CONSTRUCTION OF MIXED SAMPLING PLANS INDEXED THROUGH MAPD AND IQL WITH LINK SAMPLING PLAN AS ATTRIBUTE PLAN USING WEIGHTED POISSON DISTRIBUTION

***R. Sampath Kumar**

****R. Kiruthika**

****R. Radhakrishnan**

ABSTRACT

This paper presents the procedure for the construction and selection of mixed sampling plan with MAPD as a quality standard and link sampling plan as attribute plan using weighted Poisson distribution as a base line distribution. The plans are constructed indexed through MAPD and IQL and also compared. Tables are constructed for the easy selection of the plans.

Keywords and Phrases: Maximum allowable percent defective, Indifference quality level, Operating characteristic, Tangent intercept.

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* Assistant Professor, Department of Statistics, Government Arts College, Coimbatore.

** Assistant Professor in Statistics, Kalaignar Karunanidhi Institute of Technology, Coimbatore.

*** Associate Professor, Department of Statistics, PSG College of Arts and Science, Coimbatore.

1. INTRODUCTION

Mixed sampling plans consist of two stages of rather different nature. During the first stage the given lot is considered as a sample from the respective production process and a criterion by variables is used to check process quality. If process quality is judged to be sufficiently good, the lot is accepted. Otherwise the second stage of the sampling plan is entered and lot quality is checked directly by means of an attribute sampling plan.

There are two types of mixed sampling plans called independent and dependent plans. If the first stage sample results are not utilized in the second stage, then the plan is said to be independent otherwise dependent. The principal advantage of a mixed sampling plan over pure attribute sampling plans is a reduction in sample size for a similar amount of protection.

It is the usual practice that while selecting a sampling inspection plan, to fix the operating characteristic (OC) curve in accordance with the desired degree of discrimination. The sampling plan is in turn fixed through suitably chosen parameters. The entry parameters used in the acceptance sampling literature are acceptable quality level (AQL), indifference quality level (IQL), limiting quality level (LQL) and maximum allowable percent defective (MAPD). Several authors have provided procedures to design the sampling plans indexed through these parameters for various acceptance sampling plans.

The concept of MAPD (p_*) was introduced by Mayer (1967) and further studied by Soundararajan (1975) is the quality level corresponding to the inflection point on the OC curve. The degree of sharpness of inspection about this quality level ‘ p_*’ is measured through ‘ p_t’, the point at which the tangent to the OC curve at the inflection point cuts the proportion defective axis. One of the desired properties of an OC curve is that the decrease of $P_a(p)$ should be slower for lesser values of ‘p’ and faster for greater values of ‘p’. If we set p_* as the quality standard, the above property of the OC curve is obtained since p_* corresponds to the inflection point of the OC curve and hence

$$d^2P_a(p)/dp^2 = 0 \qquad \text{for } p = p_*$$

$$d^2P_a(p)/dp^2 < 0 \qquad \text{for } p < p_*$$

$d^2 P_a(p)/dp^2 > 0$ for p> p_*

The mixed sampling plan has been designed under two cases of significant interest. In the first case sample size n_1 is fixed and a point on the OC curve is given. In the second case plans are designed when two points on the OC curve are given. The procedure for designing the mixed sampling plans to satisfy the above mentioned conditions was provided by Schilling (1967). Using Schilling's procedure, Devaarul (2003) has constructed tables for mixed sampling plans (independent case) having various sampling plans as attribute plans. Link sampling plan comes under conditional sampling plans, which is classified as cumulative type of sampling plan. The Conditional sampling procedures have been developed to reduce the sample size and the inspection cost of the decision process using sample results from neighboring ad related lots. This is the main advantage of conditional sampling plans. But, when the process quality slowly changes the sample results from the past lots do not reveal the exact situation. Harishchandra and Srivenkataramana (1982) have developed link sampling plans. Sampath Kumar et.al (2012) and Radhakrishnan et.al (2010) have made contributions to mixed sampling plan for independent case. Sampath Kumar (2007) has constructed mixed sampling plan with various sampling plans as attribute plans using Poisson distribution as a base line distribution.

