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Effect of Variable Permeability on MHD Flow through Channel

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Introduction

The unsteady flow and heat transfer through a channel have many applications in the field of Mechanical engineering, Chemical Engineering and Aerodynamics etc. The pulsatile flow through a channel filled with porous material have practical applications in bio-fluid flow, in the dialysis of blood in artificial kidney etc. Magnetic field on the fluid flow i.e. Magnetofluiddynamic flow is used as controlling device on the flow. Many scientist have used magnetohydrodynaic flow for the pumping of blood through an artery. Sharma and Sharma (1997) have discussed unsteady flow and heat transfer between two parallel plates and obtained skin-friction and Nusselt number. Sorundalgekar and Lahurikar (2002) have worked out generalized MHD Couette flow with variable viscosity. Sharma and Sharma(2003) have worked out heat transfer through steady MHD flow between two inclined Walls and discussed numerically the effect of magnetic field on the flow and heat transfer in terms of skin friction and Nusselt number. Chamkha(2004) has workedout an unsteady MHD convective heat and mass transfer past a semi-infinite vertical permeable moving plate with heat absorption. Sharma and Sharma (2004) have studied unsteady free convection flow between horizontal parallel porous plates in the presence of heat source. Attia(2005) has studied MHD couette flow with variable physical properties. Makinde and Mhone (2006) have studied the thermally developing hartman flow in a channel of uniform width.

In the present paper an attempt has been made to study the effect of variable permeability and transverse magnetic field on fluid flow and heat transfer when the upper plate of the channel is oscillating.

Formulation of the problem

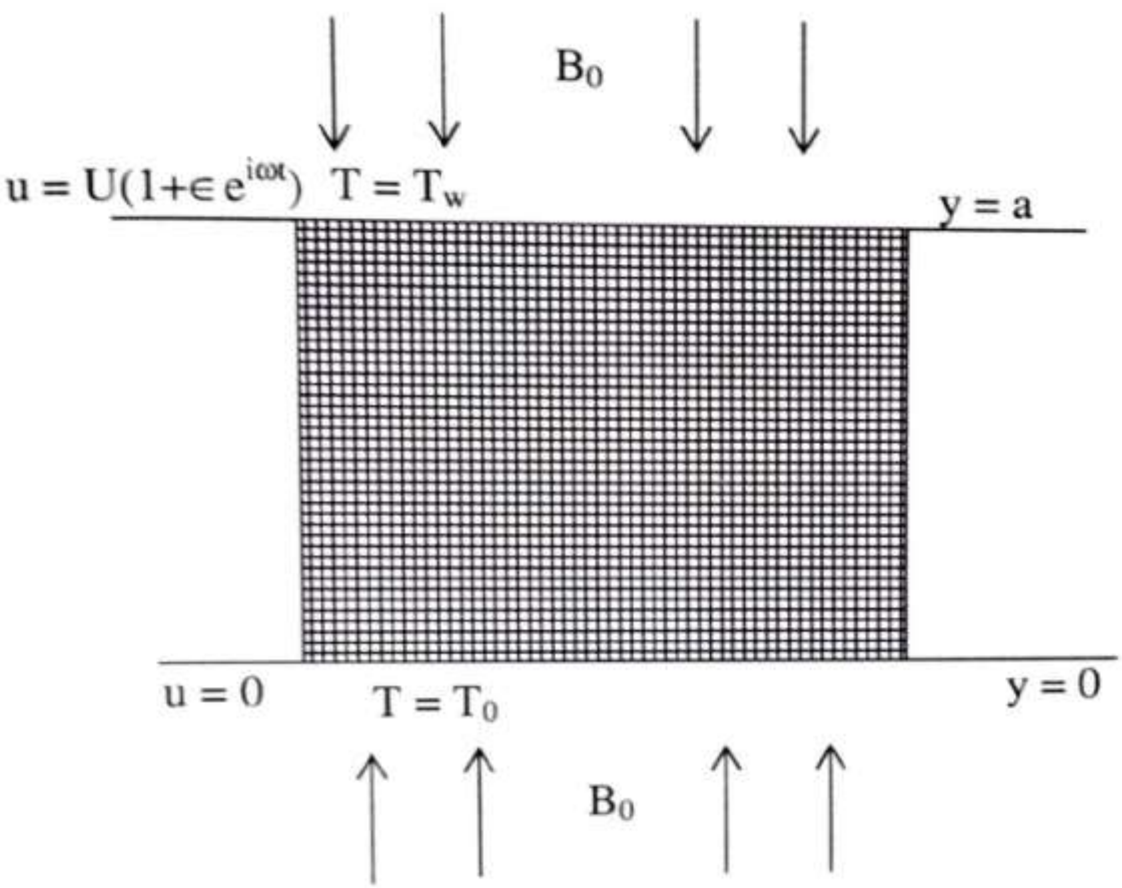
A semi infinite long channel of width a is filled with saturated medium is considered to study the effects of magnetic field under variable permeability on the fluid flow and heat transfer. The fluid is viscous incompressible electrically conducting. It is assumed that the fluid has small electrical conductivity and the electromagnetic force produced is very small. The x-axis is taken along the channel, and y-axis in the normal to the channel. The lower plate of the channel is static and kept at constant temperature T_0, while upper plate of the channel is oscillating with velocity $U(1 + \varepsilon e^{i\omega t})$ and kept at constant temperature T_w. Using Boussinesq approximation the equations governing the motion are given by

