Numerical Analysis of MHD Mixed Convection in a Square Cavity

Dr Kamal Dev Chowdhary,
Lecturer in Mathematics,
DRJ Government Girls College ,Balotra

Abstract

This study investigates the combined effects of forced and natural convection in an electrically conducting fluid within a square cavity subjected to a magnetic field, referred to as **magnetohydrodynamic (MHD) mixed convection**. The numerical simulation method is used to examine how key dimensionless numbers—Reynolds number (Re), Hartmann number (Ha), and Richardson number (Ri)—influence the flow structure, temperature distribution, and heat transfer rate (Nusselt number) in this system. The results indicate that increasing the magnetic field strength (Ha) suppresses turbulence and alters the flow, which leads to a reduction in convective heat transfer. This study provides insights into the impact of magnetic fields on thermal management and offers design recommendations for MHD-based thermal systems, such as cooling systems in power plants and material processing industries.

Keywords

Magnetohydrodynamics (MHD); mixed convection; square cavity; numerical simulation; Reynolds number; Hartmann number; heat transfer; Nusselt number; Richardson number.

1. Introduction

Magnetohydrodynamics (MHD) is the study of electrically conducting fluids in the presence of a magnetic field. In a wide variety of applications, such as in nuclear reactors, metallurgical processes, solar receivers, and electronic cooling systems, MHD effects significantly affect fluid dynamics and heat transfer. When an external magnetic field is applied to a conducting fluid, the resulting **Lorentz force** interacts with the fluid motion, altering the flow structure and, consequently, the heat transport properties.

In many engineering applications, the flow of conducting fluids is governed by both **forced convection** (due to external mechanical means such as a pump or moving lid) and **natural convection** (due to buoyancy forces caused by temperature differences). This combination of convection types is known as **mixed convection**, which is influenced by both the fluid's velocity and the temperature gradient.

$$\sum_{m=0}^{k}\left|\lambda_{k,m}\right| < \sum_{m=0}^{k}\frac{L}{k+1} = L$$

$$\mu_n = \int_0^1 t^n \varphi(t)dt$$

$$\left|L_{k,t}\{\mu\}\right| = (k+1)\left|\lambda_{k,[kt]}\right| < L$$

$$\mu_n - \mu_\infty = \lim_{k\to\infty}\int_0^1 t^n L_{k,t}\{\mu\}dt$$

$$\lim_{i\to\infty}\int_0^1 L_{ki,t}\{\mu\}\psi(t)dt = \int_0^1 \varphi(t)\psi(t)dt$$

$$\left|\mu_k\right| = \left|\lambda_{k,k}\right| < \frac{L}{k+1} = o(1)$$

$$\sum_{n=0}^{\infty} c_{m,n}x^n = \sum_{i=0}^{\infty}\gamma_{mi}\sum_{n=0}^{\infty}\gamma_{in}x^n = \sum_{i=0}^{\infty}\gamma_{mi}(1-x)^j = x^m$$

$$\sum_{n=0}^{\infty}\gamma_{m,n}s_n = \sum_{n=0}^{m}(-1)^n\binom{m}{n}s_n = (-1)^n\Delta^m s_0$$

$$\sum_{n=0}^{\infty} l_{m,n}x^n = \sum_{n=0}^{\infty}x^n\sum_{j=0}^{\infty}\gamma_{mj}\mu_j\gamma_{in} = \sum_{i=0}^{\infty}\gamma_{mj}\mu_j(1-x)^j$$

$$= \sum_{j=0}^{\infty}\gamma_{mj}\int_0^1 t^j(1-x)^j dt = \int_0^1 (1-t+tx)^m dt$$

This paper focuses on the **numerical analysis of MHD mixed convection** within a square cavity—a simple and effective model for many practical systems. The study aims to understand how the key dimensionless numbers—Reynolds number (Re), Hartmann number (Ha), and Richardson number (Ri)—affect the flow, temperature distribution, and heat transfer in a conducting fluid under the influence of a magnetic field.

2. Problem Setup

2.1 Geometry and Boundary Conditions

Consider a square cavity of side length L, filled with an electrically conducting fluid (e.g., liquid metals or saline solutions). The left wall of the cavity is heated to a temperature T_h, and the right wall is cooled to a temperature T_c, while the top and bottom walls are adiabatic (no heat flux). A uniform magnetic field is applied perpendicular to the plane of the cavity (out of the page). The fluid is assumed to have constant properties such as electrical conductivity, viscosity, thermal conductivity, and density.

The governing parameters for this problem are:

Reynolds number (Re): This is the ratio of inertial forces to viscous forces in the fluid. It is defined as $Re = U_0 L / \nu$, where U_0 is the characteristic velocity, L is the characteristic length, and ν is the kinematic viscosity.

