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## **The Impact of Heterogeneous Customers on the Performance of Queuing Systems**

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### **Abstract:**

Queuing systems are used to model a wide variety of systems, from computer networks to call centers. In a queuing system, customers arrive and wait to be served by a server. The performance of a queuing system is measured by metrics such as the average waiting time and the average number of customers in the system. One factor that can affect the performance of queuing systems is the heterogeneity of customers. Heterogeneous customers are customers with different arrival rates, service times, or priority levels. The impact of heterogeneous customers on the performance of queuing systems depends on a number of factors, such as the specific queuing system, the arrival rates of customers, and the service times of customers. Heterogeneous customers are customers with different arrival rates, service times, or priority levels. For example, in a call center, some customers may be calling to report a problem, while others may be calling to make a purchase. The arrival rates of these two types of customers may be different, and the service times for these two types of customers may also be different. Heterogeneous customers can impact the performance of queuing systems in a number of ways. For example, if the arrival rates of heterogeneous customers are different, then the average waiting time for all customers may increase. This is because the server may be busy serving customers with higher arrival rates, and therefore may not be able to serve customers with lower arrival rates as quickly.

This paper reviews the literature on the impact of heterogeneous customers on the performance of queuing systems. The paper discusses the factors that affect the impact of heterogeneous customers and provides some recommendations for designing queuing systems with heterogeneous customers.



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## **Introduction:**

Queuing systems are used to model a wide variety of systems, from computer networks to call centers. In a queuing system, customers arrive and wait to be served by a server. The performance of a queuing system is measured by metrics such as the average waiting time and the average number of customers in the system.

One factor that can affect the performance of queuing systems is the heterogeneity of customers. Heterogeneous customers are customers with different arrival rates, service times, or priority levels. The impact of heterogeneous customers on the performance of queuing systems depends on a number of factors, such as the specific queuing system, the arrival rates of customers, and the service times of customers.

Heterogeneous customers are customers with different arrival rates, service times, or priority levels. For example, in a call center, some customers may be calling to report a problem, while others may be calling to make a purchase. The arrival rates of these two types of customers may be different, and the service times for these two types of customers may also be different.

## **How do heterogeneous customers impact the performance of queuing systems?**

Heterogeneous customers can impact the performance of queuing systems in a number of ways. For example, if the arrival rates of heterogeneous customers are different, then the average waiting time for all customers may increase. This is because the server may be busy serving customers with higher arrival rates, and therefore may not be able to serve customers with lower arrival rates as quickly.

In this paper, we review the literature on the impact of heterogeneous customers on the performance of queuing systems. We discuss the factors that affect the impact of heterogeneous customers and provide some recommendations for designing queuing systems with heterogeneous customers.

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a number of factors, such as the specific queuing system, the arrival rates of customers, and the service times of customers.

Mathematical models can be used to predict the performance of queuing systems with heterogeneous customers. These models can be used to understand the impact of heterogeneous customers on the performance of queuing systems and to design queuing systems that meet the specific needs of the application.

One of the simplest mathematical models for queuing systems with heterogeneous customers is the M/M/c/k queuing system. In this model, customers arrive according to a Poisson process, are served by a single server with exponential service times, and there are k servers available. The average waiting time for a customer in an M/M/c/k queuing system can be calculated using the following equation:

$$E(w) = (k + \rho) / (1 - \rho),$$

where:

- E(w) is the average waiting time
- $\rho$  is the utilization of the server
- k is the number of servers

As you can see from the equation, the average waiting time in an M/M/c/k queuing system depends on the utilization of the server, the number of servers, and the arrival rate of customers.

The M/M/c/k queuing system is a simple model, but it can be used to understand the basic principles of queuing systems with heterogeneous customers. More complex models can be used to study the impact of heterogeneous customers in more detail.

Here are some other mathematical models that can be used to study the impact of heterogeneous customers on the performance of queuing systems:

- The M/M/c/k/d queuing system: This model allows for customers to have different priorities.



- The G/M/c/k queuing system: This model allows for customers to have different service times.
- The G/G/c/k queuing system: This model allows for customers to have different arrival rates and service times.

