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WIND ENERGY HARVESTING THROUGH MATHEMATICAL MODELING: A REVIEW

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Abstract

Every year the number of installed wind power plants in the world increases. The paper provides an overview of the concepts of wind energy technology and discusses the current status of grid-connected as well as stand-alone wind power generation worldwide. The generation and movement of wind are complicated due to the two most important factors (uneven solar heating, the Coriolis effect)have been introduced.The mathematical modeling of wind energy technology and wind power parameters is reviewed.The various wind turbine concepts with power conversion coefficients (gearbox efficiency, generator efficiency and electric efficiency) and grid for wind energy system have been discussed. Furthermore, a study of Lanchester–Betz Lawhig hlighted that no wind turbo machines could convert more than 59.26% of the kinetic energy of wind into mechanical energy.
Keywords: Wind power technology; Coriolis force; wind energy; Lanchester–Betz Law;

1. Introduction

Demand for energy and its resources, is increasing every day due to the rapid outgrowth in population and industrialization development. As the major conventional energy resources like coal, petroleum and natural gas are at the verge of getting extinct, wind energy can be considered as one of the promising environment friendly renewable energy options. Sun is the source of all energies. Renewable energy technologies provide an excellent opportunity for mitigation of greenhouse gas emission and reducing global warming through substituting conventional energy sources. To overcome the negative impacts on the environment and other problems associated with fossil fuels have forced many countries to inquire into and change to environmental friendly alternatives that are renewable to sustain the increasing energy demand [1]. To meet future energy demands efficiently, energy security and reliability must be improved and alternative energy sources must be investigated aggressively. An effective energy solution should be able to address long-term issues by utilizing alternative and renewable energy sources.Global environmental concerns and the escalating demand for energy, coupled with steady progress in renewable energy technologies, are opening up new opportunities for utilization of renewable energy resources. Wind energy is the most abundant, inexhaustible and clean of all the renewable energy resources till date [2-3].

During the last decade of the 20th century, grid-connected wind capacity worldwide has doubled approximately every three years. Due to the fast market development, wind turbine technology has experienced an important evolution over time. Due to the fast market development, wind turbine technology has experienced an important evolution over time. Since ancient past humans have attempted to harness the wind energy through diversified means and vertical axis wind turbines (VAWTs) were one of the major equipment to achieve that. In this modern time, there is resurgence of interests regarding VAWTs as numerous universities and research institutions have carried out extensive research activities and developed numerous designs based on mathematical modeling which is crucial for deducing optimum design parameters.

This article traces society's increasing preoccupation with renewability and sustainability issues over the past few decades, and addresses the question of whether the wind energy harvesting through mathematical parameters should be considered renewable and sustainable.In view of changing demand, and of environmental and socio-economical constraints, achieving such increase is extremely challenging and can be reached only by innovative and sustainable solutions for new techniques for wind energy harvesting.

2. Fundamentals of wind energy

The rising concerns over global warming, environmental pollution, and energy security have increased interest in developing renewable and environmentally friendly energy sources such as wind, solar, hydropower, geothermal, hydrogen, and biomass as the replacements for fossil fuels. Wind energy can provide suitable solutions to the global climate change and energy crisis. The utilization of wind power essentially eliminates emissions of CO_2 , SO_2 , $NO_x$and other harmful wastes as in traditional coal-fuel power plants or radioactive wastes in nuclear power plants. By further diversifying the energy supply, wind energy dramatically reduces the dependence on fossil fuels that are subject to price and supply instability, thus strengthening global energy security. During the recent three decades, tremendous growth in wind power has been seen all over the world. In 2009, the global annual installed wind generation capacity reached a record-breaking 37 GW, bringing the world total wind capacity to 158 GW. As the most promising renewable, clean, and reliable energy source, wind power is highly expected to take a much higher portion in power generation in the coming decades.

Wind energy is a converted form of solar energy which is produced by the nuclearfusion of hydrogen (H) into helium (He) in its core. The H → He fusion process creates heat and electromagnetic radiation streams out from the sun into space in all directions. Though only a small portion of solar radiation is intercepted by the earth, it provides almost all of earth’s energy needs.

3. Wind power generation and wind turbine design

Wind energy represents a mainstream energy source of new power generation and an important player in the world's energy market. As a leading energy technology, wind power’s technical maturity and speed of deployment is acknowledged, along with the fact that there is no practical upper limit to the percentage of wind that can be integrated into the electricity system. It has been estimated that the total solar power received by the earth is approximately 1.8×10^{11}MW. Of this solar input, only 2% (i.e. 3.6×10^{9}MW) is converted into wind energy and about 35% of wind energy is dissipated within 1000 m of the earth’s surface. Therefore, the available wind power that can be converted into other forms of energy isapproximately 1.26×10^{9}MW. Because this value represents 20 times the rate of the present global energy consumption, wind energy in principle could meet entire energy needs of the world. Compared with traditional energy sources, wind energy has a number of benefits and advantages. Unlike fossil fuels that emit harmful gases and nuclear power that generates radioactive wastes, wind power is a clean and environmentally friendly energy source.

