Stoneley Waves At The Interface Of Thermoelastic Diffusion Solid And Microstretch Thermoelastic Diffusion Solid

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Abstract

In this paper, dispersion equation for Stoneley waves at the interface of thermoplastic diffusion solid half space and micro stretch thermoplastic diffusion solid half space have been derived. Numerical computations are performed for a peculiar model to study the variations of phase velocity and attenuation coefficient with respect to wave number. Also the components of normal stress, normal displacement and temperature distribution are obtained and shown with the aid of graphs. Some specific cases are also deduced.

Keywords: Thermoelastic diffusion; Micro stretch; Dispersion equation; Stoneley waves; Propagation characteristics.

1. Introduction

Thermoelasticity covers a broad field of developments. It consists of the theory of heat transfer and the theory of strains and stresses due to heat flow, when coupling of temperature and strain fields occur. Thermoelasticity makes it possible to determine the stresses caused by the temperature field, and moreover to calculate the scattering of temperature due to operation of internal forces which vary with time.Besides the contradiction of infinite propagation speeds, the classical dynamic thermoelasticity theory offers unsatisfactory description of a solids response to a fast transient loading and at low temperatures. Such drawbacks have led many researchers to advance various generalized thermoelasticity theories and they proposed thermoelastic models with one or two relaxation times, model focused on two temperatures , models with absence of energy dissipation, a dual -phase-lag theory , even anomalous heat conduction described by fractional calculus or non local thermoelastic models.Surface wave propagation for thermoelastic orthotropic granular media under the influence of initial magnetic field was investigated by Ahmed and Abo-Dahab [1]. Kumar,Garg and Ahuja [2] obtained the reflection and transmission coefficients at the boundary surface of elastic and microstretch thermoelastic diffusion media.Abd-Alla, Khan, and Abo-Dahab[3] and Kumar and Kansal [4] investigated various problems surface waves in viscoelastic fibre-reinforced anisotropic media with voids and in transversely isotropic thermoelastic diffusive plate.Eringen [5] explored microcontinuum field theories. Kumar and Chawla[6] studied wave propagation at the imperfect boundary between the layers of transversely isotropic thermoelastic half-space with diffusion and an isotropic layer .In this paper, dispersion equation for Stoneley waves at the interface of thermoelastic diffusion solid half space and microstretch thermoelastic diffusion solid half space have been derived. Numerical computations are performed for a peculiar model to study the variations of phase velocity and attenuation coefficient with respect to wave number. Also the components of normal stress, normal displacement and temperature distribution are obtained and shown with the aid of graphs. Some specific cases are also deduced.

2. Basic Equations

Following [7,8,9], the governing equations for the problem under consideration can be taken

$$(\lambda+2\mu+K)grad(div\,\vec{u})-(\mu+K)curl(curl\,\vec{u})+K\,curl\,\vec{\varphi}+\lambda_0 grad\,\varphi^*$$
$$-\beta_1\left(gradT+\tau_1\,grad\,\dot{T}\right)-\beta_2\left(gradC+\tau^1 grad\dot{C}\right)=\rho\frac{\partial^2\vec{u}}{\partial t^2}, \quad (1)$$

$$(\alpha+\beta+\gamma)\nabla(div\,\vec{\varphi})-\gamma\,curl(curl\,\vec{\varphi})+K\,curl\,\vec{u}-2K\vec{\varphi}=\rho j\,\frac{\partial^2\vec{\varphi}}{\partial t^2}, \quad (2)$$

$$\alpha_0\nabla^2\varphi^*+\eta_1\left(1+\tau_1\frac{\partial}{\partial t}\right)T+\eta_2\left(1+\tau^1\frac{\partial}{\partial t}\right)C-\lambda_1\varphi^*-\lambda_0 div\,\vec{u}=\rho j_0\frac{\partial^2\varphi^*}{\partial t^2}, \quad (3)$$