The weighted Poisson distribution plays an important role in the acceptance sampling, mainly in the construction of sampling plans. Each outcome (number of defectives) is specific but can be assigned with different weights based on its importance or usage. In using weighted Poisson distribution with weights X^{α}, α =1 the range of the distribution curtailed to 1, 2, 3… from 0, 1, 2… This distribution can be viewed as a truncated Poisson distribution truncated at x =0. It will be more useful to the industries which concentrates on second's quality lots and also to the industries which has atleast one defective in the majority of the manufacturing lots. Even though the modern technologies aim at zero defective/ defects but practically it is very difficult to make the lot as zero defective lot. In this context, the application of weighted Poisson distribution in the construction of sampling plans is very relevant and it has many features /advantages also.

Radhakrishna Rao(1977) suggested a weighted Binomial distribution can be used in designing sampling plans. Sudeswari (2002) studied the construction of sampling plans using weighted Poisson distribution as a base line distribution. Mohana Priya (2008), Sampath Kumar et.al (2011), Radhakrishnan and Mohana Priya (2008 a, 2008 b) have constructed the sampling plans using weighted Poisson distribution as a base line distribution.

In this paper, mixed sampling plan (independent case) with link sampling plan as attribute plan is constructed using weighted Poisson distribution as a base line distribution. The plans indexed through MAPD and IQL are constructed separately by fixing the values of c_1, c_2 and β_j'. The mixed plans indexed through MAPD and IQL are also compared.

2. GLOSSARY OF SYMBOLS

The symbols used in this paper are as follows:

p : submitted quality of lot or process

P_a (p) : probability of acceptance for given quality p

p_* : maximum allowable percent defective

p_t : the point at which the inflection tangent of the OC curve cuts the 'p' axis

p_0 : the submitted quality level such that $P_a(p_0) = 0.50$

h_* : relative slope at p_*

n_1 : sample size for the variable sampling plan

n_2 : sample size for the attribute sampling plan

c_1 : first attributes acceptance number

c_2 : second attributes acceptance number

c_3 : third attributes acceptance number

d : number of defectives in the sample

β_j : probability of acceptance for lot quality 'p_j'

β_j' : probability of acceptance assigned to first stage for percent defective p_j

β_j'' : probability of acceptance assigned to second stage for percent defective p_j

k : variable factor such that a lot is accepted if $\bar{X} \leq A = U - k\sigma$

3. OPERATING PROCEDURE OF MIXED SAMPLING PLAN HAVING LINK SAMPLING PLAN AS ATTRIBUTE PLAN

Schilling (1967) has given the following procedure for the independent mixed sampling plan with upper specification limit (U) and standard deviation (σ).

1. Determine the parameters of the mixed sampling plan n_1, n_2, k, c_1, c_2 and c_3.
2. Take a random sample of size n_1 from the lot.
3. If a sample average $\bar{X} \leq A = U - k\sigma$, accept the lot
4. If the sample average $\bar{X} > A = U - k\sigma$, take another sample of size n_2 from the lot 'i' ad count the number of defectives d_i there i.
5. If the number of defectives $d_i \leq c_1$, accept the lot.
6. If the number of defectives $d_i < c_3$, reject the lot.
7. If $c_1 < D_i \leq c_3$, combine the total number of defectives from the immediate part and future lots, $Di=d_{i-1}+d_i+d_{i+1}$
8. If $D_i \leq c_2$, accept the lot 'i' and reject the lot if $D_i>c_3$

The OC function of the mixed sampling plan, suggested by Schilling (1967) for single sampling plan is

$$P_a(p) = P_{n_1}(\bar{x} \leq A) + P_{n_1}(\bar{x} > A)\sum_{j=0}^{c} P(j;n_2) \quad \text{.....(1)}$$

Equation (1) can be expressed as $\beta_j = \beta_j' + (1-\beta_j')\beta_j''$.
By taking the link sampling plan as attribute plan, equation (1) can be written as

$$P_a(p) = P_{n_1}(\bar{x} \leq A) + P_{n_1}(\bar{x} > A)\left(\sum_{i=1}^{c_1} P_{ri} + P_{c_1+1}\sum_{i=1}^{c_3-c_1-1} q_{ri} + P_{c_1+2}\sum_{i=1}^{c_3-c_1-2} q_{ri} + \ldots\ldots + P_{c_2}\sum_{i=1}^{c_3-c_2} q_{ri}\right) \quad \text{.....(2)}$$

Where $P_{ri} = \dfrac{e^{-np}(np)^{ri-1}}{(ri-1)!}$, ri = 1,2,……$c_2$

$q_{ri} = \dfrac{e^{-knp}(knp)^{ri-1}}{(ri-1)!}$, ri= 1,2………$c_3$-($c_1$+1)

In this paper, the probability mass function of the link sampling plan is used for k=2.