As an inexhaustible and free energy source, it is available and plentiful in most regions of the earth. In addition, more extensive use of wind power would help reduce the demands for fossil fuels, which may run out sometime in this century, according to their present consumptions. Furthermore, the cost per kWh of wind power is much lower than that of solar power. Thus, as the most promising energy source, wind energy is believed to play a critical role in global power supply in the 21st century.

4.Wind generation and Coriolis force

Wind results from the movement of air due to atmospheric pressure gradients. Wind flows from regions of higher pressure to regions of lower pressure. The larger the atmospheric pressure gradient, the higher the wind speed and thus, the greater the wind power that can be captured from the wind by means of wind energy-converting machinery.

The generation and movement of wind are complicated due to a number of factors. Among them, the most important factors are uneven solar heating, the Coriolis effect due to the earth's self-rotation, and local geographical conditions.

4.1 Uneven solar heating

Among all factors affecting the wind generation, the uneven solar radiation on the earth's surface is the most important and critical one. The unevenness of the solar radiation can be attributed to four reasons.

First, the earth is a sphere revolving around the sun in the same plane as its equator. Because the surface of the earth is perpendicular to the path of the sunrays at the equator but parallel to the sunrays at the poles, the equator receives the greatest amount of energy per unit area, with energy dropping off toward the poles. Due to the spatial uneven heating on the earth, it forms a temperature gradient from the equator to the poles and a pressure gradient from the poles to the equator. Thus, hot air with lower air density at the equator rises up to the high atmosphere and moves towards the poles and cold air with higher density flows from the poles towards the equator along the earth's surface. Without considering the earth's self-rotation and the rotation-induced Coriolis force, the air circulation at each hemisphere forms a single cell, defi ned as the meridional circulation.

Second, the earth's self-rotating axis has a tilt of about 23.5° with respect to its ecliptic plane. It is the tilt of the earth's axis during the revolution around the sun that results in cyclic uneven heating, causing the yearly cycle of seasonal weather changes.

Third, the earth's surface is covered with different types of materials such as vegetation, rock, sand, water, ice/snow, etc. Each of these materials has different reflecting and absorbing rates to solar radiation, leading to high temperature on some areas (e.g. deserts) and low temperature on others (e.g. iced lakes), even at the same latitudes.

The fourth reason for uneven heating of solar radiation is due to the earth's topographic surface. There are a large number of mountains, valleys, hills, etc. on the earth, resulting in different solar radiation on the sunny and shady sides.

4.2 Coriolis force

The earth's self-rotation is another important factor to affect wind direction and speed. The Coriolis force, which is generated from the earth's self-rotation, deflects the direction of atmospheric movements. In the north atmosphere wind is deflected to the right and in the south atmosphere to the left. The Coriolis force depends on the earth's latitude; it is zero at the equator and reaches maximum values at the poles. In addition, the amount of deflection on wind also depends on the wind speed; slowly blowing wind is deflected only a small amount, while stronger wind deflected more[3-5].

In large-scale atmospheric movements, the combination of the pressure gradient due to the uneven solar radiation and the Coriolis force due to the earth's selfrotation causes the single meridional cell to break up into three convectional cells in each hemisphere: the Hadley cell, the Ferrel cell, and the Polar cell (Fig.1).

Each cell has its own characteristic circulation pattern.In the Northern Hemisphere, the Hadley cell circulation lies between the equator and north latitude 30°, dominating tropical and sub-tropical climates. The hot air rises at the equator and flows toward the North Pole in the upper atmosphere. This moving air is deflected by Coriolis force to create the northeast trade winds. At approximately north latitude 30°, Coriolis force becomes so strong to balance the pressure gradient force. As a result, the winds are defected to the west. The air accumulated at the upper atmosphere forms the subtropical high-pressure belt and thus sinks back to the earth's surface, splitting into two components: one returns to the equator to close the loop of the Hadley cell; another moves along the earth's surface toward North Pole to form the Ferrel Cell circulation, which lies between north latitude 30° and 60°. The air circulates toward the North Pole along the earth's surface until it collides with the cold air flowing from the North Pole at approximately north latitude 60°. Under the influence of Coriolis force, the moving air in this zone is deflected to produce westerlies. The Polar cell circulation lies between the North Pole and north latitude 60°. The cold air sinks down at the North Pole and flows along the earth's surface toward the equator. Near north latitude 60°, the Coriolis effect becomes significant to force the airflow to southwest.