$$\frac{\partial \mathrm{u}}{\partial \mathrm{t}} = -\frac{1}{\rho}\frac{\partial \mathrm{P}}{\partial \mathrm{x}} + v\frac{\partial^2 \mathrm{u}}{\partial \mathrm{y}^2} - \frac{v}{\mathrm{K}}\mathrm{u} - \frac{\sigma e \mathrm{B}_0^2 \mathrm{u}}{\rho} + g\beta(\mathrm{T} - \mathrm{T}_0)$$

$$\frac{\partial \mathrm{T}}{\partial \mathrm{t}} = \frac{\mathrm{k}}{\rho \mathrm{c_p}}\frac{\partial^2 \mathrm{T}}{\partial \mathrm{y}^2} - \frac{1}{\rho \mathrm{c_p}}\frac{\partial \mathrm{q}}{\partial \mathrm{y}},$$

with the boundary conditions

$$y = z: \quad \mathrm{u} = \mathrm{U}\left(1 + \in \mathrm{e}^{i\omega t}\right),\ T = \mathrm{T_w},$$
$$\mathrm{y} = 0: \quad \mathrm{u} = 0,\ T = \mathrm{T}_0$$



Physical model of the problem

where,

$u \rightarrow$ axial velocity,
$t \rightarrow$ time
$T \rightarrow$ fluid temperature,
$p \rightarrow$ pressure
$g \rightarrow$ gravitational force,
$q \rightarrow$ radiative heat flux
$\beta \rightarrow$ coefficient of volume expansion due to temperature
$c_p \rightarrow$ specific heat at constant pressure
$k \rightarrow$ thermal conductivity,
$\rho \rightarrow$ fluid density
$K \rightarrow$ porous medium permeability coefficient.
$B_0 = (\mu_e H_0)$, the electromagnetic induction,
$\mu_e \rightarrow$ magnetic permeability,
$H_0 \rightarrow$ kinematics viscosity. Intchsity of magnetic field
$\sigma_e \rightarrow$ conductivity of the fluid,
$v \rightarrow$ kinematics viscosity.

It is assumed that both wall temperature T_0, T_w are high enough to induce radiative heat transfer. Further, it is assumed that the fluid is optically thin with a relatively low density and the radiative heat flux is given by

$$\frac{\partial q}{\partial y} = 4\alpha^2 (T - T_0)$$

where, α is the mean radiation absorption coefficient. Further, it is assumed that K, the porous medium permeability coefficient is variable, given by

$$K = K_0\left(1 + \epsilon e^{iwt}\right)$$

with this value the equation becomes

$$\frac{\partial u}{\partial t} = -\frac{1}{\rho}\frac{\partial P}{\partial x} + v\frac{\partial^2 u}{\partial y^2} - \frac{v}{K_0(1+\in e^{iwt})}u - \frac{\sigma_e B_0^2}{\rho}u + g\beta(T - T_0)$$

To convert governing equation of motion in non-dimensional form introducing following non-dimensional parameters

$$\mathrm{Re} = \frac{\mathrm{Ua}}{\mathrm{v}}, \bar{\mathrm{x}} = \frac{\mathrm{x}}{\mathrm{a}}, \bar{\mathrm{y}} = \frac{\mathrm{y}}{\mathrm{a}}, \bar{\mathrm{u}} = \frac{\mathrm{u}}{\mathrm{U}}$$

$$\theta = \frac{\mathrm{T} - \mathrm{T_0}}{\mathrm{T_w} - \mathrm{T_0}}, \mathrm{H}^2 = \frac{\mathrm{a}^2 \sigma_e \mathrm{B}_0^2}{\rho v}, \bar{\mathrm{t}} = \frac{\mathrm{U}}{\mathrm{a}},$$

$$\bar{\mathrm{P}} = \frac{\mathrm{aP}}{\rho v \mathrm{U}}, \mathrm{D}_\mathrm{a}^2 = \frac{\mathrm{K}}{\mathrm{a}^2}, \ \mathrm{Gr} = \frac{\mathrm{g}(\mathrm{T_w} - \mathrm{T_0})\mathrm{a}^2}{vU}$$

$$\mathrm{Pe} = \frac{\mathrm{Ua}\rho \mathrm{c}}{\mathrm{k}}, \mathrm{N}^2 = \frac{4\alpha^2 \mathrm{a}^2}{\mathrm{k}}$$

