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JACOBEAN INTEGRAL OF PLANAR TETHERED DUMBELL SATELLITES IN ELLIPTICAL ORBIT

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Abstract

Jacobean integral of motion of a system of two artificial cable connected satellites in elliptical orbit under several influences of general nature is obtained. Cable connecting the two satellites is light, flexible, non-conducting and elastic in nature. The influences of general nature are earth's magnetic field, solar radiation pressure, shadow of the earth, earth's oblateness and elasticity of the cable. These influences are the perturbative forces acting simultaneously on the system concerned. We do not consider nutation and wobbling of the elliptical orbit of centre of mass of the system.

Keywords: Two cable connected satellites, elliptical orbit, Elastic cable, Jacobean integral.

1. Introduction:

Physics and dynamics of a tether satellite system are very complicated due to undesirable factors like, flexibility, centrifugal force and a variety of perturbations of general nature. To predict the dynamics of the tethered satellites we have two options, analytical analysis of the tethered satellites by selecting the tethered model and numerical simulation to reveal the dynamical behaviours. We are interested in first option in this paper by taking the dumbbell model of tethered satellite in planar motion.

Two cable-connected satellites, rather than single one, have the advantage of transporting personnel from one spaceship to the other, of repairing one when something goes wrong with it, creation of an artificial gravity on board a space station, lifting of spacecraft to higher orbit, use as space elevator, space escalator, interplanetary transfer and many more.

During the past few decades, tether systems satellites have attracted the attention of researchers from around the glove. The main features of the tether satellite systems from conventional spacecraft are their long length, variable configuration, and their ability to interact with the Earth's magnetic field. The first experiments with using them were conducted in the mid 1960s, and currently more than twenty programs have been realised. With this connection, the task of developing physical and mathematical models and the dynamics of space tether systems taking into account the motion of its centre of mass is very important.

Beletsky and Novikova (1969) are the pioneer workers in the area of researches related to a cable connected satellites system. They studied about motion of a system of two cable-connected satellites in the central gravitational field of force relative to its centre of mass. This study assumed that the two satellites are moving in the plane of the centre of mass. This problem was further investigated in two and three dimensional cases by Singh and Demin (1972) and Singh (1973). Das et al. (1976) studied the effect of magnetic force on the motion of a system of two cable-connected satellites in orbit. Kumar and Bhattacharya (1995) studied the stability of equilibrium positions of two cable connected satellites under the influence of solar radiation pressure, earth's oblateness and earth's magnetic field. Sinha and Singh (1987) investigated the effect of solar radiation pressure on the motion and stability of the system of two inter-connected satellites when their centre of mass moves in circular orbit. Singh et al. (2001) studied non-linear effects on the motion and stability of an inter connected satellites system orbiting around an oblate earth. Kumar and Srivastava (2006) studied evolutional and non-evolutional motion of a system of two cable-connected artificial satellites under some perturbative forces. Kumar and Prasad (2015) studied about nonlinear planer oscillation of a cable-connected satellites system and non-resonance. Kumar and Kumar (2016) studied equilibrium positions of a cable-connected satellites system under several influences.

The work is a physical and mathematical idealization of real space system. We establish Jacobean integral of motion of the system under the influence of shadow of the earth, solar radiation pressure, oblateness of the earth and earth's magnetic field in elliptical orbit. The influence of the above mentioned perturbations on the system has been studied singly and by a combination of any two or three of them by various workers, but never conjointly all at a time. Therefore, these could not give a real picture of motion of the system. This fact has initiated the present research work. Central attractive force of the earth will be the main force and all other forces, being small enough are considered here as perturbing forces. Since masses of the satellites are small and distances between the satellites and other celestial bodies are very large, the gravitational forces of attraction between the satellites and other celestial bodies including the sun have been neglected. The satellites are considered as charged material particles.