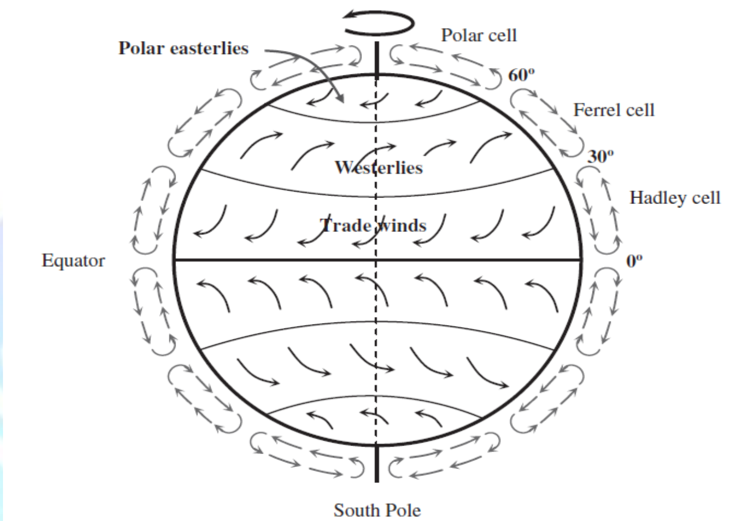

Fig.1: Idealized atmospheric circulations.

4.3 Local geography

The roughness on the earth's surface is a result of both natural geography and manmade structures. Frictional drag and obstructions near the earth's surface generally retard with wind speed and induce a phenomenon known as wind shear. The rate at which wind speed increases with height varies on the basis of local conditions of the topography, terrain, and climate, with the greatest rates of increases observed over the roughest terrain. A reliable approximation is that wind speed increases about 10% with each doubling of height.

In addition, some special geographic structures can strongly enhance the wind intensity. For instance, wind that blows through mountain passes can form mountain jets with high speeds.

5.Mathematical modeling of Wind energy Technology

Wind energy is a special form of kinetic energy in air as it flows. Wind energy can be either converted into electrical energy by power converting machines or directly used for pumping water, sailing ships, or grinding gain.

5.1 Wind power

Kinetic energy exists whenever an object of a given mass is in motion with a translational or rotational speed [6]. When air is in motion, the kinetic energy in moving air can be determined as

$$E_k = \frac{1}{2} m\bar{u}^2 \quad (1)$$

where$m$ is the air mass and $\bar{u}$is the mean wind speed over a suitable time period. The wind power can be obtained by differentiating the kinetic energy in wind with respect to time, i.e.:

$$P_w = \frac{dE_k}{dt} = \frac{1}{2} m\bar{u}^2 \quad (2)$$

However, only a small portion of wind power can be converted into electrical power. When wind passes through a wind turbine and drives blades to rotate, the corresponding wind mass flowrate is

$$\dot{m} = \rho A\bar{u} \quad (3)$$

wherepis the air density and A is the swept area of blades, as shown in Fig. 3.2 Substituting (3) into (2), the available power in wind P_w can be expressed as

$$P_w = \frac{1}{2}\rho A\bar{u}^3 \quad (4)$$

An examination of equation (4) reveals that in order to obtain a higher wind power, it requires a higher wind speed, a longer length of blades for gaining a larger swept area, and a higher air density. Because the wind power output is proportional to the cubic power of the mean wind speed, a small variation in wind speed can result in a large change in wind power.

5.2 Blade swept area

As shown in Fig. 2 the blade swept area can be calculated from the formula:

$$A = \pi[(l+r)^2 - r^2] = \pi l(l+2r) \quad (5)$$

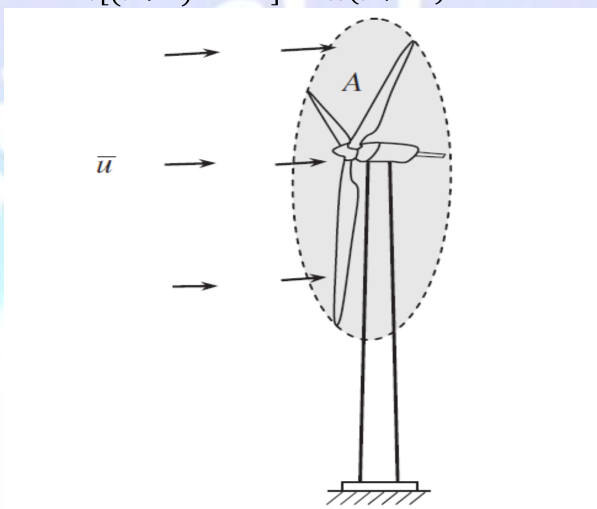

Fig. 2: Swept area of wind turbine blades.

where$l$ is the length of wind blades and r is the radius of the hub. Thus, by doubling the length of wind blades, the swept area can be increased by the factor up to 4. When $l >> 2r$, $A \approx \pi l^2$.