The dimensionless governing equations of motion and energy are

$$\begin{aligned}\mathrm{Re}\frac{\partial u}{\partial t}\left(1 + \epsilon e^{iwt}\right) = \quad & -\left(1 + \epsilon e^{iwt}\right)\frac{\partial P}{\partial x} + \left(1 + \epsilon e^{iwt}\right)\frac{\partial^2 u}{\partial y^2} \\ & -s^2 u - H^2 u\left(1 + \epsilon e^{iwt}\right) + \mathrm{Gr}\,\theta\left(1 + \epsilon e^{iwt}\right)\end{aligned}$$

and

$$\mathrm{Pe}\frac{\partial \theta}{\partial \mathrm{t}} = \frac{\partial^2 \theta}{\partial y^2} + N^2 \theta$$

Corresponding boundary conditions are

$$\begin{aligned} &\mathrm{y} = 1: \quad \mathrm{u} = 1 + \epsilon \mathrm{e}^{\mathrm{i}\omega \mathrm{t}}, \quad \theta = 1 \\ &\mathrm{y} = 0: \quad \mathrm{u} = 0, \qquad\qquad \theta = 0 \end{aligned}$$

For the sake of brevity the bars are dropped immediately. Here

U → mean flow velocity

Gr → Grashoff Number

H → Hartmann Number

N → radiation parameter

Pe → Peclet Number

Re → Reynolds Number,

Da → Darcy Number

$\mathrm{s} = \left(\frac{1}{\mathrm{Da}}\right)$ → porous medium shape factor parameter

Method of Solution

The governing equation of motion is nonlinear coupled partial differential equation. For its solution, let us consider

$$-\frac{\partial \mathrm{P}}{\partial \mathrm{x}} = \lambda\left(1+\in \mathrm{e}^{it}\right)$$

$u(y,t) = u_0(y) + \epsilon u_1(y)e^{iwt}$

and

$$\theta(\mathrm{y},\mathrm{t}) = \theta_0(\mathrm{y})+\in \theta_1(\mathrm{y})\mathrm{e}^{\mathrm{iwt}}$$

where

$$\lambda \rightarrow \text{ constant, w} \rightarrow \text{ frequency of the oscillations.}$$

On substituting the above expressions in the equations, we get

$$\begin{aligned}\mathrm{Re}[\epsilon u_1(y)e^{iwt}iw][1+\epsilon e^{iwt}] = {} & (1+\epsilon e^{iwt})(1+\epsilon e^{iwt})\lambda \\ & +(1+\epsilon e^{iwt})\left(\frac{d^2u_0}{dy^2}+\epsilon\frac{d^2u_1}{dy^2}e^{iwt}\right) \\ & -s^2(u_0+\epsilon u_1 e^{iwt}) \\ & -\mathrm{H}^2(\mathrm{u}_0+\epsilon \mathrm{u}_1\mathrm{e}^{\mathrm{iwt}})(1+\epsilon \mathrm{e}^{\mathrm{iwt}}) \\ & +\mathrm{Gr}(\theta_0+\epsilon\theta_1\mathrm{e}^{\mathrm{it}})(1+\epsilon \mathrm{e}^{\mathrm{iwt}})\end{aligned}$$

and

$$\mathrm{Pe}[\in \theta_1 iwe^{\mathrm{iwt}}] = \frac{\mathrm{d}^2\theta_0}{\mathrm{dy}^2}+\in \mathrm{e}^{\mathrm{iwt}}\frac{\mathrm{d}^2\theta_1}{\mathrm{dy}^2}+\mathrm{N}^2\left(\theta_0+\in \theta_1\mathrm{e}^{\mathrm{iwt}}\right)$$

On comparing the coefficients of like powers of ϵ, we have

zero-order equations

$$\begin{aligned}&\frac{\mathrm{d}^2\mathrm{u}_0}{\mathrm{dy}^2}-(\mathrm{s}^2+\mathrm{H}^2)\mathrm{u}_0 = -\lambda-\mathrm{Gr}\theta_0 \\ &\frac{\mathrm{d}^2\theta_0}{\mathrm{dy}^2}+\mathrm{N}^2\theta_0 = 0\end{aligned}$$

