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FITTING OF POISSON DISTRIBUTION BY USING RECURRENCE RELATION METHOD BETWEEN RAINFALL AND GROUND WATER LEVELS – A CASE STUDY

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Abstract—Present paper deals with the application of 'Distribution Theory' to analyze and predict Rainfall (RF) and Ground water levels (GWLs) in Anantapuramu district based on the data collected from January 2007 to December 2016. Through with Poisson distribution by using recurrence relation method, for the purpose of analysis the district is divided into five zones or Revenue Divisions (RD) namely, 1. Anantapuramu RD 2. Penukonda RD 3. Kadiri RD 4. Kalyandurg RD 5. Dharmavaram RD. We have estimated the Poisson distribution by using recurrence relation method values and compared among them by using the data. Further, validation of the fitted distribution identified the best suitable zone. i.e., Residual Analysis or Error Analysis or Residual Sum of Squares (RSS) or Error Sum of Squares (ESS) or least Mean Square Error (MSE) value of the zone and forecast on the Rainfall and Ground water levels of this district. We also calculate Critical Difference (C.D) test and conclusions are drawn based on the results obtained.

Keywords—*Rainfall, Ground Water Level, Residual Sum of Squares, Validation of the distribution, Critical Difference test.*

1. INTRODUCTION

Earlier we have discussed in the previous paper [1] the method of curve fitting is the best for estimating trend in the time series analysis. The nature of the curve that is appropriate for the given data can be satisfactorily decided either by a theoretical understanding of the data or by observing the scatter diagram that is constructed for the given data.

The methods of fitting Straight Line, Second Degree Parabola, Exponential Curve and Power Curves by least squares method was discussed in the earlier research paper [1].

Linear, Parabolic, Exponential and Power Curve projections generally assume that growth or decline continues without limit. While these trends continue for some time they are not continue forever. There are a number of situations in which there is an asymptote to growth or decline. There are three types of Growth Curves or Models is there, that is:
1. Modified Exponential Model [2, 6]
2. Gompertz Model [3, 5]
3. Logistic Model. These models also discussed earlier papers in 'Time Series Analysis and Forecasting' concept [4].

We have discussed 'Distribution Theory' for different distributions like. Binomial Distribution-Direct and Recurrence Relation Method, Negative Binomial Distribution Recurrence Relation Method and Poisson distribution direct method already we will analyze; now I will fit **Poisson distribution by using recurrence relation method** in this paper.

The data is collected on Average Rainfall and Average Ground Water Levels are given in the following Table-1.1 for a ready reference [1, 2, 3, 4, 5, 6, 7, 8 and 9].

Table-1.1
Average Rainfall and Average Ground water levels data from 2007 to 2016

Year	Zone-I		Zone-II		Zone-III		Zone-IV		Zone-V	
	RF (in mm)	GWL	RF (in mm)	GWL	RF (in mm)	GWL	RF (in mm)	GWL	RF (in mm)	GWL
2007	65.60	10.57	58.20	22.58	67.20	14.23	52.00	14.97	60.50	17.03
2008	53.90	9.96	77.90	20.73	65.20	9.27	61.30	10.88	62.70	9.09
2009	45.40	12.17	50.60	17.53	46.30	11.08	57.10	9.58	38.70	10.24
2010	53.90	12.74	71.50	15.02	70.80	12.03	64.60	8.58	56.30	11.79
2011	39.50	12.69	42.30	15.20	48.90	11.48	31.80	8.93	36.60	12.84
2012	43.20	14.98	43.40	20.49	45.30	16.08	40.50	13.76	41.90	13.22
2013	35.00	15.94	52.30	23.03	47.10	18.69	34.80	16.98	38.10	14.30
2014	31.10	15.87	30.30	23.40	27.10	21.16	37.10	18.92	22.80	16.30
2015	44.10	14.90	62.60	26.88	66.30	25.80	46.00	19.26	54.30	17.66
2016	33.50	15.57	33.40	27.27	32.30	15.35	25.70	19.51	30.10	16.15

2. STATISTICAL ANALYSIS

Some of the Preliminary Statistical analysis is done for the data provided in the above table -1.1, such as yearly averages of Rainfall and Ground water levels are calculated and Karl-Pearson's Correlation Coefficient (r) is calculated between Average Rainfall(X) and Average Ground water levels (Y) Zonal wise[1, 2 and 3].