5.3 Air density

Another important parameter that directly affects the wind power generation is thedensity of air [7], which can be calculated from the equation of state:

$$\rho = \frac{P}{RT} \quad (6)$$

Where p is the local air pressure, R is the gas constant (287 J/kg-K for air), and T is the local air temperature in K.

The hydrostatic equation states that whenever there is no vertical motion, the difference in pressure between two heights is caused by the mass of the air layer:

$$dP = -\rho g dz \quad (7)$$

where$g$ is the acceleration of gravity. Combining eqns (6) and (7), yields

$$\frac{dP}{P} = \frac{g}{RT}dz \quad (8)$$

The acceleration of gravity g decreases with the height above the earth's surface z :

$$g = g_0\left(1 - \frac{4z}{D}\right) \quad (9)$$

where$g_0$is the acceleration of gravity at the ground and D is the diameter of the earth. However, for the acceleration of gravity g, the variation in height can be ignored because D is much larger than 4z.

In addition, temperature is inversely proportional to the height. Assume that d T /d z = c, it can be derived that

$$P = P_0(\frac{T}{T_0})^{-glcR} \quad (10)$$

wherep_0 and $T_0$are the air pressure and temperature at the ground, respectively.
Combining equations (6) and (10), it gives

$$\rho = \rho_0(\frac{T}{T_0})^{-(glcR+1)} = \rho_0(1+\frac{cz}{T_0})^{-(glcR+1)} \quad (11)$$

This equation indicates that the density of air decreases nonlinearly with the height above the sea level.

6.Various wind turbine concepts

A modern wind turbine is an energy-converting machine to convert the kinetic energy of wind into mechanical energy and in turn into electrical energy. In the recent three decades, remarkable advances in wind turbine design have been achieved along with modern technological developments. It has been estimated that advances in aerodynamics, structural dynamics, and micrometeorology may contribute to a 5% annual increase in the energy yield of wind turbines.

Various wind turbine concepts have been developed and built for maximizing the wind energy output, minimizing the turbine cost, and increasing the turbine efficiency and reliability.

6.1 Wind turbine classification

Wind turbines can be classified according to the turbine generator configuration, airflow path relatively to the turbine rotor, turbine capacity, the generator-driving pattern, the power supply mode, and the location of turbine installation.

6.2 Horizontal-axis and vertical-axis wind turbines

When considering the configuration of the rotating axis of rotor blades, modern wind turbines can be classified into the horizontal-axis and vertical-axis turbines. Most commercial wind turbines today belong to the horizontal-axis type, in which the rotating axis of blades is parallel to the wind stream. The advantages of this type of wind turbines include the high turbine efficiency, high power density, low cut-in wind speeds, and low cost per unit power output.

Several typical vertical-axis wind turbines are shown in Fig.3The blades of the vertical-axis wind turbines rotate with respect to their vertical axes that are perpendicular to the ground. A significant advantage of vertical-axis wind turbine is that the turbine can accept wind from any direction and thus no yaw control is needed. Since the wind generator, gearbox, and other main turbine components can be set up on the ground, it greatly simplifies the wind tower design and construction, andconsequently reduces the turbine cost. However, the vertical-axis wind turbinesmust use an external energy source to rotate the blades during initialization. Because the axis of the wind turbine is supported only on one end at the ground, its maximum practical height is thus limited. Due to the lower wind power efficiency, vertical-axis wind turbines today make up only a small percentage of wind turbines.

6.3 Upwind and downwind wind turbines

Based on the configuration of the wind rotor with respect to the wind fl owing direction, the horizontal-axis wind turbines can be further classified as upwind and downwind wind turbines. The majority of horizontal-axis wind turbines being used today are upwind turbines, in which the wind rotors face the wind. The main advantage of upwind designs is to avoid the distortion of the flow field as the wind passes though the wind tower and nacelle.

For a downwind turbine, wind blows first through the nacelle and tower and then the rotor blades. This configuration enables the rotor blades to be made more flexible without considering tower strike. However, because of the influence of the distorted unstable wakes behind the tower and nacelle, the wind power output generated from a downwind turbine fluctuates greatly. In addition, the unstable flow field may result in more aerodynamic losses and introduce more fatigue loads on the turbine [8-10]. Furthermore, the blades in a downwind wind turbine may produce higher impulsive or thumping noise.