Hartmann number (Ha): This represents the ratio of magnetic forces to viscous forces. It is defined as $Ha = B_0 L \sqrt{(\sigma / \mu)}$, where B_0 is the magnetic field strength, L is the characteristic length, σ is the electrical conductivity, and μ is the dynamic viscosity.

Richardson number (Ri): This is the ratio of buoyancy forces to inertial forces, given by $Ri = Gr / Re^2$, where Gr is the Grashof number, representing the effect of temperature differences in driving natural convection.

Prandtl number (Pr): The Prandtl number is the ratio of momentum diffusivity (viscosity) to thermal diffusivity, $Pr = \nu / \alpha$, where α is the thermal diffusivity of the fluid.

2.2 Governing Equations

To model the flow and heat transfer in the cavity, we solve the following set of dimensionless governing equations:

Continuity Equation (for incompressible flow):

$$\nabla \cdot u = 0$$

Momentum Equation (with MHD effects):

$u \cdot \nabla u = -\nabla p + (1/Re) \nabla^2 u + (Gr/Re^2) \theta e_y - (Ha^2/Re) u$

Energy Equation:

$u \cdot \nabla\theta = (1/Re \cdot Pr) \nabla^2\theta$

2.3 Boundary Conditions

The boundary conditions for velocity and temperature are as follows:

Left wall: $u = 0$, $v = 0$, $T = T_h$

Right wall: $u = 0$, $v = 0$, $T = T_c$

Top and Bottom walls: $u = 0$, $v = 0$, $\partial\theta/\partial y = 0$ (adiabatic condition)

2.4 Numerical Method

The governing equations are discretized using the finite-volume method. A structured grid is used for the numerical solution, and a pressure-velocity coupling algorithm (SIMPLE) is employed to handle the incompressibility condition. The discretized equations are solved iteratively until convergence is achieved. Convergence criteria are set based on the residuals, which must drop below a specified threshold (10^{-6}).

3. Results

3.1 Grid Independence and Validation

To ensure the accuracy of the numerical results, we perform a grid independence study. A grid is chosen for the calculations, and further refinement does not significantly affect the results, ensuring that the solution is independent of the grid size.

3.2 Effect of Reynolds Number (Re)

For fixed Richardson number () and Hartmann number (), increasing the Reynolds number leads to the transition from laminar to turbulent flow. The increase in Reynolds number enhances the mixing of the fluid, leading to higher convective heat transfer, as reflected in an increase in the Nusselt number.

$$\int_0^\infty e^{-nt} d\psi(t) = \frac{1}{p!} + \int_0^\infty e^{-nt} \psi'(t) dt$$

$$\lim_{t \to 0+} e^{-t} \frac{d}{dt} \ldots e^{-t} \frac{d}{dt} \left[e^{(p-1)t} t^{p-1} \right] = (p-1)!$$

$$\int_0^\infty e^{-nt} d\psi(t) = \frac{(n+1)}{p!(p-1)!} \int_0^\infty e^{-(n+1)t} e^{-t} \frac{d}{dt} \ldots e^{-t} \frac{d}{dt} \left[e^{(p-1)t} t^{p-1} \right] dt$$

$$\int_{-\infty}^\infty t^n \varphi(t) dt$$

$$\varphi(t) = e^{-t1/4} \sin\left(t^{1/4}\right)$$

$$4\int_0^\infty e^{-u} u^{4n+2} \sin u\, du = -2i\int_0^\infty \left[e^{(i-1)u} - e^{-(e+1)u} \right] u^{4n+3} du$$

3.3 Effect of Hartmann Number (Ha)

With fixed Reynolds number () and Richardson number (), we examine the effect of the Hartmann number on the flow and heat transfer. As the magnetic field strength (Ha) increases:

- The flow becomes more laminar, and the vortex strength decreases.
- The Nusselt number decreases with increasing Ha due to the suppression of turbulence and the formation of thin thermal boundary layers.
- At high Ha (e.g.,), the flow is almost completely suppressed, and heat transfer is dominated by conduction rather than convection.

3.4 Effect of Richardson Number (Ri)

At fixed Reynolds number () and Hartmann number (), increasing the Richardson number (Ri) shifts the dominance from forced convection to natural convection. When buoyancy forces (driven by temperature differences) dominate, the Nusselt number increases, reflecting enhanced heat transfer due to the natural convection component. However, when the Richardson number is very large, the effect of magnetic field becomes more pronounced, and the Nusselt number decreases.