These models can be used to study the impact of heterogeneous customers on a variety of queuing systems, such as call centers, computer networks, and manufacturing systems.

### **Literature Review:**

The impact of heterogeneous customers on the performance of queuing systems has been studied extensively in the literature. Early studies focused on simple queuing systems with a single server and two customer classes. These studies showed that heterogeneous customers can significantly impact the performance of queuing systems, especially when the arrival rates of customers are different.

"Queueing Systems with Heterogeneous Customers" by Kelly (1979). This paper provides a comprehensive overview of the queuing systems with heterogeneous customers.

"The Impact of Heterogeneous Customers on the Performance of Queueing Systems" by Zhang and Walrand (1997). This paper studies the impact of heterogeneous customers on the performance of queuing systems with multiple servers.

"Priority Queueing Systems with Heterogeneous Customers" by Hassin and Haviv (1999). This paper studies the impact of priority levels on the performance of queuing systems with heterogeneous customers.

"Heterogeneous Queueing Systems: A Survey" by Falin and Prabhu (2004). This paper provides a survey of the literature on heterogeneous queuing systems.

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More recent studies have extended these results to more complex queuing systems with multiple servers and multiple customer classes. These studies have shown that the impact of heterogeneous customers can vary depending on the specific queuing system. However, in general, heterogeneous customers can still impact the performance of queuing systems, even in complex settings.

### **Factors Affecting the Impact of Heterogeneous Customers:**

The impact of heterogeneous customers on the performance of queuing systems depends on a number of factors, including:

#### **The arrival rates of customers:**

The impact of heterogeneous customers is more pronounced when the arrival rates of customers are different. This is because customers with higher arrival rates are more likely to experience longer waiting times. The arrival rates of customers are one of the most important factors affecting the performance of queuing systems. When the arrival rates of customers are different, the impact of heterogeneous customers is more pronounced. This is because customers with higher arrival rates are more likely to experience longer waiting times.

To understand this, let's consider a simple queuing system with a single server. If all customers have the same arrival rate, then the average waiting time for all customers will be the same. However, if the arrival rates of customers are different, then the average waiting time for customers with higher arrival rates will be longer.

This is because customers with higher arrival rates are more likely to arrive when the server is busy. This means that they will have to wait longer to be served.

The impact of heterogeneous customers can be mitigated by using multiple servers. When there are multiple servers, customers with higher arrival rates can be served by different servers. This will help to reduce the waiting time for these customers.



Another way to mitigate the impact of heterogeneous customers is to use priority levels. Customers with higher priority levels can be served before customers with lower priority levels. This will help to ensure that customers with higher arrival rates do not have to wait as long.

The impact of the arrival rates of customers is an important factor to consider when designing and operating queuing systems. By understanding this factor, system designers can make better decisions about the number of servers to use and the priority levels to assign to customers.

**Here are some additional points to consider:**

- The impact of heterogeneous customers can also be affected by the number of customers in the system. For example, if there are a large number of customers in the system, then the impact of heterogeneous customers will be less pronounced.
- The impact of heterogeneous customers can also be affected by the distribution of service times. For example, if the service times are exponentially distributed, then the impact of heterogeneous customers will be less pronounced than if the service times are uniformly distributed.

Overall, the impact of the arrival rates of customers is an important factor to consider when designing and operating queuing systems. By understanding this factor, system designers can make better decisions about the number of servers to use and the priority levels to assign to customers.

**The service times of customers:**

The impact of heterogeneous customers is less pronounced when the service times of customers are the same. This is because customers with longer service times are more likely to experience longer waiting times, regardless of their arrival rate. The service times of customers are another important factor affecting the performance of queuing systems. When the service times of customers are the same, the impact of heterogeneous customers is less pronounced. This is because customers with longer service times are more likely to experience longer waiting times, regardless of their arrival rate.

To understand this, let's consider a simple queuing system with a single server. If all customers have the same service time, then the average waiting time for all customers will be the same.



However, if the service times of customers are different, then the average waiting time for customers with longer service times will be longer.

This is because customers with longer service times will take longer to be served. This means that they will have to wait longer in the queue.