.

To forecast **Rainfall** and **Ground Water Levels** through **Poisson distribution by using recurrence relation method** for different zones we can consider given as follows:

The Probability Mass Function of Poisson distribution is given by

$$P(X = x) = \frac{e^{-\lambda}\lambda^x}{x!} \quad where\ x = 1,2,3, ..., \quad \ldots\ldots (2.1)$$

In Poisson distribution the parameter λ is equal to the Arithmetic mean.

$$\bar{x} = \frac{\sum_{i=1}^{n} f_i x_i}{N} = \lambda = \hat{\lambda}$$

Substitute this $\hat{\lambda}$ in the above Poisson distribution equation (2.1) we will get the Probability Mass Function of a Poisson distribution has

$$P(X=x)=\frac{e^{-\hat{\lambda}}\hat{\lambda}^{x}}{x!} \quad where\ x=1,2,3,...,\qquad \ldots\ldots\ (2.2)$$

To find the Expected Frequencies: To find the Expected Frequencies, we use the following probability Mass function of Poisson distribution by using recurrence relation method is given by

$$P(x+1)=\frac{\lambda}{x+1}P(x) \qquad \ldots\ldots\ (2.3)$$

The fitted Poisson distribution by using recurrence relation method for Average RF and Average GWLs:

A: For Average Rainfall

Zone-I

The Probability Mass Function of Poisson distribution is given by

$$P(X=x)=\frac{e^{-4.96}(4.96)^{x}}{x!}$$

By using recurrence relation method is given by

$$P(x+1)=\frac{4.96}{x+1}P(x)$$

Zone-II

The Probability Mass Function of Poisson distribution is given by

$$P(X=x)=\frac{e^{-5.03}(5.03)^{x}}{x!}$$

By using recurrence relation method is given by

$$P(x+1)=\frac{5.03}{x+1}P(x)$$

Zone-III

The Probability Mass Function of Poisson distribution is given by

$$P(X=x)=\frac{e^{-5.04}(5.04)^{x}}{x!}$$

By using recurrence relation method is given by

$$P(x+1)=\frac{5.04}{x+1}P(x)$$

Zone-IV

The Probability Mass Function of Poisson distribution is given by

$$P(X = x) = \frac{e^{-4.92}(4.92)^x}{x!}$$

By using recurrence relation method is given by

$$P(x+1) = \frac{4.92}{x+1}P(x)$$

Zone-V

The Probability Mass Function of Poisson distribution is given by

$$P(X = x) = \frac{e^{-4.98}(4.98)^x}{x!}$$

By using recurrence relation method is given by

$$P(x+1) = \frac{4.98}{x+1}P(x)$$

B: For Average Ground water levels

Zone-I

The Probability Mass Function of Poisson distribution is given by

$$P(X = x) = \frac{e^{-5.91}(5.91)^x}{x!}$$

By using recurrence relation method is given by

$$P(x+1) = \frac{5.91}{x+1}P(x)$$

Zone-II

The Probability Mass Function of Poisson distribution is given by

$$P(X = x) = \frac{e^{-5.84}(5.84)^x}{x!}$$

By using recurrence relation method is given by

$$P(x+1) = \frac{5.84}{x+1}P(x)$$

Zone-III

The Probability Mass Function of Poisson distribution is given by

$$P(X = x) = \frac{e^{-6.15}(6.15)^x}{x!}$$

By using recurrence relation method is given by

$$P(x+1) = \frac{6.15}{x+1}P(x)$$

Zone-IV

The Probability Mass Function of Poisson distribution is given by

$$P(X = x) = \frac{e^{-6.12}(6.12)^x}{x!}$$

By using recurrence relation method is given by

$$P(x+1) = \frac{6.12}{x+1} P(x)$$

Zone-V

The Probability Mass Function of Poisson distribution is given by

$$P(X = x) = \frac{e^{-5.83}(5.83)^x}{x!}$$

By using recurrence relation method is given by

$$P(x+1) = \frac{5.83}{x+1} P(x)$$

$where\ x = 1,2,3 \ldots$ Substitute in the above equations we can get the values of $p(1), p(2), p(3), \ldots, multiplying\ these\ p(1), p(2), p(3), \ldots,\ values\ by\ the\ N = \sum_{i=1}^{n} f_i$ we get the required Expected Frequencies, these are denoted by $f(1), f(2), f(3), \ldots,$

