Vol.09 Issue-02, (July - December, 2017) ISSN: 2394-9309 (E) / 0975-7139 (P)
Aryabhatta Journal of Mathematics and Informatics (Impact Factor- 5.856)

ADJACENT DEGREE POLYNOMIAL OF GRAPHS

Dr Ajitha V
Associate Professor
P.G Department of Mathematics
Mahatma Gandhi College, Iritty-670703,
Kerala, India

Silja C
Research Scholar
Mary Matha Arts & Science College
Mananthavady - 670 645
Kerala, India

Abstract

We introduce a special type of polynomial associated to every (p,q)-graph G. The adjacent degree polynomial of a graph G depends the sum of the degrees of the vertices in the neighborhood of all vertices in G. This paper mainly focuses on adjacent degree polynomial of graphs and shall attempt to compute the adjacent degree index of some special type of graphs. We also establish some fundamental properties of adjacent degree index of certain graphs.

Keywords

Adjacent degree polynomial of a graph, adjacent degree index of a graph.

1. Introduction

By a $(p,q)-$graph $G=(V(G),E(G))$ we mean a finite undirected simple graph with $q > 1$ and having no isolated vertex. Let $v \in V(G)$ and the neighborhood of v is the set $N_G(v)=N(v)=\{u \in V(G): uv \in E(G)\}$. The degree of $v \in V(G)$ is equal to the cardinality of $N_G(v)$ and is denoted by $d_G(v)$. δ is the minimum degree of a graph. or various graph theoretic notations and terminology we follow F. Harrary [1] and for basic number theoretic results we refer [4].

2. Adjacent degree polynomial of graphs

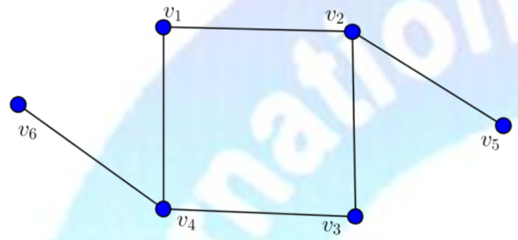
Definition 2.1 Let G be a $(p,q)-$graph with vertex set $V(G)=\{v_1.v_2, \ \cdots, \ v_p\}$.

Let $\sum_{v_i \in V(G)} |N(v_i)| = k_i$ for $1 \le i \le p$.

Then the polynomial defined by $\mu_G(x) = \sum_{i=1}^{p} x^{k_i}$ is called the adjacent degree polynomial associated with G and $\mu_G(q)(\bmod p)$ is called the adjacent degree index of G and is denoted by $In_{adj}(G)$.

The following are some simple observations which follow immediately from the definition of a adjacent degree polynomial.

Example :



$\mu_G(x) = 2x^3 + 2x^5 + 2x^6$.

Observation 2.2 Sum of the coefficient of $\mu_G(x)$ is the number of vertices of G .**Observation 2.3** For an Euler graph G, $\mu_G(x)$ is an even function.

Observation 2.4 For a k-regular graph G, $\mu_G(x) = px^{k^2}$ and thus for a complete graph K_n, $\mu_{K_n}(x) = nx^{(n-1)^2}$.

Observation 2.5 For Star graphs K_{1n}, $\mu_{K_{1n}}(x) = (n+1)x^n$

Observation 2.6 For Paths P_n, $\mu_{P_2}(x) = 2x, \mu_{P_3}(x) = 3x^2$

For $n \geq 4$, $\mu_{P_n}(x) = 2x^2 + 2x^3 + (n-4)x^4$

Observation 2.7 For Cycles C_n, $\mu_{C_n}(x) = nx^4$

Observation 2.8 For Complete Bipartite graphs K_{mn}, $\mu_{K_{mn}}(x) = (n+m)x^{mn}$

Observation 2.9 For Wheels W_{1n}, $\mu_{W_{1n}}(x) = nx^{6+n} + x^{3n}$

Theorem 2.10 : Let G is a k-regular graph of order n then adjacent degree polynomial of G^c is $\mu_{G^c}(x) = nx^{(n-1-k)^2}$.

Vol.09 Issue-02, (July - December, 2017) ISSN: 2394-9309 (E) / 0975-7139 (P)
Aryabhatta Journal of Mathematics and Informatics (Impact Factor- 5.856)

Proof : Let G be a k-regular graph of order n then $\mu_G(x) = nx^{k^2}$. G^c is (n-1-k) -regular graph of order n. So $\mu_{G^c}(x) = nx^{(n-1-k)^2}$.

3. Adjacent degree index of graphs

Theorem 3.1 : For star graphs $K_{1n}, In_{adj}(K_{1n}) = 0$

Proof : Star graphs K_{1n} has $n+1$ vertices and $n+1$ edges.

$p = n+1, q = n$ and $\mu_{K_{1n}}(x) = (n+1)x^n$

$$In_{adj}(K_{1n}) = \mu_{K_{1n}}(q)(\bmod p) = (n+1)n^n(\bmod n+1) = 0$$

Theorem 3.2 : For Paths P_n, $In_{adj}(P_n) = n-4$, for $n \geq 4$

Proof For Paths P_n has n vertices and $n-1$ edges.