4. CONSTRUCTION OF MIXED SAMPLING PLAN HAVING LINK SAMPLING PLAN AS ATTRIBUTE PLAN USING WEIGHTED POISSON DISTRIBUTION

The detailed procedure adopted in this paper for the construction of mixed sampling plan having link sampling as attribute plan using weighted Poisson distribution indexed through MAPD is given below:

- Assume that the mixed plan is independent
- Decide the sample size n_1 (for variable sampling plan) to be used.
- Calculate the acceptance limit for the variable sampling plan as

  $A = U - [z(p_*\ ) + \{z(\beta_*^{'}) / \sqrt{n_1}\}]\sigma$, where z is standard normal variate corresponding

  to'‘t' such that t = $\int_{z(t)}^{\infty} \frac{1}{\sqrt{2\pi}} e^{\frac{-u^2}{2}} du$

- Split the probability of acceptance β_* as β_*' and β_*''such that $\beta_* = \beta_*' + (1- \beta_*')\ \beta_*''$. Fix the value of β_*' .
- Determine β_*'', the probability of acceptance assigned to the attribute plan associated with the second stage sample as $\beta_*'' = (\beta_* - \beta_*')/ (1- \beta_*')$.
- Determine the appropriate second stage sample of n_2 from the relation

$$\beta_*'' = \sum_{i=1}^{c_1} P_{ri} + P_{c_i+1} \sum_{i=1}^{c_3-c_1-1} q_{ri} + P_{c_1+2} \sum_{i=1}^{c_3-c_1-2} q_{ri} + \ldots\ldots\ldots\ldots\ldots + P_{c_2} \sum_{i=1}^{c_3-c_2} q_{ri}$$

Where $P_{ri} = \frac{e^{-np}(np)^{ri-1}}{(ri-1)!}$, ri = 1, 2,……$c_2$

$$q_{ri} = \frac{e^{-knp}(knp)^{ri-1}}{(ri-1)!}$$, ri= 1, 2………c_3-(c_1+1)

Using the above procedure, tables have been constructed to facilitate easy selection of mixed sampling plan using link sampling plan as attribute plans indexed through MAPD.

4.1 CONSTRUCTION OF TABLES

The OC function of weighted Poisson distribution for single sampling plan is given by,

$$P_a(p) = \frac{x^{\alpha} p(x,\alpha)}{\sum_{x=0}^{\infty} x^{\alpha} p(x;\alpha)}$$; x=0, 1, 2………… …………………………………………………

(3)

The probability of acceptance for link sampling plan under weighted Poisson distribution when α = 1 is used in this paper for determining the second stage probabilities and is given by

$$P_a(p) = \sum_{i=1}^{c_1} P_{ri} + P_{c_i+1} \sum_{i=1}^{c_3-c_1-1} q_{ri} + P_{c_1+2} \sum_{i=1}^{c_3-c_1-2} q_{ri} + \ldots\ldots\ldots\ldots + P_{c_2} \sum_{i=1}^{c_3-c_2} q_{ri} = \beta_j'' \quad \ldots\ldots\ldots\ldots\ldots (4)$$

Where $q_{ri} = \frac{e^{-knp}(knp)^{ri-1}}{(ri-1)!}$, ri = 1, 2,……$c_2$, $q_{ri} = \frac{e^{-knp}(knp)^{ri-1}}{(ri-1)!}$, ri= 1, 2………$c_3$-($c_1$+1)

Using the equation (4) the inflection point (p_*) is obtained by using $\frac{d^2 p_a(p)}{dp^2} = 0$ and $\frac{d^3 p_a(p)}{dp^3} \neq 0$.

The relative slope of the OC curve h_* is given by, $h_* = \left(\frac{-p}{P_a(p)}\right)\frac{dP_a(p)}{dp}$ at $p=p_*$.