3. VALIDATION OF THE FITTED DISTRIBUTION

Validation of the fitted distribution is necessary to check the suitability of the distribution for the given data this is done by considering X = Years and Y = Average RF or Average GWL given in table-1.1 and estimated the Average RF (Y) or Average GWL (Y) denoted by $\hat{y}$. The estimated Average RF and Average GWLs are given in the following tables.

Table-3.1

Estimated Average RF $\hat{y}$ for Poisson distribution by using recurrence relation method

Year	Zone-I		Zone-II		Zone-III		Zone-IV		Zone-V	
	Actual	Estimates	Actual	Estimates	Actual	Estimates	Actual	Estimates	Actual	Estimates
2007	65.60	13.36	58.20	15.68	67.20	15.50	52.00	18.04	60.50	13.26
2008	53.90	31.16	77.90	41.80	65.20	41.32	61.30	45.09	62.70	30.94
2009	45.40	53.42	50.60	67.93	46.30	67.15	57.10	72.14	38.70	53.04
2010	53.90	66.78	71.50	83.60	70.80	82.64	64.60	90.18	56.30	66.30
2011	39.50	66.78	42.30	83.60	48.90	82.64	31.80	90.18	36.60	66.30
2012	43.20	53.42	43.40	67.93	45.30	67.15	40.50	72.14	41.90	53.04
2013	35.00	40.07	52.30	47.03	47.10	46.49	34.80	49.60	38.10	39.78
2014	31.10	26.71	30.30	31.35	27.10	30.99	37.10	31.56	22.80	26.52
2015	44.10	13.36	62.60	15.68	66.30	15.50	46.00	18.04	54.30	13.26
2016	33.50	4.45	33.40	10.45	32.30	10.33	25.70	9.02	30.10	4.42

Table-3.2

Estimated Average GWL $\hat{y}$ for Poisson distribution by using recurrence relation method

Year	Zone-I		Zone-II		Zone-III		Zone-IV		Zone-V	
	Actual	Estimates	Actual	Estimates	Actual	Estimates	Actual	Estimates	Actual	Estimates
2007	10.57	2.71	22.58	4.24	14.23	1.55	14.97	1.41	17.03	2.77
2008	9.96	8.12	20.73	12.73	9.27	4.66	10.88	4.24	9.09	8.32
2009	12.17	16.25	17.53	25.46	11.08	9.31	9.58	8.48	10.24	16.63
2010	12.74	24.37	15.02	38.18	12.03	13.97	8.58	12.72	11.79	23.57
2011	12.69	28.43	15.20	44.55	11.48	17.07	8.93	15.55	12.84	27.72
2012	14.98	28.43	20.49	42.43	16.08	17.07	13.76	15.55	13.22	26.34
2013	15.94	24.37	23.03	36.06	18.69	15.52	16.98	14.14	14.30	22.18
2014	15.87	17.60	23.40	25.46	21.16	12.41	18.92	11.31	16.30	16.63
2015	14.90	12.19	26.88	16.97	25.80	7.76	19.26	7.07	17.66	11.09
2016	15.57	6.77	27.27	10.61	15.35	4.66	19.51	4.24	16.15	6.93

In the above tables -3.1 and 3.2 for the validation of the distribution, Residual Analysis or Error Analysis or Residual Sum of Squares (RSS) or Error Sum of Squares (ESS) or Mean Square Errors (MSE's) are calculated zone wise by considering

$$Residual\ Sum\ of\ Squares\ (RSS) = \sum_{i=1}^{n}(y_i - \hat{y}_i)^2 \qquad \text{….. (3.1)}$$

Where y_i *or* o_i represents actual or observed values given in table-1.1 and $\hat{y}_i$ *or* $\hat{e}_i$ is the estimated values through fitted distribution is given in tables- 3.1 and 3.2. Residual Sum of Squares was calculated and is given in the following table.

Table-3.3

RSS values for Average RF for Poisson distribution by using recurrence relation method.