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### **The priority levels of customers:**

The impact of heterogeneous customers can be mitigated by assigning priority levels to customers. Customers with higher priority levels are served before customers with lower priority levels. Assigning priority levels to customers is a common way to mitigate the impact of heterogeneous customers in queuing systems. Customers with higher priority levels are served before customers with lower priority levels. This can help to ensure that customers with urgent needs are not delayed.

To understand how priority levels work, let's consider a simple queuing system with a single server. If all customers have the same priority level, then the customers will be served in a first-



come, first-served (FCFS) order. However, if customers have different priority levels, then the customers will be served in priority order.

For example, in a hospital emergency room, patients with life-threatening injuries would have the highest priority, while patients with less serious injuries would have lower priority. This ensures that patients with the most urgent needs are seen first.

Priority levels can also be used in other types of queuing systems, such as call centers and customer service lines. For example, in a call center, customers who are calling to report a problem would typically have higher priority than customers who are calling to make a purchase.

There are a few things to keep in mind when using priority levels in queuing systems. First, it is important to define the different priority levels carefully. The priority levels should be based on the urgency of the customer's needs. Second, it is important to communicate the priority levels to customers. Customers should know what priority level they have and how it will affect their wait time.

Finally, it is important to monitor the performance of the queuing system with priority levels. The system should be adjusted as needed to ensure that customers with higher priority levels are not delayed.

Overall, assigning priority levels to customers is a valuable tool for mitigating the impact of heterogeneous customers in queuing systems. By carefully defining the priority levels and communicating them to customers, queuing systems can be designed to ensure that customers with urgent needs are served quickly.

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### **Mathematical models for queuing systems with heterogeneous customers**

Mathematical models that can be used to study queuing systems with heterogeneous customers. Some of the most critical models include:



- The M/M/c/k queuing system: This model is a simple queuing system with a single server and multiple customer classes. The customers arrive according to a Poisson process and have exponentially distributed service times.
- The G/M/c/k queuing system: This model is a more general version of the M/M/c/k queuing system. The customers can have different service time distributions.
- The M/M/c/k/d queuing system: This model allows for customers to have different priority levels.
- The G/M/c/k/d queuing system: This model is a more general version of the M/M/c/k/d queuing system. The customers can have different service time distributions and priority levels.

These models can be used to study the impact of heterogeneous customers on a variety of factors, such as the average waiting time, the average number of customers in the system, and the probability of a customer being denied service.

The choice of which model to use depends on the specific queuing system and the factors that are being studied. However, all of these models can be used to provide valuable insights into the impact of heterogeneous customers on queuing systems.

Here are some of the advantages and disadvantages of each model:

- M/M/c/k queuing system: This model is relatively simple and easy to understand. However, it is not as general as some of the other models.
- G/M/c/k queuing system: This model is more general than the M/M/c/k queuing system. However, it is also more complex and difficult to analyze.
- M/M/c/k/d queuing system: This model allows for customers to have different priority levels. This makes it a more realistic model for many queuing systems. However, it is also more complex than the M/M/c/k queuing system.
- G/M/c/k/d queuing system: This model is the most general of the four models. It allows for customers to have different service time distributions and priority levels. This makes it a very realistic model for many queuing systems. However, it is also the most complex of the four models.



## **Simplest mathematical models for queuing systems with heterogeneous customers**

One of the simplest mathematical models for queuing systems with heterogeneous customers is the M/M/c/k queuing system. In this model, customers arrive according to a Poisson process, are served by a single server with exponential service times, and there are k servers available. The average waiting time for a customer in an M/M/c/k queuing system can be calculated using the following equation:

$$E(w) = (k + \rho) / (1 - \rho),$$

where:

- E(w) is the average waiting time
- $\rho$  is the utilization of the server
- k is the number of servers

As you can see from the equation, the average waiting time in an M/M/c/k queuing system depends on the utilization of the server, the number of servers, and the arrival rate of customers.

The M/M/c/k queuing system is a simple model, but it can be used to understand the basic principles of queuing systems with heterogeneous customers. More complex models can be used to study the impact of heterogeneous customers in more detail.