The inflection tangent of the OC curve cuts the 'p' axis at $p_t = p_* + (p_*/h_*)$. The values of n_2p_*, h_*, n_2p_t and $R= p_t / p_*$ are calculated for the specified $\beta_*' = 0.25$ using C program and presented in Table 1.

4.2 SELECTION OF THE PLAN

Table 1 is used to construct the plans when MAPD (p_*) and tangent intercept (p_t) are given. For any given values of c_1, c_2, p_t and p_* one can find the ratio $R = p_t / p_*$. Corresponding to the value of c_1 and c_2 find the value in Table1 under the column R which is equal to or just greater than the specified ratio, the corresponding value of c_3 is noted. From this c_1,c_2 and c_3 values one can determine the value of 'n' using $n_2 = n_2 p_*/ p_*$.

Example 1: Given $c_1 = 4$, $c_2 = 8$, $p_* = 0.06$, p_t =0.15and $\beta_*' = 0.25$. Find the ratio $R= p_t /p_* = 2.5$. Using Table 1, corresponding to $c_1 = 4$, c_2=8 select the value of R equal to or just greater than this ratio is 2.6239 which is associated with $c_1 = 4$, c_2=8, $c_3 = 12$ and $n_2 = n_2 p_*/ p_* =$

(3.6448/0.06) = 61. The Mixed Sampling plan with link sampling plan as attribute plan is, n_2=61, c_1 = 4, c_2=8 and c_3=12 and the OC curve is presented in Figure1.

5. SELECTION OF MIXED SAMPLING PLAN HAVING LINK SAMPLING PLAN AS ATTRIBUTE PLAN INDEXED THROUGH IQL

The general procedure given in section 4 is used for designing the mixed sampling plan having link sampling plan as attribute plan indexed through IQL (p_0). For the specified values of β_0=0.50 and β_0' =0.25, the n_2p_0 values are calculated for different values of c_1, c_2 and c_3 using C program and presented in Table1.

5.1 SELECTION OF THE PLAN

Table 1 is used to construct the plans when IQL (p_0), c_1, c_2, c_3 values are given. For any specified values of p_0, c_1 ,c_2 and c_3 one can determine n_2 value using n_2= n_2p_0 / p_0.

Example 2: Given p_o= 0.07, c_1 = 4, c_2 = 7, c_3 = 11 and β_0' =0.25. Using Table 1, find n_2= n_2p_0/p_0= 4.8788/0.07=70. For a fixed β_0' =0.25, the mixed sampling plan with link sampling plan as attribute plan is n_2= 70, c_1= 4, c_2=7 and c_3=11.

6. COMPARISON OF PLANS INDEXED THROUGH MAPD AND IQL

In this section the mixed sampling plan with link sampling plan as attribute plan indexed through MAPD is compared with the mixed sampling plan with link sampling plan as attribute plan indexed through IQL by fixing the parameters c_1,c_2 and β_0'.

For the specified values of p_* and p_t with the assumption β_*' = 0.25 one can find the values of c_3 and n_2 indexed through MAPD as in section 4. By fixing the values of c_1, c_2 and n_2, find the value of p_1 by equating $P_a(p) = \beta_0$= 0.50. Using β_0' =0.25 ,c_1, c_2 and p_0 one can find the value of n_2 using $n_2 = n_2p_0/p_0$ from Table 1. For different combinations of c_1,c_2, p_* and p_t, the values of n_2, c_3(indexed through MAPD) and n_2, c_3 (indexed through IQL) are calculated and presented in Table 2.

Table 2: Comparison of plans

Given Values				Through MAPD	Through IQL
c_1	c_2	p_*	p_t	n_2,c_3	n_2,c_3
1	3	0.02	0.10	59,6	70,6
1	4	0.04	0.08	54,7	62,7
2	5	0.04	0.14	41,7	49,7
3	5	0.05	0.14	48,8	58,8
3	7	0.06	0.12	58,11	65,11
4	8	0.06	0.15	61,12*	70,12*