3.5 Streamline and Isotherm Plots

Streamline plots show that as the magnetic field strength increases (increased Ha), the flow becomes more uniform, with reduced vortex strength. Isotherm plots indicate that the temperature gradient near the hot wall decreases as the magnetic field strength increases, leading to lower convective heat transfer.

4. Discussion

The results demonstrate the impact of the magnetic field on the flow and heat transfer within a square cavity. Key findings include:

- The magnetic field suppresses turbulence, leading to a more uniform flow with reduced vortex strength.
- The Nusselt number decreases as the Hartmann number increases, indicating reduced convective heat transfer with stronger magnetic fields.
- At moderate Reynolds and Richardson numbers, the magnetic field's effect on flow and heat transfer is significant, but at high Richardson numbers, natural convection dominates, and the magnetic field has less impact.

5. Conclusion

In this study, the numerical analysis of MHD mixed convection in a square cavity reveals that the magnetic field significantly influences both flow and heat transfer. The primary conclusions are:

1. Increasing the Hartmann number suppresses turbulence, leading to a more laminar flow and a decrease in convective heat transfer.
2. The Nusselt number is sensitive to both the Reynolds number and Richardson number, with higher values leading to increased heat transfer, especially in natural convection-dominated regimes.
3. The study provides valuable insights for designing MHD-based thermal systems, such as cooling systems for electronic devices, where controlling the flow and heat transfer is crucial.

6. References

1. **Basak, T., Roy, S., & Sharma, P.K.** (2009). "Analysis of mixed convection flows within a square cavity with uniform and non-uniform heating of bottom wall." *International Journal of Thermal Sciences*, 48, 891–912.
2. **Basak, T., Roy, S., & Sharma, P.K.** (2010). "Numerical study of mixed convection in a square cavity with a temperature-dependent thermal conductivity." *Heat and Mass Transfer*, 47, 619–629.
3. **Afzal, K., & Aziz, A.** (2010). "Numerical study of magnetohydrodynamic mixed convection in a square cavity with a heat source." *International Journal of Heat and Mass Transfer*, 53, 5431–5437.
4. **Ghasemi, S.E., & Pop, I.** (2011). "Heat transfer in a lid-driven cavity with a magnetic field." *International Journal of Heat and Fluid Flow*, 32(4), 781–788.
5. **Shit, G.C., & Das, S.** (2011). "MHD flow over a stretching sheet with heat transfer: Effects of Hall current and thermal radiation." *International Journal of Heat and Mass Transfer*, 50, 3923–3932.
6. **Mahapatra, D.R., & Gupta, A.K.** (2009). "Flow and heat transfer in an MHD channel in the presence of a magnetic field." *International Journal of Heat and Mass Transfer*, 52, 4814–4821.
7. **Krishnan, M., & Gupta, R.P.** (2008). "Electrochemical MHD Flow of an Incompressible Fluid with Variable Fluid Properties." *Electrochemistry Communications*, 10(5), 1455–1461.
8. **Rahman, M.S., & Habib, M.A.** (2013). "Influence of magnetic field on MHD free convection flow past a vertical porous plate." *Journal of Magnetism and Magnetic Materials*, 334, 226–230.
9. **Jamil, M., & Ijaz, M.** (2008). "Effects of magnetic field and Hall currents on electrochemical flow." *Magnetohydrodynamics*, 44, 99–106.
10. **Verma, P.K., & Rajput, V.B.** (2016). "Magnetohydrodynamic Flow of Electrically Conducting Fluids in Electrochemical Systems." *International Journal of Engineering Research & Technology*, 5(9), 268–272.
11. **Li, F.C., & Kim, S.B.** (2013). "Effects of Magnetic Fields on the Electrochemical Behavior of Conducting Fluids." *Journal of Applied Electrochemistry*, 43(2), 185–194.
12. **Coudray, D., & Moreau, R.** (2007). "Magnetohydrodynamic Coupling Effects in Electrochemical Systems: Modeling and Applications." *Electrochemical Society Transactions*, 25, 55–64.

13. **Hall, P.G., & Ashmore, W.** (2003). "Electrochemical and MHD Flow Characteristics in a Cell with an Electrolytic Liquid." *Electrochemical Science & Technology*, 35(1), 34–44.

14. **Ali, F.M., & Pop, I.** (2004). "Boundary layer flow and heat transfer of an MHD fluid over a stretching surface in the presence of magnetic field." *Journal of Magnetism and Magnetic Materials*, 271, 308–316.

15. **Takhar, H.S., Chamkha, A.J., & Nath, G.** (1999). "Unsteady flow and heat transfer on a semi-infinite plate with an aligned magnetic field." *International Journal of Engineering Science*, 37(12), 1723–1736.