Type of the Distribution	Zone-I	Zone-II	Zone-III	Zone-IV	Zone-V
Poisson distribution	6158.81	8622.37	8512.68	8015.60	6912.55

Table-3.4

RSS values for Average GWLs for Poisson distribution by using recurrence relation method.

Type of the Distribution	Zone-I	Zone-II	Zone-III	Zone-IV	Zone-V
Poisson distribution	804.56	2892.20	747.48	741.08	967.45

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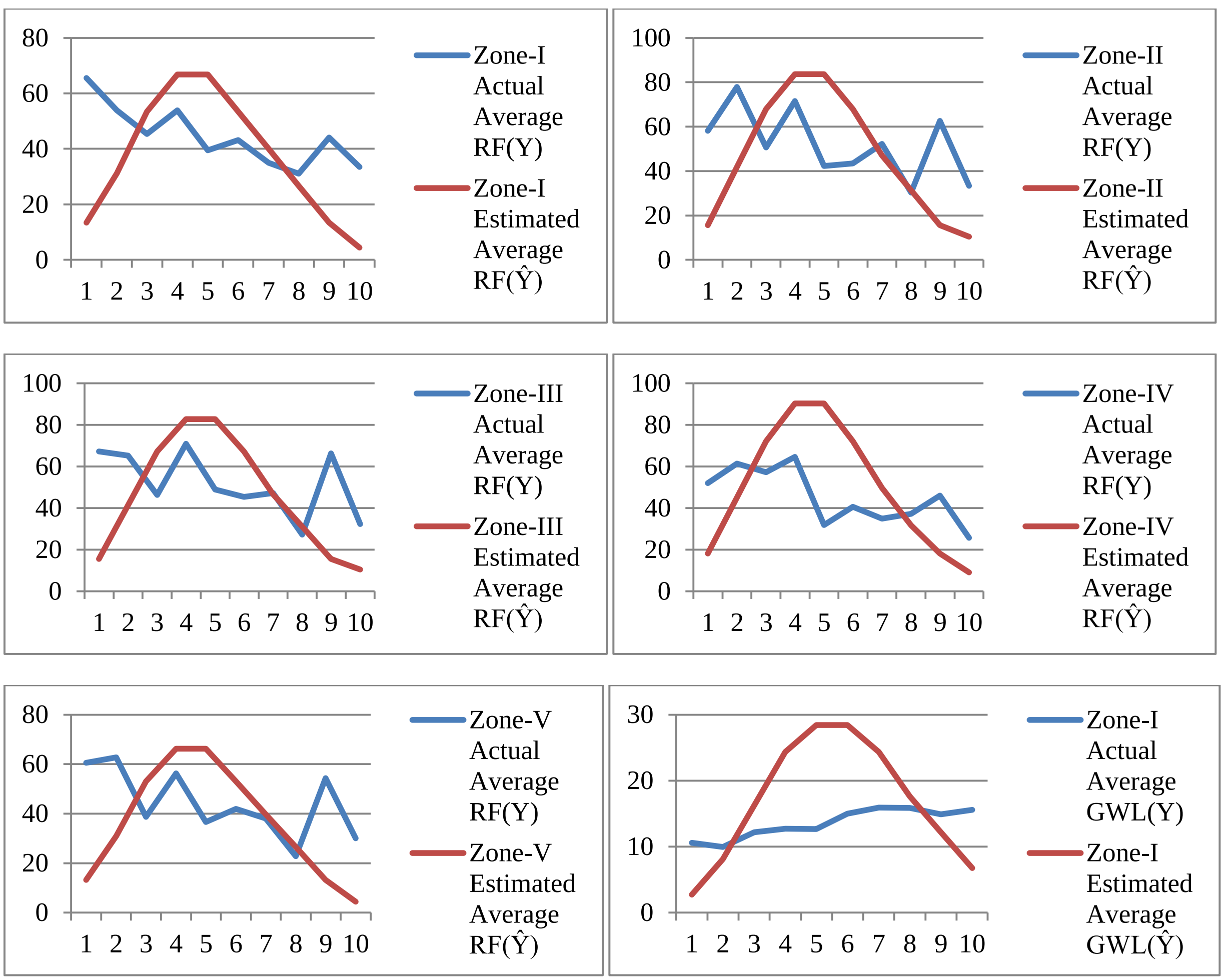
CONCLUSIONS:

By Comparing Residual Sum of Squares values for Average RF and Average GWLs through Poisson distribution by using recurrence relation method under consideration, for RF of zone-I is least and GWLs for zone-IV Residual Sum of Squares values is least. Next to zone-I, zone-V has least Residual Sum of Square value in RF and GWLs zone-III is least. Further, the behaviors of RF and GWL through this distribution in different zones are represented in the following Figure-3.1. Similar conclusions can be drawn from the following graphs also.