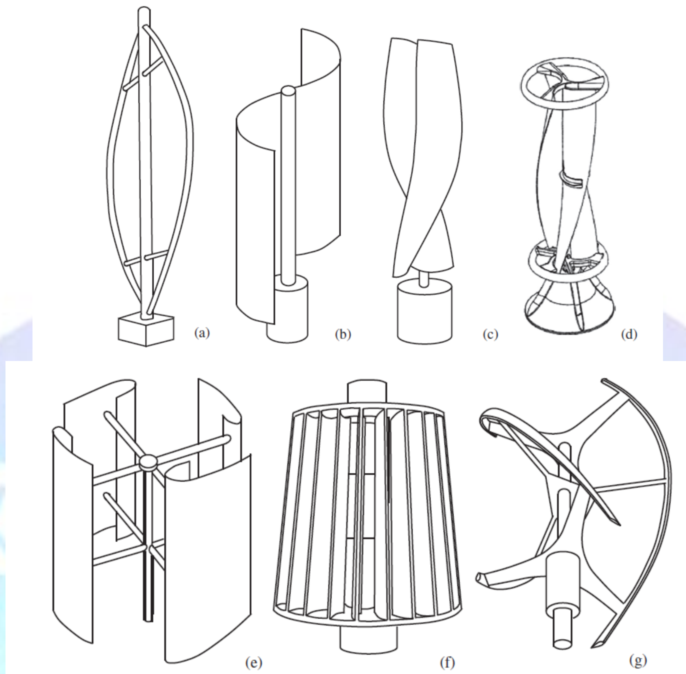

Fig. 3: Several typical types of vertical-axis wind turbines: (a) Darrius; (b) Savonius; (c) Solar wind; (d) Helical (e) Noguchi (f) Maglev; (g) Cochrane.

6.4 Wind turbine capacity

Wind turbines can be divided into a number of broad categories in view of their rated capacities: micro, small, medium, large, and ultra-large wind turbines. Though a restricted definition of micro wind turbines is not available, it is accepted that a turbine with the rated power less than several kilowatts can be categorized as micro wind turbine. Micro wind turbines are especially suitable in locations where the electrical grid is unavailable. They can be used on a per-structure basis, such as street lighting, water pumping, and residents at remote areas, particularly in developing countries. Because micro wind turbines need relatively low cut-in speeds at start-up and operate in moderate wind speeds, they can be extensively installed in most areas around the world for fully utilizing wind resources and greatly enhancing wind power generation availability.

Small wind turbines usually refer to the turbines with the output power less than 100 kW. Small wind turbines have been extensively used at residential houses, farms, and other individual remote applications such as water pumping stations, telecom sites, etc., in rural regions. Distributed small wind turbines can increase electricity supply in the regions while delaying or avoiding the need to increase the capacity of transmission lines.

The most common wind turbines have medium sizes with power ratings from 100 kW to 1 MW. This type of wind turbines can be used either on-grid or off-grid systems for village power, hybrid systems, distributed power, wind

Power plants, etc.

Megawatt wind turbines up to 10 MW may be classified as large wind turbines. In recent years, multi-megawatt wind turbines have become the mainstream of the international wind power market. Most wind farms presently use megawatt wind turbines, especially in offshore wind farms.

Ultra-large wind turbines are referred to wind turbines with the capacity more than 10 MW. This type of wind turbine is still in the earlier stages of research and development.

6.5 Direct drive and geared drive wind turbines

According to the drivetrain condition in a wind generator system, wind turbines can be classified as either direct drive or geared drive groups. To increase the generator rotor rotating speed to gain a higher power output, a regular geared drive wind turbine typically uses a multi-stage gearbox to take the rotational speed from the low-speed shaft of the blade rotor and transform it into a fast rotation on the high-speed shaft of the generator rotor. The advantages of geared generator systems include lower cost and smaller size and weight. However, utilization of a gearbox can significantly lower wind turbine reliability and increase turbine noise level and mechanical losses [11].

By eliminating the multi-stage gearbox from a generator system, the generator shaft is directly connected to the blade rotor. Therefore, the direct-drive concept is more superior in terms of energy efficiency, reliability, and design simplicity.

6.6 On-grid and off-grid wind turbines

Wind turbines can be used for either on-grid or off-grid applications. Most medium-size and almost all large-size wind turbines are used in grid tied applications. One of the obvious advantages for on-grid wind turbine systems is that there is no energy storage problem.

As the contrast, most of small wind turbines are off-grid for residential homes, farms, telecommunications, and other applications. However, as an intermittent power source, wind power produced from off-grid wind turbines may change dramatically over a short period of time with little warning. Consequently, off-grid wind turbines are usually used in connection with batteries, diesel generators, and photovoltaic systems for improving the stability of wind power supply.

