposal problem (Richter, 1996a, 1996b, 1997) could be extended to the more realistic integer problem. The characteristics of the cost function developed in Richter and Dobos were further studied in Dobos and Richter (2000). They showed that the minimum cost function is partly piecewise convex and partly piecewise concave function of the waste disposal rate. In a follow-up paper, Dobos and Richter (2003) extended their earlier work (Dobos & Richter, 2000) assuming finite repair and production rates with a single repair cycle and a single production cycle per time interval. In a subsequent paper, Dobos and Richter (2004) generalized the work they presented in Dobos and Richter (2003) by assuming that a time interval to consist of multiple repair and production cycles. Steering their earlier work into a new direction, Dobos and Richter (2006) investigated the production-recycling model in Dobos and Richter (2004) for quality considerations. In this work, they assumed that the quality of used collected (returned) items is not always suitable for further recycling, i.e., not all used items can be reused. Dobos and Richter (2006) showed that when considering quality, a mixed strategy is more economical than the pure strategies of their earlier work. Apart from the above surveyed works, other researchers have developed models along the same lines as Richter and Dobos, but with different assumptions. Some of the recent works, but not limited to, are those of Teunter (2003), Inderfurth, Lindner, and Rachaniotis (2005), Konstantaras and Papachristos (2006), and Jaber and Rosen (2008). The work of Richter has limitations. Two of these limitations are addressed in this paper.

First, this paper modifies the method that Richter adopted for calculating the holding costs. Richter assumed that collected items are transferred from shop I to shop 2 in m batches to be repaired at the start of each time interval. Richter considered a general time interval ignoring the effect of the first time interval. The first time interval has no repair batches, as there is nothing manufactured before that is to be collected and repaired in this interval. This assumption results in a residual inventory and thus overestimates the holding cost. Second, this paper accounts for switching costs (e.g., production loss, deterioration in quality, additional labor) when alternating between production and recovery runs. When shifting from producing (performing) one product (job) to another in the same facility, the facility may incur additional costs referred to as switching costs.

2. NOTATION AND ASSUMPTIONS :

In this section, the present study develops EOQ repair and waste disposal inventory model. The following notations and assumptions are used throughout this paper.

Notation :

- n : number of newly manufactured batcher in are interval of length T.
- m : number of repaired batches in an interval of length T
- d : demand rate (unit per unit of time)
- h : holding cost per unit per unit of time for shop 1.
- u : holding cost per unit per unit of time for shop 2.
- α : waste disposal rate, where $0 < \alpha < 1$
- β : repair rate of used items, where $\alpha + \beta = I$ and $0 < \beta < 1$.
- Q : batch size for interval T which includes n newly manufactured and m repaired batches.
- r : repair setup cost per batch
- s : manufacturing set up cost per batch
- a : fixed cost per trip(monetary unit)
- b : variable cost per unit transported per distance travelled
- D : distance travelled
- μ : proportion of demand returned (0 < μ < 1)
- θ : social cost from vehicle emission
- v : average velocity (km/h)
- γ : cost to dispose waste to the environment(mu/unit)
- δ : proportion of waste produced per lot Q.

Assumptions :

- 1. Infinite manufacturing and recovery rates.
- 2. Repaired items are as good as new.
- 3. Demand is known, constant and independent.
- 4. Lead time is zero.
- 5. Single product case.

W.Ritha and I.Antonitte Vinoline

- 6. No shortage.
- 7. Infinite planning horizon.
- 8. Unlimited storage capacity is available.

3. **MODEL FORMULATION**

Ritcher introduced repair and waste disposal model in which demand is satisfied by manufacturing "new" and repairing "used" items of a certain product. There are n batches of newly manufactured items and m batches of repaired items in some collection time interval T. Ritcher assumed instantaneous manufacturing and repair rates and a considered repaired items to be as good as new. This paper assumes demand is supplied by n newly manufactured batches and m repaired batches in some collection time interval of length T. Ritchers work, as well as the other studies in the literature which did not account for T. Ritcher, when there are no items to be repaired, perhaps because all these studies assumed an infinite planning horizon. Modelling the total cost expression for shops 1 and 2 and considering the cost to dispose waste to the environment, the optimal total cost consists of the following elements.