First-order equations

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$$\frac{d^2u_1}{dy^2} - (s^2 + H^2 - \mathrm{Rei}\omega)u_1 = -2\lambda - \frac{d^2u_0}{dy^2} + H^2u_0 - \mathrm{Gr}\theta_0 - \mathrm{Gr}\theta_1$$
$$\frac{d^2\theta_1}{dy^2} + (N^2 - Pei\omega)\theta_1 = 0$$

Second-order equations

$$\frac{d^2u_1}{dy^2} - (H^2 + \mathrm{Re}i\omega)u_1 = -\lambda - \mathrm{Gr}\theta_1$$

with the corresponding boundary conditions

$$\begin{aligned} &\mathrm{y}=0: \quad \mathrm{u}_0 = 0, \quad \mathrm{u}_1 = 0, \theta_0 = 0, \theta_1 = 0 \\ &\mathrm{y}=1: \quad \mathrm{u}_0 = 1, \quad \mathrm{u}_1 = 1, \theta_0 = 1, \theta_1 = 0 \end{aligned}$$

The solution of the equation is

$$\theta_0(\mathrm{y}) = \mathrm{c}_1\cos \mathrm{Ny} + \mathrm{c}_2\sin \mathrm{Ny}$$

On applying boundary condition, $\theta_0(0) = 0$ and $\theta_0(1) = 1$, we get

$$c_1 = 0 \text{ and } c_2 = \frac{1}{\sin \mathrm{N}}$$

Thus, under the prescribed boundary conditions the solution of $\theta_0(\mathrm{y})$ is given by

$$\theta_0(y) = \frac{1}{\sin N}\sin \mathrm{Ny}$$

Now consider the equation

$$\frac{d^2\theta_1}{dy^2} + (N^2 - \mathrm{Pe}i\omega)\theta_1 = 0$$

or

$$\frac{d^2\theta_1}{dy^2} + \mathrm{A}_1^2\theta_1 = 0$$

where

$$A_1 = \sqrt{N^2 - \mathrm{Pei}\omega}$$

The solution of the equation is given by

$$\theta_1(y) = c_3\cos A_1y + c_4\sin A_1y$$

On applying boundary conditions $\theta_1(0) = 0$ and $\theta_1(1) = 0$, we get

$$c_3 = 0 \text{ and } c_4 = 0$$

Thus, under the prescribed boundary conditions the solution

$$\theta_1(\mathrm{y}) = 0$$

Now, we solve equation

$$\frac{d^2u_0}{dy^2} - (s^2 + H^2)u_0 = -\lambda - \mathrm{Gr}\,\theta_0$$

with the help of the equation , the above equation can be rewritten as

$$\frac{d^2u_0}{dy^2} - (s^2 + H^2)u_0 = -\lambda - \frac{\sin Ny}{\sin N}Gr$$

or

$$\frac{d^2u_0}{dy^2} - A_2^2u_0 = -\lambda - \frac{\sin Ny}{\sin N}, \text{ where}$$

$$\mathrm{A}_2 = \sqrt{\mathrm{s}^2 + \mathrm{H}^2}$$

The complementary function is given by

$$\mathrm{C\cdot F} = \mathrm{c}_5\cosh \mathrm{A}_2\mathrm{y} + \mathrm{c}_6\sinh \mathrm{A}_2\mathrm{y}$$

and Particular Integral is

$$\begin{aligned} P.I \quad &= \frac{1}{m^2 - A_2^2}\left(-\lambda - \frac{\sin Ny}{\sin N}\mathrm{Gr}\right) \\ &= \frac{1}{\mathrm{m}^2 - \mathrm{A}_2^2}(-\lambda) + \frac{1}{\mathrm{m}^2 - \mathrm{A}_2^2}\left(-\frac{\sin Ny}{\sin \mathrm{N}}\mathrm{Gr}\right) \\ &= \frac{1}{\mathrm{A}_2^2}\lambda + \frac{1}{\mathrm{N}^2 + \mathrm{A}_2^2}\frac{\sin Ny}{\sin N}\mathrm{Gr} \end{aligned}$$