6.7 Onshore and offshore wind turbines

Onshore wind turbines have a long history on its development. There are a number of advantages of onshore turbines, including lower cost of foundations, easier integration with the electrical-grid network, lower cost in tower building and turbine installation, and more convenient access for operation and maintenance.

Offshore wind turbines have developed faster than onshore since the 1990s due to the excellent offshore wind resource, in terms of wind power intensity and continuity. A wind turbine installed offshore can make higher power output and operate more hours each year compared with the same turbine installed onshore. In addition, environmental restrictions are more lax at offshore sites than at onshore sites. For instance, turbine noise is no long an issue for offshore wind turbines.

6.8 Wind turbine configuration

Most of the modern large wind turbines are horizontal-axis turbines with typically three blades [9]. As shown in Fig.4, a wind turbine is comprised of a nacelle, which is positioned on the top of a wind tower, housing the most turbine components inside. Three blades (not shown) mounted on the rotor hub, which is connected via the main shaft to the gearbox. The rotor of the wind generator is connected to the output shaft of the gearbox. Thus, the slow rotating speed of the rotor hub is increased to a desired high rotating speed of the generator rotor.

Using the pitch control system, each blade is pitched individually to optimize the angle of attack of the blade for allowing a higher energy capture in normal operation and for protecting the turbine components (blade, tower, etc.) from damaging in emergency situations. With the feedback information such as measured instantaneous wind direction and speed from the wind vane, the yaw control system provides the yaw orientation control for ensuring the turbine constantly against the wind.

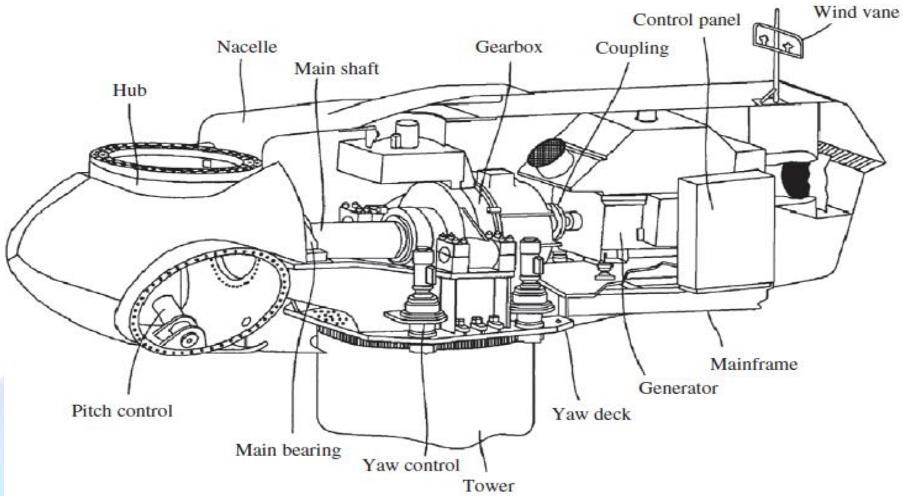

Fig.4 A horizontal-axis wind turbine configuration.

7. Wind power parameters

7.1 Power coefficient

The conversion of wind energy to electrical energy involves primarily two stages: in the first stage, kinetic energy in wind is converted into mechanical energy to drive the shaft of a wind generator. The critical converting devices in this stage are wind blades. For maximizing the capture of wind energy, wind blades need to be carefully designed.

The power coefficient C_p deals with the converting efficiency in the first stage, defined as the ratio of the actually captured mechanical power by blades to the available power in wind [4]:

$$C_P = \frac{P_{me,out}}{P_W} = \frac{P_{me,out}}{(1/2)\rho A \bar{u}^3} \qquad (12)$$

Because there are various aerodynamic losses in wind turbine systems, for instance, blade-tip, blade-root, profile, and wake rotation losses, etc., the real power coefficient C_p is much lower than its theoretical limit, usually ranging from 30 to 45%.

7.2 Total power conversion coefficient and effective power output

In the second stage, mechanical energy captured by wind blades is further converted into electrical energy via wind generators [12]. In this stage, the converting efficiency is determined by several parameters

- **Gearbox efficiency η_{gear}** – The power losses in a gearbox can be classified as load-dependent and no-load power losses. The load-dependent losses consist of gear tooth friction and bearing losses and no-load losses consist of oil churning, windage, and shaft seal losses. The planetary gearboxes, which are widely used in wind turbines, have higher power transmission efficiencies over traditional gearboxes.
- **Generator efficiency η_{gen}** – It is related to all electrical and mechanical losses in a wind generator, such as copper, iron, load, windage, friction, and other miscellaneous losses.
- **Electric efficiency η_{ele}** – It encompasses all combined electric power losses in the converter, switches, controls, and cables.