Setup cost for shop 1 and shop 2 is

Repair setup cost per batch = r. Therefore setup cost per m repair batches is mr. Manufacturing setup cost per batch = s, setup cost for n manufacturing batches is ns.

 \therefore Total setup cost = mr + ns

Holding cost for the first

Holding cost for the first shop is
$$= \begin{bmatrix} \frac{h}{2} \left(\frac{\alpha Q}{n} \cdot \frac{\alpha T}{n} \cdot n \right) + \frac{h}{2} \left(\frac{\beta Q}{m} \cdot \frac{\beta T}{m} \cdot m \right) \end{bmatrix}$$
$$= \begin{bmatrix} \frac{h}{2} \left(\frac{\alpha Q}{n} \cdot \frac{\alpha T}{n} \cdot n \right) + \frac{h}{2} \left(\frac{\beta Q}{m} \cdot \frac{\beta T}{m} \cdot m \right) \end{bmatrix}$$
$$= \frac{h}{2} \begin{bmatrix} \frac{\alpha^2 Q^2}{dn} + \frac{\beta^2 Q^2}{dm} \end{bmatrix} \text{ since } T = \frac{Q}{d}$$
Holding cost for the second shop is
$$= u \begin{bmatrix} \frac{Q}{d} \quad \frac{\beta Q}{2} - \frac{1}{2} \frac{\beta Q}{m} \frac{\beta Q}{dm} (m-1)m \end{bmatrix}$$
$$= \frac{u}{2d} \begin{bmatrix} \beta Q^2 - \frac{\beta^2 Q^2}{2} (m-1) \end{bmatrix}$$
$$= \frac{u\beta Q^2}{2d} \begin{bmatrix} 1 - \frac{\beta}{m} (m-1) \end{bmatrix}$$

Transportation cost per cycle is

$$C_t(Q) = 2a + bDQ + bD\mu Q$$

Emission cost from transportation per cycle is

$$C_e(Q) = 2\theta \frac{D}{V}$$

The number 2 refers to a roundtrip waste produced by the inventory system per cycle is

$$C_{w}(Q) = \gamma_{0} + \gamma Q(\delta + \mu)$$

where γ_0 is the fixed cost per waste disposal activity.

The total cost K(Q)= setup cost + holding cost + transportation cost + emission cost + waste disposal cost. Environmentally responsible repair and waste Disposal inventory models

$$= \left[\left(\mathbf{mr} + \mathbf{ns} \right) + \frac{h}{2} \left[\frac{\alpha^2 Q^2}{dn} + \frac{\beta^2 Q^2}{dm} \right] + \frac{u\beta Q^2}{2d} \left(1 - \frac{\beta}{m} (m-1) \right) \right]$$
$$+ \left[2a + bDQ + bD\mu Q + 2\theta \frac{D}{V} + \gamma_0 + \gamma Q(\delta + \mu) \right]$$

The total cost per unit of time is given by dividing the above expression by T.