$$K^*\nabla^2 T=\beta_1 T_0\left(\frac{\partial}{\partial t}+\varepsilon\tau_0\frac{\partial^2}{\partial t^2}\right)div\,\vec{u}+\eta_1 T_0\left(\frac{\partial}{\partial t}+\varepsilon\tau_0\frac{\partial^2}{\partial t^2}\right)\varphi^*+\rho C^*\left(\frac{\partial}{\partial t}+\tau_0\frac{\partial^2}{\partial t^2}\right)T$$
$$+aT_0\left(\frac{\partial}{\partial t}+\gamma_1\frac{\partial^2}{\partial t^2}\right)C, \quad (4)$$

$$D\beta_2\nabla^2(div\,\vec{u})+D\eta_2\nabla^2\varphi^*+Da\nabla^2\left(1+\tau_1\frac{\partial}{\partial t}\right)T+\left(\frac{\partial}{\partial t}+\varepsilon\tau^0\frac{\partial^2}{\partial t^2}\right)C$$
$$-Db\nabla^2\left(1+\tau^1\frac{\partial}{\partial t}\right)C=0. \quad (5)$$

and constitutive relations are

$$t_{ij}=\lambda u_{k,k}\delta_{ij}+\mu\left(\frac{\partial u_i}{\partial x_j}+\frac{\partial u_j}{\partial x_i}\right)+K\left(\frac{\partial u_j}{\partial x_i}-\varepsilon_{ijl}\varphi_l\right)+\lambda_0\delta_{ij}\varphi^*-\beta_1\left(1+\tau_1\frac{\partial}{\partial t}\right)T\delta_{ij}$$
$$-\beta_2\left(1+\tau^1\frac{\partial}{\partial t}\right)C\delta_{ij}, \quad (6)$$

$$m_{ij} = \alpha\varphi_{k,k}\delta_{ij} + \beta\frac{\partial\varphi_i}{\partial x_j} + \gamma\frac{\partial\varphi_j}{\partial x_i} + b_0\varepsilon_{kij}\frac{\partial\varphi^*}{\partial x_k} \quad , \tag{7}$$

$$\lambda_k^* = \alpha_0\frac{\partial\varphi^*}{\partial x_k} + \; b_0\varepsilon_{kjm}\frac{\partial\varphi_j}{\partial x_m}, \tag{8}$$

where

$K,\alpha_0,\lambda_{1,}\lambda_0,\beta,\gamma,b_0,\alpha, \rightarrow$ material constants , $\vec{\varphi} = (\varphi_1,\varphi_2,\varphi_3) \rightarrow$ microrotation vector, $\varphi^* \rightarrow$ scalar microstretch function., $\beta_1 = (3\lambda + 2\mu + K)\alpha_{t1}$, $\beta_2 = (3\lambda + 2\mu + K)\alpha_{c1}, \eta_1 = (3\lambda + K + 2\mu)\alpha_{t2}$, $\eta_1 = (3\lambda + K + 2\mu)\alpha_{c2}, \alpha_{t2,}\alpha_{t1} \rightarrow$ coefficients of thermal expansion(linear), $\alpha_{c1},\alpha_{c2} \rightarrow$ coefficients of diffusion expansion (linear), $j \rightarrow$ microintertia, $j_o \rightarrow$ microinertia of the microelements, m_{ij} $\rightarrow$ components of couple stress tensors respectively, $\lambda_i^* \rightarrow$ microstress tensor, $\tau^1,\tau^0 \rightarrow$ diffusion relaxation times with $0 \le \tau^0 \le \tau^1$, $\tau_1,\tau_0 \rightarrow$ thermal relaxation times with $0 \le \tau_0 \le \tau_1, K^* \rightarrow$ coefficient of the thermal conductivity.

Here for coupled thermoelastic model $\tau_0 = \gamma_1 = \tau^0 = \tau_1 = \tau^1 = 0$,

Lord-Shulman model requires $\begin{matrix}\tau^1 = \tau_1 = 0, \\ \tau_0 = \gamma_1, \varepsilon = 1.\end{matrix}$ $\varepsilon = 1, \gamma_1 = \tau_0$,

Green-Lindsay (G-L) model requires

$\varepsilon = 0,\ \gamma_1 = \tau^0$ where $\tau^0 > 0$.