  Therefore, the total power conversion efficiency from wind to electricity η_t is the production of these parameters, i.e.:

$$\eta_t = C_P \eta_{gear}\, \eta_{gen}\, \eta_{ele} \qquad (13)$$

The effective power output from a wind turbine to feed into a grid becomes

$$P_{eff} = C_P \eta_{gear}\, \eta_{gen}\, \eta_{ele}\, P_W = \eta_t P_w = \frac{1}{2}(\eta_t \rho A \bar{u}^3) \qquad (14)$$

7.3 Lanchester–Betz Law

The theoretical maximum efficiency of an ideal wind turbo machine was derived by Lanchester in 1915 and Betz in 1920. It was revealed that no wind turbo machines could convert more than 16/27 (59.26%) of the kinetic energy of wind into mechanical energy. This is known as Lanchester–Betz limit (or Lanchester– Betz law) today [2]. As shown in Fig.5, $\bar{u}_1$ and $\bar{u}_4$ are mean velocities far upstream and downstream from the wind turbine; $\bar{u}_2$ and $\bar{u}_3$ are mean velocities just in front and back of the wind rotating blades, respectively. By assuming that there is no change in the air velocity right across the wind blades (i.e. $\bar{u}_2$=$\bar{u}_3$) and the pressures far upstream and downstream from the wind turbine are equal to the static pressure of the undisturbed airflow (i.e. $p_1 = p_4 = p$), it can be derived that

$$P_2 - P_3 = \frac{1}{2}\rho(\bar{u}_1^2 - \bar{u}_4^2) \quad (15)$$

$$\bar{u}_2 = \bar{u}_3 = \frac{1}{2}(\bar{u}_1 + \bar{u}_4) \quad (16)$$

Thus, the power output of mechanical energy captured by wind turbine blades is

$$P_{me,out} = \frac{1}{2}\rho A\bar{u}_2(\bar{u}_1^2 - \bar{u}_4^2) = \frac{1}{2}\rho A\bar{u}_1^3 4a(1-a)^2 \quad (17)$$

Where a is the axial induction factor, defined as

$$a = \frac{\bar{u}_1 - \bar{u}_2}{\bar{u}_1} \quad (18)$$

Substitute equation (17) into (12) (where $\bar{u}_1 = \bar{u}_2$), yields

$$C_P = 4a(1-a)^2 \quad (19)$$

This indicates that the power coefficient is only a function of the axial induction factor *a.* It is easy to derive that the maximum power coefficient reaches its maximum value of 16/27 when *a* = 1/3 (see Fig.6).

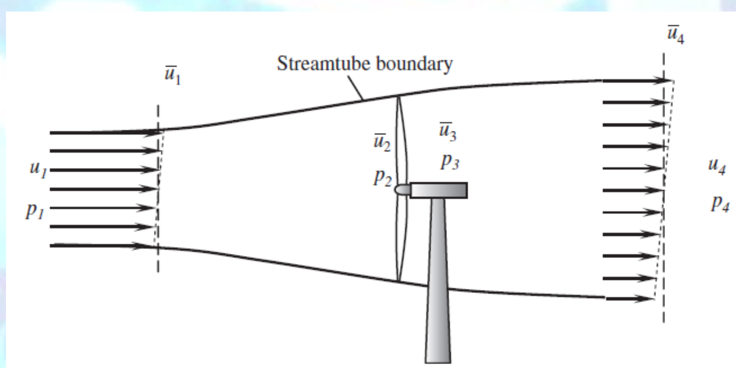

Fig.5 Airflow through a wind turbine.

7.4 Power curve

As can be seen from equation (14), the effective electrical power output from a wind turbine P_{eff} is directly proportional to the available wind power P_w and the total effective wind turbine efficiency η_t . The power curve of a wind turbine displays the power output (either the real electrical power output or the percentage of the rated power) of the turbine as a function of the mean wind speed. Power curves are usually determined from the field measurements.

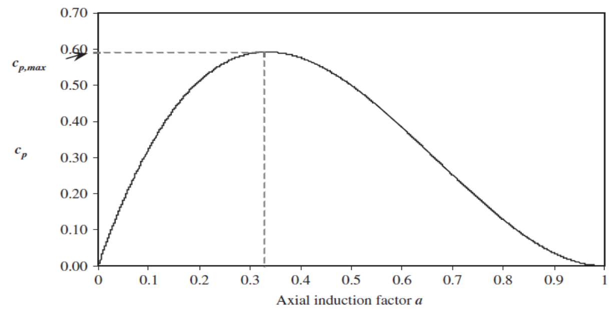

Fig.6: Power coefficient as a function of axial induction factor *a*.