* OC Curves are drawn

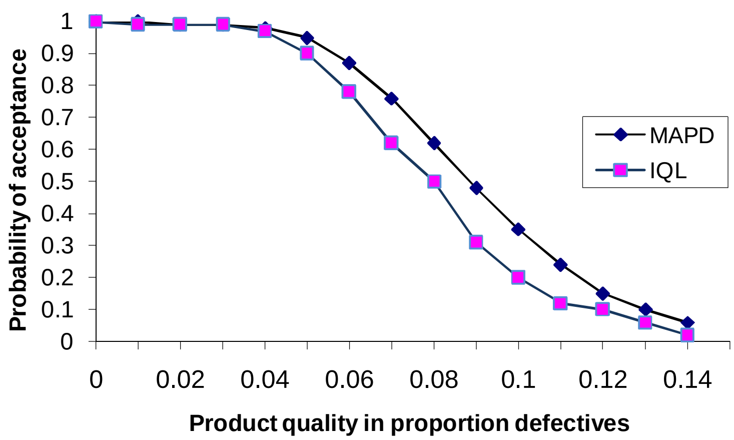

Figure 1: OC Curves for LSP with n=61 (MAPD), n=70 (IQL) , c_1 =4, c_2 = 8, c_3 = 12

7. CONCLUSION

In this paper the procedure for constructing mixed sampling plans with link sampling plan attribute plan indexed through MAPD and IQL with weighted Poisson distribution as the baseline distribution are presented. Suitable tables are also provided for the easy selection of the plans for the engineers who are working on the floor of the assembly. It is concluded from the study that the second sample size required for mixed sampling plan with link sampling plan as attribute plan indexed through MAPD is less than that of the second stage sample size of the mixed sampling plan with link sampling plan as attribute plan indexed through IQL, justified by Sampath Kumar (2008). These plans definitely help the producers, because of the lesser sample size which directly result in lesser sampling cost and indirectly reduces the total cost of the product. The different sampling plans can also be constructed by changing the first stage probabilities (β_*' and β_0') and can be compared for their efficiency.

REFERENCES

1. **Devaarul, S (2003).** 'Certain Studies Relating to Mixed Sampling Plans and Reliability based Sampling Plans', Ph.D., Dissertation, Department of Statistics, Bharathiar University, Coimbatore, Tamil Nadu, India.
2. **Harishchandra, K. and Srivenkataramana, T.** (1982). 'Link sampling for attributes', Communications in Statistics, 'Theory and Methods', Vol.11, No.16, pp.1855-1868.
3. **Mayer, P.L. (1967).** 'A note on sum of Poisson probabilities and an application', '*Industrial Quality Control*', Vol.19, No.5, pp.12-15.
4. **Mohana Priya, L. (2008).** 'Construction and Selection of Acceptance Sampling Plans based on Weighted Poisson Distribution', Ph.D., Dissertation, Department of Statistics, Bharathiar University, Coimbatore, Tamil Nadu, India.
5. **Radhakrishna Rao, C. (1977).** 'A natural example of a weighted binomial distribution', The American Statistician, Vol.31, No.1
6. **Radhakrishnan, R. and Mohana Priya, L. (2008 a).** 'Selection of single sampling plan using conditional weighted Poisson distribution', '*International Journal of Statistics and Systems*', Vol.3, No.1, pp. 91-98.
7. **Radhakrishnan, R. and Mohana Priya, L. (2008 b).** 'Comparison of CRGS plans using Poisson and weighted Poisson distribution', '*Probstat Forum'*, Vol.1, No.1, pp.50-61.
8. **Radhakrishnan, R. Sampath Kumar, R. and Malathi, M. (2010).** 'Selection of mixed sampling plan with TNT- **(**n_1, n_2; 0) plan as attribute plan indexed through MAPD and MAAOQ', '*International Journal of Statistics and system*', Vol.5, No.4, pp.477-484.
9. **Sampath Kumar, R. (2007).** 'Construction and Selection of Mixed Variables-Attributes Sampling Plans' - Ph.D., Dissertation, Department of Statistics, Bharathiar University, Coimbatore, Tamil Nadu, India.
10. **Sampath Kumar, R. Kiruthika, R. and Radhakrishnan, R. (2011).** 'Comparison of mixed sampling plan indexed through MAPD and LQL with single sampling plan as attribute plan using weighted Poisson distribution', '*Journal of statistics science'*, Vol.3, No.2,pp. 165-173.
11. **Sampath Kumar, R.Vijayakumar, R. and Radhakrishnan, R. (2012).** 'Selection of mixed sampling plans with repetitive group sampling plan as attribute plan indexed

through MAPD and AQL using intervened random effect Poisson distribution', '*Global journal of Management science and technology'*, Vol.1, No. 4.