Thus the complete solution is

$$\begin{aligned} \mathrm{u}_0(\mathrm{y}) \quad &= \mathrm{C\cdot F + P\cdot I} \\ &= c_5\cosh \mathrm{A}_2\mathrm{y} + \mathrm{c}_6\sinh \mathrm{A}_2\mathrm{y} + \frac{1}{\mathrm{A}_2^2}\lambda + \frac{1}{\mathrm{N}^2 + \mathrm{A}_2^2}\frac{\sin Ny}{\sin N}Gr \end{aligned}$$

on applying the boundary conditions, $\mathrm{u}_0(0) = 0, \mathrm{u}_0(1) = 1$

$$c_5 = \frac{-\lambda}{A_2^2}, c_6 = \frac{1}{\sinh A_2}\left[1 + \frac{\lambda}{A_2^2}\cosh A_2 - \frac{\lambda}{A_2^2} - \frac{\mathrm{Gr}}{N^2 + A_2^2}\right]$$

Then under the prescribed boundary conditions the mean flow velocity profile is given by

$$\begin{aligned} u_0(y) = & \frac{\lambda}{A_2^2}\cosh A_2 y + \frac{1}{\sinh A_2}\left[1 + \frac{\lambda}{A_2^2}(\cosh A_2 - 1) - \frac{\mathrm{Gr}}{N^2 + A_2^2}\right]\sinh A_2 y \\ & + \frac{1}{A_2^2}\lambda + \frac{1}{N^2 + A_2^2}\frac{\sin Ny}{\sin N}\mathrm{Gr} \\ = & -A_3\cosh A_2 y + (A_5 + A_6 + A_7)\sinh A_2 y + A_3 + A_4 \sin Ny \end{aligned}$$

where

$$\begin{aligned} A_3 &= \frac{\lambda}{A_2^2} \\ A_4 &= \frac{\mathrm{Gr}}{(N^2 + A_2^2)\sin N} \\ A_5 &= \frac{1}{\sinh A_2} \\ A_6 &= \frac{A_3}{\sinh A_2}(\cos hA_2 - 1) \\ A_7 &= -\frac{A_4 \sin N}{\sinh A_2} \end{aligned}$$

Now consider the equation

$$\frac{d^2u_1}{dy^2} - (H^2 + \mathrm{Re}\, i\omega)u_1 = -\lambda - \mathrm{Gr}\,\theta_1$$

or $\frac{d^2u_1}{dy^2} - A_8^2 u_1 = -\lambda - \mathrm{Gr}\,\theta_1$

where

$$A_8 = \sqrt{H^2 + \mathrm{Re}\, i\omega}$$

with the help of equation , this can be written as

$$\frac{d^2u_1}{dy} - A_8^2 u_1 = -\lambda$$

Its solution is given by

$$
\begin{aligned}
u_1(y) &= C\cdot F + P.I \\
&= c_7\cosh A_8 y + c_8\sinh A_8 y + \frac{1}{A_8^2}\lambda
\end{aligned}
$$

In view of the boundary condition, $u_1(0) = 0, u_1(0) = 1$

we get

$$
\begin{aligned}
c_7 &= -\frac{1}{A_8^2}\lambda \\
c_8 &= \frac{1}{\sinh A_8}\left[1 + \frac{\lambda}{A_8^2}(\cosh A_8 - 1)\right]
\end{aligned}
$$

Then under the prescribed boundary conditions the expression of transient velocity profile is given by

$$
u_1(y) = -\frac{1}{A_8^2}\lambda\cosh A_8 y + \frac{1}{\sinh A_8}\left[1 + \frac{\lambda}{A_8^2}(\cosh A_8 - 1)\right]\sinh A_8 y
$$

$$
+\frac{1}{A_8^2}\lambda
$$

$$
= -A_9\cosh A_8 y + (A_{10} + A_{11})\sinh A_8 y + A_9
$$

where $A_9 = \frac{\lambda}{A_8^2}$

$$
A_{10} = \frac{1}{\sinh A_8}
$$

$$
A_{11} = A_9.A_{10}(\cosh A_8 - 1)
$$

Hence, the expression of velocity profile and temperature distribution are derived and given by

$$
\begin{aligned}
u(y,t) = \; & (u_0(y) + \in u_1(y)e^{iwt} \\
& = -A_3\cosh A_2 y + (A_5 + A_6 + A_7)\sinh A_2 y \\
& +A_3 + A_4\sin N y + \in \left[-A_9\cosh A_8 y + (A_{10} + A_{11})\sinh A_{8_y} + A_9\right]e^{iwt}
\end{aligned}
$$

and

$$
\begin{aligned}
\theta(y,t) = \theta_0(y) \quad & + \in \theta_1(y)e^{\mathrm{iwt}} \\
& = \theta_0(y) \\
& = \frac{1}{\sin N}\sin N y
\end{aligned}
$$

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