2. Treatment of the problem and Jacobean integral:

We write the equations of motion of one of the two satellites when the centre of mass moves along Keplerian elliptical orbit in Nechvile's (1926) co ordinate system as

$$X''-2Y'-3X\rho=-\frac{A}{\rho}\cos i-\gamma\frac{R^3}{\mu}\left(\frac{B_1}{m_1}-\frac{B_2}{m_2}\right)\cos\in\cos(\nu-\alpha)+\frac{12k_2}{R^2}\rho X$$

$$-\lambda_\alpha\frac{R^3}{\mu}\left[\rho-l_0\left(X^2+Y^2\right)^{-1/2}\right]X$$

and

$$Y''+2X'=-\frac{A\rho'}{\rho^2}\cos i+\gamma\frac{R^3}{\mu}\left(\frac{B_1}{m_1}-\frac{B_2}{m_2}\right)\cos\in\sin(\nu-\alpha)-\frac{3k_2}{R^2}\rho Y$$

$$-\lambda_\alpha \frac{R^3}{\mu}\left[\rho - l_0\left(X^2+Y^2\right)^{-1/2}\right]Y \tag{1}$$

With the condition of constraint

$$X^2+Y^2 \le \frac{l_0^{\,2}}{\rho^2} \tag{2}$$

Also,

$$\rho = \frac{1}{1+e\cos v}\,, \quad \lambda_\alpha = \left[\frac{m_1+m_2}{m_1 m_2}\right]\frac{\lambda}{l_0}\,,\ k_2 = \frac{\bar{\varepsilon}\,\mathrm{R_e}^{\,2}}{3}\,,\ \bar{\varepsilon} = \alpha_R - \frac{m}{2}\,,\ m = \frac{\Omega^2\,\mathrm{R_e}}{g_e}$$

$$A = \left(\frac{m_1}{m_1+m_2}\right)\left(\frac{Q_1}{m_1}-\frac{Q_2}{m_2}\right)\frac{\mu_E}{\sqrt{\mu p}} \tag{3}$$

Where m_i (i = 1, 2) are the masses of the two satellites. μ denotes the product of mass of the earth and gravitational constant. λ denotes modulus of elasticity for the connecting cable. Q_1, Q_2 are the charges developed on the two satellites respectively. γ is a shadow function which depends on the illumination of the system of satellites by the sun rays. If the system is affected by the shadow of the earth then γ is equal to zero on the other hand if the system is not within the said shadow then γ is equal to one. We have denoted B_1, B_2 as the absolute values of the forces due to the direct solar radiation pressure on m_1 and m_2 respectively. Also ℓ_0 is the original length of the cable, α_R denotes the earth's oblateness, Ω denotes the angular velocity of the earth's rotation, $\mathrm{R_e}$ is for equatorial radius of the earth and g_e is the acceleration due to gravity. e is the eccentricity of the orbit of centre of mass of the system, v is the independent variable in place of time t is called true anomaly. i is inclination of the orbit with the equatorial plane, $\in$ is inclination of the oscillatory plane of the masses m_1 and m_2 with the orbital plane of the centre of mass of the system and α is the inclination of the ray. The magnetic moment of the earth's dipole is represented by μ_E and p is the focal parameter. The prime represent differentiation with respect to v.

If motion of one of the satellites we determined with the help of equation (1), motion of the other satellite of mass m_2 can be determined by

$$m_1\bar{\rho}_1 + m_2\bar{\rho}_2 = 0 \tag{4}$$

$\bar{\rho}_j\,(j=1,2)$ is the radius vector of the particle $m_j\,(j=1,2)$ with respect to the centre of mass of the system.

For the analysis of the small secular and long periodic effects of the solar pressure and the effects of the earth's shadow on the system, the periodic terms in the equations of motion (1) may be averaged with respect to ν as

(i) From θ_2 to $(2\pi - \theta_2)$ for a period, when the system is under the influence of sun rays directly i.e. $\gamma = 1$.

and

(ii) From $-\theta_2$ to $+\theta_2$ for a period, when the system is under the influence of shadow of the earth i.e. $\gamma = 0$.