Now diff (1), w.r.t Q,

(ie) $\frac{d^2 K}{dQ^2} \text{ for every } Q > 0 \text{ and it has a unique minimum and derived by setting its first derivative equal to zero (ie)}$ $<math display="block">\frac{dK}{dQ} = 0.$ $\frac{dK}{dQ} = -\frac{d}{Q^2} (mr + ns) + \frac{h}{2} \left(\frac{\alpha^2}{n} + \frac{\beta^2}{m} \right) + \frac{u\beta}{2} \left(1 - \beta + \frac{\beta}{m} \right) - \frac{2ad}{Q^2} + 0 + 0 - \frac{2\theta Dd}{VQ^2} - \gamma_0 \frac{d}{Q^2} + 0 = 0$ $= -\frac{1}{Q^2} \left[d(mr + ns) + 2ad + \frac{2\theta Dd}{VQ^2} - \gamma_0 d \right] + \frac{1}{2} \left[h \left(\frac{\alpha^2}{n} + \frac{\beta^2}{m} \right) + u\beta \left(1 - \beta + \frac{\beta}{m} \right) \right] = 0 \qquad \dots (2)$ $\frac{1}{Q^2} \left[d(mr + ns) + 2ad + \frac{2\theta Dd}{V} + \gamma_0 d \right] = \frac{1}{2} \left[h \left(\frac{\alpha^2}{n} + \frac{\beta^2}{m} \right) + u\beta \left(1 - \beta + \frac{\beta}{m} \right) \right]$ $\Rightarrow 2 \left[d(mr + ns) + 2ad + \frac{2\theta Dd}{V} + \gamma_0 d \right] = Q^2 \left[h \left(\frac{\alpha^2}{n} + \frac{\beta^2}{m} \right) + u\beta \left(1 - \beta + \frac{\beta}{m} \right) \right]$ $\Rightarrow Q^2 = \frac{2d(mr + ns) + 4ad + \frac{4\theta Dd}{V} + 2\gamma_0 d}{h \left(\frac{\alpha^2}{n} + \frac{\beta^2}{m} \right) + u\beta \left(1 - \beta + \frac{\beta}{m} \right)}$ Г

$$Q = \sqrt{\frac{2d(mr+ns) + 4ad + \frac{4\theta Dd}{V} + 2\gamma_0 d}{h\left(\frac{\alpha^2}{n} + \frac{\beta^2}{m}\right) + u\beta\left(1 - \beta + \frac{\beta}{m}\right)}} \qquad \dots (3)$$

Now diff (2) w.r.t. Q,

$$\frac{d^{2}K}{dQ^{2}} = \frac{2d}{Q^{3}}(mr + ns) + \frac{4ad}{Q^{3}} + \frac{4\theta Dd}{VQ^{3}} + 2\gamma_{0}\frac{d}{Q^{3}}$$
$$= \frac{1}{Q^{3}}\left[2d(mr + ns) + 4ad + \frac{4\theta Dd}{V} + 2\gamma_{0}d\right] \qquad \dots (4)$$

Subs the value of Q in (3), we get

$$\frac{d^{2}K}{dQ^{2}} = \frac{2d(mr+ns) + 4ad + \frac{4\theta Dd}{V} + 2\gamma_{0}d}{\left[\sqrt{\frac{2d(mr+ns) + 4ad + \frac{4\theta Dd}{V} + 2\gamma_{0}d}{h\left(\frac{\alpha^{2}}{n} + \frac{\beta^{2}}{m}\right) + u\beta\left(1 - \beta + \frac{\beta}{m}\right)}}\right]^{3/2}} > 0$$

The optimal value of Q^0 is

$$Q^{0} = \sqrt{\frac{2d(mr+ns) + 4ad + \frac{4\theta Dd}{V} + 2\gamma_{0}d}{h\left(\frac{\alpha^{2}}{n} + \frac{\beta^{2}}{m}\right) + u\beta\left(1 - \beta + \frac{\beta}{m}\right)}}$$

Subs the value of Q^0 in (1)

$$K(Q) = \frac{d}{Q^{0}}(mr + ns) + \frac{Q^{0}}{2} \left[h\left(\frac{\alpha^{2}}{n} + \frac{\beta^{2}}{m}\right) + \frac{u\beta}{2} \left(1 - \beta + \frac{\beta}{m}\right) \right] + \frac{2ad}{Q^{0}} + bdD + bD\mu + 2\theta \frac{D}{V} \frac{d}{Q^{0}} + \gamma_{0} \frac{d}{Q^{0}} + \gamma(\delta + \mu)d$$

$$= \frac{1}{Q^{0}} \left[d\left(mr + ns\right) + 2ad + 2\theta \frac{Dd}{V} + \gamma_{0}d \right] + \frac{Q^{0}}{2} \left[h\left(\frac{\alpha^{2}}{n} + \frac{\beta^{2}}{m}\right) \frac{u\beta}{2} \left(1 - \beta + \frac{\beta}{m}\right) \right] + bdD + bd\mu D$$

$$K(Q) = l_{1} + l_{2} + l_{3} \qquad \dots (5)$$

where
$$l_1 = \frac{1}{Q^0} \left[d\left(mr + ns\right) + 2ad + 2\theta \frac{Dd}{V} + \gamma_0 d \right]$$