$p = n, q = n-1$ and $\mu_{P_n}(x) = 2x^2 + 2x^3 + (n-4)x^4$

$$In_{adj}(P_n) = \mu_{P_n}(q)(\bmod p) = 2(n-1)^2 + 2(n-1)^3 + (n-4)(n-1)^4(\bmod n)$$

$$= 2-2+n-4(\bmod n) = (n-4) \bmod n.$$

Theorem 3.3 : For k-regular graph G, $In_{adj}(G) = 0$

Proof : If G has n vertices then G has $\frac{nk}{2}$ edges.

$p = n, q = \frac{nk}{2}$ and $\mu_G(x) = px^{k^2}$

$$In_{adj}(G) = \mu_G(q)(\bmod p) = n(\tfrac{nk}{2})^n(\bmod n) = 0$$

Theorem 3.4 : For Cycles, $In_{adj}(C_n) = 0$

Proof : For Cycles C_n has n vertices and n edges.

$p = n, q = n$ and $\mu_{C_n}(x) = nx^4$

$$In_{adj}(C_n) = \mu_{C_n}(q)(\bmod p) = n(n)^4(\bmod n) = 0$$

Theorem 3.5 : For Complete Bipartite graphs K_{mn}, $In_{adj}(K_{mn}) = 0$

Proof : For Complete Bipartite graphs K_{mn} has $m+n$ vertices and mn edges.

$p = m+n, q = mn$ and $\mu_{K_{mn}}(x) = (n+m)x^{mn}$

$$In_{adj}(K_{mn}) = \mu_{K_{mn}}(q)(\bmod p) = (n+m)(mn)^{mn}(\bmod m+n) = 0$$

Theorem 3.6 : For Wheels W_{1n}, $In_{adj}(W_{1n}) = -(-2)^{6+n} + (-2)^{3n}(\bmod n+1)$

Proof : For Wheels W_{1n} has $n+1$ vertices and $2n$ edges.

$p = n+1, q = 2n$ and $\mu_{W_{1n}}(x) = nx^{6+n} + x^{3n}$

$$In_{adj}(W_{1n}) = \mu_{W_{1n}}(q)(\bmod p) = n(2n)^{6+n} + (2n)^{3n}(\bmod n+1) = -(-2)^{6+n} + (-2)^{3n}(\bmod n+1)$$

Corollary 3.7 : For Wheels W_{1n}, $In_{adj}(W_{1n}) = -2^6 + 1 \bmod (n+1)$,If $n+1$ is prime.

Proof : For Wheels W_{1n} $In_{adj}(W_{1n}) = -(-2)^{6+n} + (-2)^{3n}(\bmod n+1)$

If $n+1$ is prime, $(-2)^n = 1 \bmod (n+1)$ then

$$In_{adj}(W_{1n}) = -(-2)^{6+n} + (-2)^{3n}(\bmod n+1)$$

$$= -[(-2)^6(-2)^n] + (-2)^{3n}(\bmod n+1) = -[-2^6] + 1(\bmod n+1) = -2^6 + 1 \bmod (n+1)$$

Theorem 3.6: Let G be (p,q) graph with $p = q$ then $In_{adj}(G)$ is the constant term of $\mu_G(x)$.

Proof : Let $\mu_G(x) = a_0 + a_1 x + ... + a_k x^k$ For Wheels

$$In_{adj}(G) = \mu_G(q)(\bmod p) = a_0 + a_1 q + ... + a_k q^k$$

$$= a_0 + a_1 p + ... + a_k p^k (\bmod p) = a_0$$

Corollary 3.6: Let G be (p,q) graph with $p = q$, $\delta > 0$ then $In_{adj}(G) = 0$.

Proof : Since there is no isolated vertex $|N_G(v)| \neq 0$ for any vertex v in G.

Then the constant term of $\mu_G(x)$ =0. By above theorem $In_{adj}(G) = 0$.

Theorem 3.6: Let G be (p,q) graph then $\mu_G(x) = 2mx \iff G \cong mK_2$, $m > 0$

Proof : Suppose $\mu_G(x) = 2mx$ then there is only one adjacent vertex and which have degree one so which is mK_2.Conversely suppose $G \cong mK_2$ using the definition clearly we get $\mu_G(x) = 2mx$.

Theorem 3.6: For any integer n with $In_{adj}(G) = n$ there always exist a graph G

Proof : Let n any integer, consider the Path P_{n+4}, $In_{adj}(P_{n+4}) = n$.

REFERENCES

[1] F. Harary Graph Theory, Addition-Wesley, Reading, Mass, 1972

[2] Sergio R.Canoy, Jr. Mhelmar A Labendia, Polynomial Representation and Degree Sequence of a Graph, Int. Jonl of Math. Analysis, Vol.8, 2014.no.29.

[3] Graph Polynomials and Their Representations, Technische Universitat Bergakademie Freiberg, 1765.

[4] David M. Burton, Elementary Number Theory, Second Edition, Wm. C. Brown Company Publishers, 1980.

[5] J.A. Gallian, A Dynamic Survey of Graph Labeling, Electronic Journal of Combinatorics, 17 (2010), DS6.