As shown in Fig.7, the wind turbine starts to produce usable power at a low wind speed, defined as the cut-in speed. The power output increases continuously with the increase of the wind speed until reaching a saturated point, to which the power output reaches its maximum value, defined as the rated power output. Correspondingly, the speed at this point is defined as the rated speed. At the rated speed, more increase in the wind speed will not increase the power output due to the activation of the power control. When the wind speed becomes too large to potentially damage the wind turbine, the wind turbine needs to shut down immediately to avoid damaging the wind turbine. This wind speed is defined as the cut-out speed. Thus, the cut-in and cut-out speeds have defined the operating limits of the wind turbine.

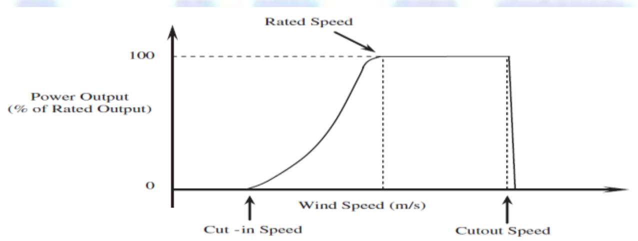

Fig.7: Typical wind turbine power curve.

7.5 Tip speed ratio

The tip speed ratio is an extremely important factor in wind turbine design, which is defined as the ratio of the tangential speed at the blade tip to the actual wind speed, i.e.:

$$\lambda = \frac{(l+r)\omega}{\bar{u}} \quad (20)$$

Where *l* is the length of the blade, *r* is the radius of the hub, and ꙍis the angular speed of blades.

If the blade angular speed *w* is too small, most of the wind may pass undisturbed though the blade swept area making little useful work on the blades [10]. On the contrary, if ꙍis too large, the fast rotating blades may block the wind flow reducing the power extraction. Therefore, there exists an optimal angular speed at which the maximum power extraction is achieved. For a wind turbine with *n* blades, the optimal angular speed can be approximately determined as:

$$\omega_{opt} \approx \frac{2\pi}{n}\frac{\bar{u}}{L} \quad (21)$$

where*L* is the length of the strongly disturbed air stream upwind and downwind of the rotor.

Substituting eqn (21) into (20), the optimal tip speed ratio becomes

$$\lambda_{opt} \approx \frac{2\pi}{n}\left(\frac{l+r}{L}\right) \quad (22)$$

Empirically, the ratio (*l*+ *r*)/ *L* is equal to about 2. Thus, for three-blade wind turbines (i.e. *n* = 3),$\lambda_{opt} \approx \frac{4\pi}{3}$.

If the aerofoil blade is designed with care, the optimal tip speed ratio may be about 25–30% higher than the calculated optimal values above [6]. Therefore, a wind turbine with three blades would have an optimal tip speed ratio

$$\lambda_{opt} = \frac{4\pi}{3}(1.25{\sim}1.30) \approx 5.24{\sim}5.45 \qquad (23)$$

7.6 Wind turbine capacity factor

Due to the intermittent nature of wind, wind turbines do not make power all the time. Thus, a capacity factor of a wind turbine is used to provide a measure of the wind turbines actual power output in a given period (e.g. a year) divided by its power output if the turbine has operated the entire time. A reasonable capacity factor would be 0.25–0.30 and a very good capacity factor would be around 0.40 [5].In fact, wind turbine capacity factor is very sensitive to the average wind speed.

8. Conclusions

The most promising energy source, wind energy is believed to play a critical role in global power supply in the 21st century.Renewable energy sources like wind energy (commonly recognized to be a clean and environmentally friendly) is indigenous and can help in reducing the dependency on fossil fuels. Its mature technology and comparatively low cost make it promising as an important primary energy source in the future. Due to the fast market development, wind turbine technology has experienced an important evolution over time. From this survey it known that the vertical axis wind turbine appears to be advantageous to the horizontal axis wind turbine in energy harvesting. Again from this review one can revealed the cut-in and cut-out speeds have defined the operating limits of the wind turbine and also the wind turbine capacity factor is very sensitive to the average wind speed.Based on these mathematical formulations one can analyses that the wind power output is proportional to the cubic power of the mean wind speed, a small variation in wind speed can result in a large change in wind power and the density of air decreases nonlinearly with the height above the sea level. The tip speed ratio is an extremely important factor in wind turbine design. Therefore, there exist optimal angular speeds at which the maximum power extraction is achieved and the wind energy is harvested.

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