12. **Schilling, E.G (1967).** 'A General Method for Determining the Operating Characteristics of Mixed Variables'-'Attributes sampling plans Single side specifications, S.D.known', Ph.D., Dissertation- Rutgers- The State University, New Brunswick, New Jersy.

13. **Soundararajan, V. (1975).** 'Maximum allowable percent defective (MAPD) single sampling inspection by attribute plan', '*Journal of Quality Technology*', Vol.7, No.4, pp.173-182.

14. **Subramani, K .(1991).** 'Studies on Designing Attribute Acceptance Sampling Plans with Emphasis on Chain Sampling plans', Ph.D Thesis, Bharathiar University, Coimbatore, Tamil Nadu, India.

15. **Sudeswari (2002).** 'Designing of Sampling Plan using Weighted Poisson distribution. M.Phil., Dissertation, Department of Statistics, PSG College of Arts and Science, Coimbatore.

Table 1: Various characteristics of the mixed sampling plan when (p_*, β_*) and (p_0, β_0) are known for a specified $\beta_*' = 0.25$, $\beta_0=0.50$ and $\beta_0' =0.25$

c_1	c_2	c_3	n_2p_0	β_*	β_*''	n_2p_*	h_*	n_2p_t	$R = p_t/p_*$
1	2	2	1.0989	0.6618	0.5491	0.5944	0.2006	3.5874	5.9849
1	2	3	1.2020	0.5976	0.4635	0.9068	0.2116	5.1922	5.7258
1	3	4	1.5459	0.6933	0.5911	1.0422	0.2696	4.9079	4.7092
1	3	5	1.8861	0.8042	0.7389	1.1314	0.2188	6.3023	5.5703
1	3	6	2.2274	0.9161	0.8881	1.1740	0.2450	5.9658	5.0816
1	4	5	2.0451	0.7523	0.6697	1.3902	0.4267	4.6482	3.3435
1	4	6	2.4879	0.7924	0.7232	1.7437	0.6538	4.4107	2.5295
1	4	7	2.9182	0.8023	0.7364	2.1483	0.9899	4.3185	2.0102
2	4	5	2.3975	0.6984	0.5979	1.6104	0.2832	5.6864	3.5310
2	4	6	2.5734	0.7619	0.6825	1.6383	0.3414	6.4370	3.9291
2	4	7	2.8132	0.8601	0.8135	1.6063	0.3010	6.9428	4.3222
2	5	5	2.4242	0.8409	0.7879	1.1667	0.3074	4.9621	4.2531
2	5	6	2.6465	0.8623	0.8164	1.3947	0.3374	5.5283	3.9637
2	5	7	2.9546	0.9016	0.8688	1.6291	0.3950	5.7534	3.5316

2	5	8	3.3108	0.9230	0.8973	1.9697	0.3601	7.4355	3.7749
3	5	8	3.6153	0.7869	0.7159	2.3840	0.3533	7.1011	2.9786
3	5	9	3.7841	0.8768	0.8357	2.1789	0.3192	9.0050	4.1328
3	6	8	3.6962	0.7788	0.7051	2.4299	0.3666	9.0581	3.7278
3	6	9	3.9011	0.7963	0.7284	2.6705	0.4991	8.0211	3.0036
3	7	8	3.7115	0.7799	0.7065	2.4610	0.3719	9.0784	3.6889
3	7	9	3.9713	0.8037	0.7383	2.7154	0.4960	8.1899	3.0161
3	7	10	4.3102	0.8227	0.7636	3.0587	0.6999	7.4288	2.4287
3	7	11	4.6764	0.8345	0.7793	3.4498	0.9661	7.0207	2.0351
4	7	11	4.8788	0.9570	0.9427	2.5273	0.3456	9.8400	3.8934
4	7	12	5.0991	0.8191	0.7588	3.6097	0.6546	9.1241	2.5277
4	8	11	4.9386	0.8241	0.7655	3.3503	0.5679	9.2498	2.7608
4	8	12	5.1995	0.8406	0.7875	3.6448	0.6158	9.5636	2.6239
4	8	13	5.5141	0.8499	0.7999	4.0137	0.8478	8.7479	2.1795