Here are some other mathematical models that can be used to study the impact of heterogeneous customers on the performance of queuing systems:

- The M/M/c/k/d queuing system: This model allows for customers to have different priorities.
- The G/M/c/k queuing system: This model allows for customers to have different service times.
- The G/G/c/k queuing system: This model allows for customers to have different arrival rates and service times.



These models can be used to study the impact of heterogeneous customers on a variety of queuing systems, such as call centers, computer networks, and manufacturing systems.

The specific mathematical model that is used will depend on the specific queuing system and the factors that are being studied. However, all of these models can be used to understand the factors affecting the impact of heterogeneous customers on the performance of queuing systems.

### **Recommendations for Designing Queuing Systems with Heterogeneous Customers:**

When designing queuing systems with heterogeneous customers, it is important to consider the factors that affect the impact of heterogeneous customers. Some recommendations for designing queuing systems with heterogeneous customers include:

- Use priority levels to mitigate the impact of heterogeneous customers.
- Consider the arrival rates of customers when designing the queuing system.
- Consider the service times of customers when designing the queuing system.
- Use a queuing system that is appropriate for the specific application.

The following are some recommendations for designing queuing systems with heterogeneous customers:

- Use priority levels: Assigning priority levels to customers is a common way to mitigate the impact of heterogeneous customers. Customers with higher priority levels are served before customers with lower priority levels. This can help to ensure that customers with urgent needs are not delayed.

The average waiting time for a customer with priority level  $d$  in an  $M/M/c/k/d$  queuing system can be calculated using the following equation:

$$E(w_d) = \frac{(k + \rho)(d - 1)}{(c - d + 1)(1 - \rho)},$$

where:

- $E(w_d)$  is the average waiting time for a customer with priority level  $d$



- $\rho$  is the utilization of the server
- $k$  is the number of servers
- $c$  is the number of customer classes
- $d$  is the priority level
- Use multiple servers: Using multiple servers can help to reduce the waiting time for all customers, regardless of their priority level. This is because customers can be served by different servers, and therefore the server is not as likely to be overloaded.

The average waiting time for a customer in an M/M/c/k queuing system can be calculated using the following equation:

$$E(w) = \frac{(k + \rho)}{(1 - \rho)},$$

where:

- $E(w)$  is the average waiting time for all customers
- $\rho$  is the utilization of the server
- $k$  is the number of servers
- Use queuing models: Queuing models can be used to predict the performance of queuing systems with heterogeneous customers. This information can be used to design queuing systems that meet the specific needs of the application.

The M/M/c/k/d queuing model is a queuing model that can be used to study the impact of heterogeneous customers on queuing systems. The model can be used to calculate the average waiting time, the average number of customers in the system, and the probability of a customer being denied service.

These are just a few of the recommendations for designing queuing systems with heterogeneous



customers. The specific recommendations that are most appropriate will depend on the specific queuing system and the factors that are being considered.

## Conclusion:

The impact of heterogeneous customers on the performance of queuing systems depends on a number of factors. By understanding these factors, engineers can design queuing systems with heterogeneous customers that meet the specific needs of their applications. The impact of heterogeneous customers on the performance of queuing systems can be significant. The factors that can affect the impact of heterogeneous customers include the arrival rates, service times, number of servers, priority levels, and distribution of service times of customers. The impact of heterogeneous customers is more pronounced when the arrival rates of customers are different. This is because customers with higher arrival rates are more likely to experience longer waiting times. The impact of heterogeneous customers is less pronounced when the service times of customers are the same. This is because customers with longer service times are more likely to experience longer waiting times, regardless of their arrival rate. The impact of heterogeneous customers can be mitigated by using multiple servers. This is because customers can be served by different servers, and therefore the server is not as likely to be overloaded. The impact of heterogeneous customers can be mitigated by assigning priority levels to customers. Customers with higher priority levels are served before customers with lower priority levels. The impact of heterogeneous customers can also be affected by the distribution of service times. For example, if the service times are exponentially distributed, then the impact of heterogeneous customers will be less pronounced than if the service times are uniformly distributed.

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