 $l_2 = \frac{Q^0}{2} \left[h\left(\frac{\alpha^2}{n} + \frac{\beta^2}{m}\right) + u\beta \left(1 - \beta + \frac{\beta}{m}\right) \right]$
 $l_3 = bdD + bd\mu D$

Now

 l_1

$$l_{2} = \frac{\left[2d\left(mr+ns\right)+4ad+4\theta\frac{Dd}{V}+2\gamma_{0}d\right]^{1/2}}{\left[h\left(\frac{\alpha^{2}}{n}+\frac{\beta^{2}}{m}\right)+u\beta\left(1-\beta+\frac{\beta}{m}\right)\right]^{1/2}} \times \frac{1}{2}\left[h\left(\frac{\alpha^{2}}{n}+\frac{\beta^{2}}{m}\right)+u\beta\left(1-\beta+\frac{\beta}{m}\right)\right]^{1/2}}$$
$$= \frac{1}{\sqrt{2}}\left[h\left(\frac{\alpha^{2}}{n}+\frac{\beta^{2}}{m}\right)+u\beta\left(1-\beta+\frac{\beta}{m}\right)\right]^{1/2}} \times \left[d\left(mr+ns\right)+2ad+2\theta\frac{Dd}{V}+\gamma_{0}d\right]^{1/2}}$$
$$l_{2} = \sqrt{\frac{1}{2}\left[h\left(\frac{\alpha^{2}}{n}+\frac{\beta^{2}}{m}\right)+u\beta\left(1-\beta+\frac{\beta}{m}\right)\right]}\left[d\left(mr+ns\right)+2ad+2\theta\frac{Dd}{V}+\gamma_{0}d\right]} \qquad \dots (7)$$

Subs the value of l_1 , l_2 , l_3 in (3) we get

$$K(Q) = \sqrt{\frac{1}{2} \left[h\left(\frac{\alpha^2}{n} + \frac{\beta^2}{m}\right) + u\beta\left(1 - \beta + \frac{\beta}{m}\right) \right] \left[d\left(mr + ns\right) + 2ad + 2\theta\frac{Dd}{V} + \gamma_0 d \right]} + \sqrt{\frac{1}{2} \left[h\left(\frac{\alpha^2}{n} + \frac{\beta^2}{m}\right) + u\beta\left(1 - \beta + \frac{\beta}{m}\right) \right] \left[d\left(mr + ns\right) + 2ad + 2\theta\frac{Dd}{V} + \gamma_0 d \right]} + bdD + bd\mu D$$

$$= 2\sqrt{\frac{1}{2} \left[h\left(\frac{\alpha^2}{n} + \frac{\beta^2}{m}\right) + u\beta\left(1 - \beta + \frac{\beta}{m}\right) \right] \left[d\left(mr + ns\right) + 2ad + 2\theta\frac{Dd}{V} + \gamma_0 d \right]} + bdD + bd\mu D$$

$$= 2\left[h\left(\frac{\alpha^2}{n} + \frac{\beta^2}{m}\right) + u\beta\left(1 - \beta + \frac{\beta}{m}\right) \right] \left[d\left(mr + ns\right) + 2ad + 2\theta\frac{Dd}{V} + \gamma_0 d \right]^{1/2} + bdD + bd\mu D$$
The enting lated part is

The optimal total cost is

$$K(Q) = 2\left[h\left(\frac{\alpha^2}{n} + \frac{\beta^2}{m}\right) + u\beta\left(1 - \beta + \frac{\beta}{m}\right)\right] \left[d\left(mr + ns\right) + 2ad + 2\theta\frac{Dd}{V} + \gamma_0 d\right]^{1/2} + bdD + bd\mu D \qquad \dots (8)$$

CONCLUSION

This paper studies the environmentally responsible repair and waste disposal inventory models. The model is very helpful to reduce the cost of remanufacturing option provides inventory cost benefits for a wide range of system by giving the manufacturer the opportunity to balance the workload between two alternative ways to produce a product.

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