Fig-3.1

Behavior of RF and GWLs Actual and Estimated values for Poisson distribution by using recurrence relation method in Zone –I, II, III, IV and V



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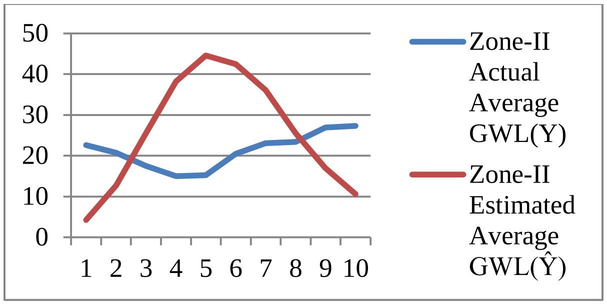

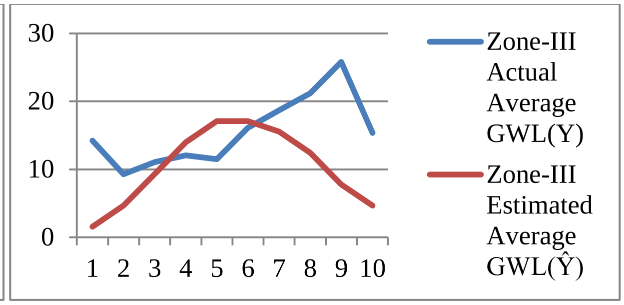

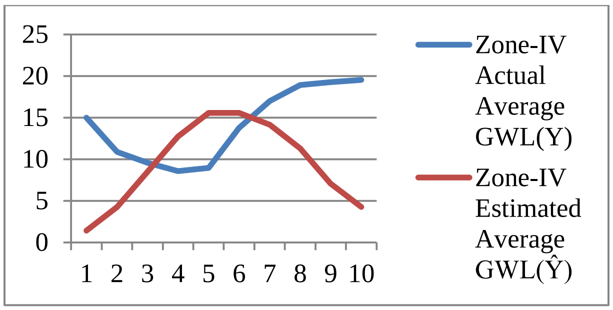

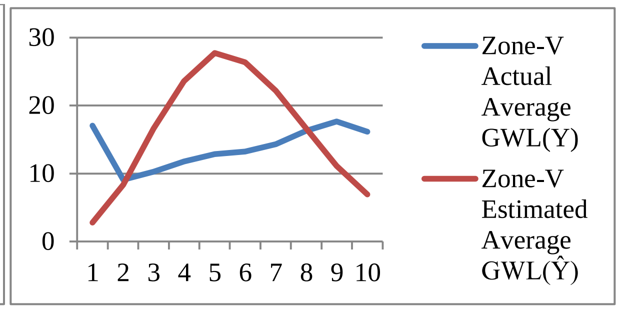

Note: In the above graphs x-axis represents years in the last decade i.e. from 2007 to 2016.
On y-axis Average RF measured in Mille Meters or Average GWLs measured in Meters.

4. FURTHER STATISTICAL ANALYSIS

Now, we proceed to analyze the given estimates in tables-3.1 and 3.2 using ANOVA two-way classification by considering rows as different years and columns as different zones and the following Null Hypothesis are formed and tested.

H_{01}: There is no significant difference between different years of Average RF in Anantapuramu District [1, 2, 3, 4, 5, 6, 7, 8, 9 and 10].

H_{02}: There is no significant difference between Average RF of different zones in Anantapuramu District [1, 2, 3, 4, 5, 6, 7, 8, 9 and 10].