The averaged values of the secular terms due to the periodic forces in the equations of motion (1) can be deduced as given below

$$\frac{1}{2\pi}\int_0^{2\pi}\frac{1}{\rho}d\nu = 1, \frac{1}{2\pi}\int_0^{2\pi}\rho d\nu = \frac{1}{(1-e^2)^{1/2}}, \frac{1}{2\pi}\int_0^{2\pi}\frac{1}{\rho^2}d\nu = \left(1+\frac{1}{2}e^2\right)$$
$$\frac{1}{2\pi}\int_0^{2\pi}\rho^2 d\nu = \frac{1}{(1-e^2)^{3/2}}, \frac{1}{2\pi}\int_0^{2\pi}\rho\rho' d\nu = 0$$
$$\frac{1}{2\pi}\int_0^{2\pi}\rho^3 d\nu = \frac{(2+e^2)}{2(1-e^2)^{5/2}}, \frac{1}{2\pi}\int_0^{2\pi}\rho'\rho^{-2} d\nu = 0$$
$$\frac{1}{2\pi}\int_0^{2\pi}\rho^4 d\nu = \frac{(2+3e^2)}{2(1-e^2)^{7/2}}, \quad \frac{1}{2\pi}\int_0^{2\pi}\rho'\rho^{-1} d\nu = 0$$
$$\frac{1}{2\pi}\int_0^{2\pi}\gamma\left(\frac{B_1}{m_1}-\frac{B_2}{m_2}\right)\cos\in\cos(\nu-\alpha)\,d\nu = -\left(\frac{B_1}{m_1}-\frac{B_2}{m_2}\right)\frac{\cos\in\cos\alpha\sin\theta_2}{\pi}$$

and

$$\frac{1}{2\pi}\int_0^{2\pi}\gamma\left(\frac{B_1}{m_1}-\frac{B_2}{m_2}\right)\cos\in\sin(\nu-\alpha)\,d\nu = \left(\frac{B_1}{m_1}-\frac{B_2}{m_2}\right)\frac{\cos\in\sin\alpha\sin\theta_2}{\pi} \tag{5}$$

Using (5), equations (1) may be written as

$$X''-2Y'-\frac{3X}{(1-e^2)^{1/2}} = -A\cos i + \frac{R^3}{\mu}\left(\frac{B_1}{m_1}-\frac{B_2}{m_2}\right)\frac{\cos\in\cos\alpha\sin\theta_2}{\pi} + \frac{12k_2}{R^2}\frac{1}{(1-e^2)^{1/2}}X$$
$$-\lambda_\alpha\frac{1}{(1-e^2)^{1/2}}\frac{R^3}{\mu}\left[1-l_0\left(X^2+Y^2\right)^{-1/2}\right]X$$
$$Y''+2X' = \frac{R^3}{\mu}\left(\frac{B_1}{m_1}-\frac{B_2}{m_2}\right)\frac{\cos\in\sin\alpha\sin\theta_2}{\pi} - \frac{3k_2}{R^2}\frac{1}{(1-e^2)^{1/2}}Y - \lambda_\alpha\frac{R^3}{\mu}\frac{1}{(1-e^2)^{1/2}}\left[1-l_0\left(X^2+Y^2\right)^{-1/2}\right]Y \tag{6}$$

The condition of constraint given by (2) takes the form

$$X^2+Y^2 \le \frac{l_0}{\left(1+\frac{1}{2}e^2\right)^{1/2}} \tag{7}$$

Equations (6) do not contain the time explicitly. Therefore, Jacobian integral of the problem exists over here.

Multiplying the first and second equations of (6) by 2 X' and 2Y' respectively and adding them and then integrating we get the final equation as

$$X'^2+Y'^2-\frac{3X^2}{\left(1-e^2\right)^{1/2}}=-2AX\cos i+\frac{2}{\pi}\frac{R^3}{\mu}\left(\frac{B_1}{m_1}-\frac{B_2}{m_2}\right)\cos\in\sin\theta_2\left(X\cos\alpha+Y\sin\alpha\right)$$

$$+\frac{3k_2}{R^2}\frac{1}{\left(1-e^2\right)^{1/2}}\left(4X^2-Y^2\right)-\frac{\lambda_\alpha}{\left(1-e^2\right)^{1/2}}\frac{R^3}{\mu}\left[\left(X^2+Y^2\right)-2l_0\left(X^2+Y^2\right)^{1/2}\right] \quad (8)$$

Result and Discussions:

Aim of the present paper is to obtain Jacobean integral of motion of a system of two cable-connected satellites under the influence of several perturbative forces like shadow of the earth, solar radiation pressure, oblateness of the earth and earth's magnetic field and elasticity of the cable in elliptical orbit. The cable connecting the two satellites is light, flexible, non-conducting and elastic in nature. The satellites are considered as charged material particles. As the body of the satellites is made up of metal, the satellites cut magnetic lines of force of the earth during the motion. According to Lorentz force charges get developed on the two satellites. But magnitude of the charges is very small. Thus, electrostatic interaction between the satellites is not taken into account. Motion of the system is studied relative to the centre of mass.

3. Conclusion:

Equation (8) is the required Jacobean integral of motion of the system. It is also known as the first integral of motion of the system. It has wide applications in the further studies of the problems related to two elastic cable connected artificial satellites.

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References:

1. Beletsky, V. V. & Novikova, E. T. (1969): About the relative motion of two connected bodies. KosmicheskieIssledovania, 7(6), 377-384.

2. Das, S. K., Bhattacharya, P. K. & Singh, R. B. (1976): Effect of magnetic force on the motion of a system of two cable-connected satellites in orbit, Proceedings of National Academy of Sciences India, 46(A), 287-299.

3. Kumar, S. and Bhattacharya, P. K. (1995): Stability of equilibrium positions of two cable-connected satellites. Proc. Workshop on Sp. Dyn. And Celes. Mech., Muz., India, Eds. K. B. Bhatnagar and B. Ishwar, 71-74.

4. Kumar, S. & Srivastava, U. K. (2006): Evolutional and non-evolutional motion of a system of two cable-connected artificial satellites under Some perturbative forces. Celestial Mechanics, Recent Trends: Narosa Publishing House, New Delhi, Chennai, Mumbai, Kolkata, Ed. B. Ishwar, 187-192.

5. Kumar, S. & Prasad, J.D. (2015): Non – linear planer oscillation of a cable – connected satellites system and non – resonance, Indian Journal of Theoretical Physics, Kolkata, India, 63(1, 2), 01 – 14.

6. Kumar, S. & Kumar, S. (2016): Equilibrium positions of a cable- connected satellites system under several influences, International Journal of Astronomy and Astrophysics, China, 06, 288-292.

7. Nechvile,V.(1926): Surunenouvelicforme des equations differentielies du problem restriintelliptique, Academy Paris Compte Rendus,182, 310-322.

8. Singh, R. B. & Demin, V. G. (1972): Two-dimensional motion of two connected bodies in the central gravitational field of force, Celestial Mechanics, 06, 268-277.

9. Singh, R. B. (1973): Three - dimensional motion of a system of two cable - connected satellites in orbit, Astronautica Acta, 18, 301-308.

10. Sinha, S. K. & Singh, R. B. (1987): Effect of solar radiation pressure on the motion and stability of the system of two inter connected satellites when their centre of mass moves in circular orbit, Astrophysics and Space Science, 129, 233-245.

11. Singh, B. M., Narayan, A. & Singh, R. B. (2001): Non- linear effect on the motion and stability of an inter connected satellites system orbiting around an oblate earth, Proceedings of National Academy of Sciences India, 71, 225-235