H_{03}: There is no significant difference between different years of Average Ground Water Levels in Anantapuramu District [1, 2, 3, 4, 5, 6, 7, 8, 9 and 10].

H_{04}: There is no significant difference between Average Ground Water Levels of different zones in Anantapuramu District [1, 2, 3, 4, 5, 6, 7, 8, 9 and 10].

Table-4.1
ANOVA Two-way Table for RF

Source of variation	d.f	S.S	M.S.S	F-cal
Rows (years)	9	31094.73	3454.97	251.3304
Columns (Zones)	4	1410.143	352.5357	25.64507
Error	36	494.8821	13.74672	
Total	49	32999.75		

By comparing F-calculated value of Rows (Years) with F-critical value at 5% level of significance we reject the H_{01} i.e. There is a significant difference between different years of Average RF in Anantapuramu District. Similarly by comparing F-calculated value of Columns (Zones) with F-critical value at 5% level of significance we reject the H_{02} i.e. There is a significant difference between Average RF of different zones in Anantapuramu District.

Table-4.2
ANOVA Two-way Table for GWLs

Source of variation	d.f	S.S	M.S.S	F-cal
Rows (years)	9	3310.501	367.8334	26.31873
Columns (Zones)	4	1680.526	420.1316	30.0607
Error	36	503.1398	13.97611	
Total	49	5494.167		

By comparing F-calculated value of Rows (Years) with F-critical value at 5% level of significance we reject the H_{03} i.e. There is a significant difference between different years of Average GWLs in Anantapuramu District. Similarly by comparing F-calculated value of Columns (Zones) with F-critical value at 5% level of significance we reject the H_{04} i.e. There is a significant difference between Average GWLs of different zones in Anantapuramu District.

Since F-cal value related to Rows (Years) in RF and Columns (Zones) in GWL is high so there is a necessity for Critical Difference (C.D) Test for sub-grouping various years and columns using the following formula.

C.D. = $\sqrt{2 \times Error M.S.S/m} \times t_{0.01}$ for error d.f. in tables (4.1) and (4.2) … (4.1)

Where m represents number of replicates in each zone and as well as year.

5. CRITICAL DIFFERENCE (C.D) TEST: Average RF for Years

Table-5.1
Year wise Aggregate Average RF for Poisson distribution estimates by using recurrence relation method

Year	2007	2008	2009	2010	2011	2012	2013	2014	2015	2016
Average	15.168	38.062	62.736	77.9	77.9	62.736	44.594	29.426	15.168	7.734

Table 5.2
If we can arranged Ascending Order

Year	2016	2007	2015	2014	2008	2013	2009	2012	2010	2011
Average	7.734	15.168	15.168	29.426	38.062	44.594	62.736	62.736	77.9	77.9

S.E = $\sqrt{2 \times Error\ M.S.S/m}$
= 2.34
1% l.o.f C.D = 2.58×2.34 = 6.04

2016 $\quad$ $\overline{2007 \quad 2015}$ $\quad$ 2014 $\quad$ 2008 $\quad$ 2013 $\quad$ $\underline{2009 \quad 2012}$ $\quad$ $\overline{2010 \quad 2011}$

Above notation indicates that 2007-2015, 2009-2012, 2010-2011 years Average RF come under one category and 2016, 2014, 2008, 2013 year Average RF different category because there is no Significant Difference in average RF.

CRITICAL DIFFERENCE (C.D) TEST: Average GWLs for Zones

Table-5.3
Year wise Aggregate Average GWLs for Poisson distribution estimates by using recurrence relation method

Zones	Zone-I	Zone-II	Zone-III	Zone-IV	Zone-V
Average	16.924	25.669	10.398	9.471	16.218

Table 5.4
If we can arranged Ascending Order

Zones	Zone-IV	Zone-III	Zone-V	Zone-I	Zone-II
Average	9.471	10.398	16.218	16.924	25.669

S.E = $\sqrt{2 \times Error\ M.S.S/m}$
= 1.67

1% l.o.f C.D = 2.58×1.67 = 4.31

Zone-IV Zone-III Zone-V Zone-I Zone-II

Above notation indicates that Zone-IV, Zone-III Average GWLs come under one category and Zone-V, Zone-I Average GWLs come under one category and Zone-II different category because there is no Significant Difference in average GWLs.

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