3 Thermoelastic with diffusion

Following [8], the governing equations for a homogeneous isotropic thermoelastic diffusion solid not possessing body forces, heat sources and mass diffusion sources are given by

$$(\bar{\lambda} + 2\bar{\mu})\nabla(div\,\vec{\bar{u}}) - \bar{\mu}curl(curl\,\vec{\bar{u}}) - \bar{\beta}_1\left(grad\bar{T} + \bar{\tau}_1\,grad\,\dot{\bar{T}}\right) - \bar{\beta}_2\left(grad\bar{C} \;+\; \bar{\tau}^1 grad\dot{\bar{C}}\right) = \rho\frac{\partial^2\vec{\bar{u}}}{\partial t^2} \quad , \tag{9}$$

$$\overline{K^*}\nabla^2\bar{T} = \bar{\beta}_1\bar{T}_0\left(\frac{\partial}{\partial t} + \bar{\varepsilon}\bar{\tau}_0\frac{\partial^2}{\partial t^2}\right)div\,\vec{\bar{u}} + \rho\bar{C}^*\left(\frac{\partial}{\partial t} + \bar{\tau}_0\frac{\partial^2}{\partial t^2}\right)\bar{T} + \bar{a}\bar{T}_0\left(\frac{\partial}{\partial t} + \bar{\gamma}_1\frac{\partial^2}{\partial t^2}\right)\bar{C}, \tag{10}$$

$$\bar{D}\beta_2\nabla^2(div\,\vec{\bar{u}}) + \bar{D}\bar{a}\nabla^2\left(1 + \bar{\tau}_1\frac{\partial}{\partial t}\right)\bar{T} + \left(\frac{\partial}{\partial t} + \bar{\varepsilon}\bar{\tau}^0\frac{\partial^2}{\partial t^2}\right)\bar{C} - \bar{D}\bar{b}\nabla^2\left(1 + \bar{\tau}^1\frac{\partial}{\partial t}\right)\bar{C} = 0,$$

(11)

and constitutive relations are

$$t_{ij} = \bar{\lambda}\bar{u}_{k,k}\delta_{ij} + \bar{\mu}\left(\frac{\partial \bar{u}_i}{\partial x_j} + \frac{\partial \bar{u}_j}{\partial x_i}\right) - \bar{\beta}_1\delta_{ij}(\bar{T} + \bar{\tau}_1\dot{\bar{T}} \quad - \bar{\beta}_2\delta_{ij}\left(\bar{C} + \bar{\tau}^1\dot{\bar{C}}\right), \qquad (12)$$

4. Problem formulation

We take a homogeneous isotropic generalized thermoelastic diffusion half-space M_1 overlying a homogeneous isotropic microstretch generalized thermoelastic diffusion half-space M_2 connecting at the interface $x_3 = 0$. The origin of the coordinate system (x_1, x_2, x_3) is taken at any point on the plane horizontal surface $x_3 = 0$. We choose the x_1-axis in the direction of wave propagation in such a way that all the particles on a line parallel to the x_2 axis are equally displaced. Therefore, all field quantities are independent of the x_2 coordinate. Medium M_2 occupies the region $-\infty < x_3 \le 0$ and the region $0 \le x_3 < \infty$ is occupied by the half-space of M_1. The plane $x_3 = 0$ represents the interface between two media M_1 and M_2.

We define all the quantities with attached bar for medium M_1 and without bar for medium M_2.

For the two dimensional problem, we take

For medium M_2

$$\vec{u}(x_1, x_3, t) = (u_1, 0, u_3), \ \vec{\varphi} = (0, \varphi_2, 0), \varphi^* = \varphi^*(x_1, x_3, t),$$
$$\mathrm{T} = \mathrm{T}(x_1, x_3, t), C = C(x_1, x_3, t), \qquad (13)$$

and, for medium M_1

$$\vec{\bar{u}}(x_1, x_3, t) = (\bar{u}_1, 0, \bar{u}_3), \ \bar{\mathrm{T}} = \bar{\mathrm{T}}(x_1, x_3, t), \bar{C} = \bar{C}(x_1, x_3, t), \qquad (14)$$

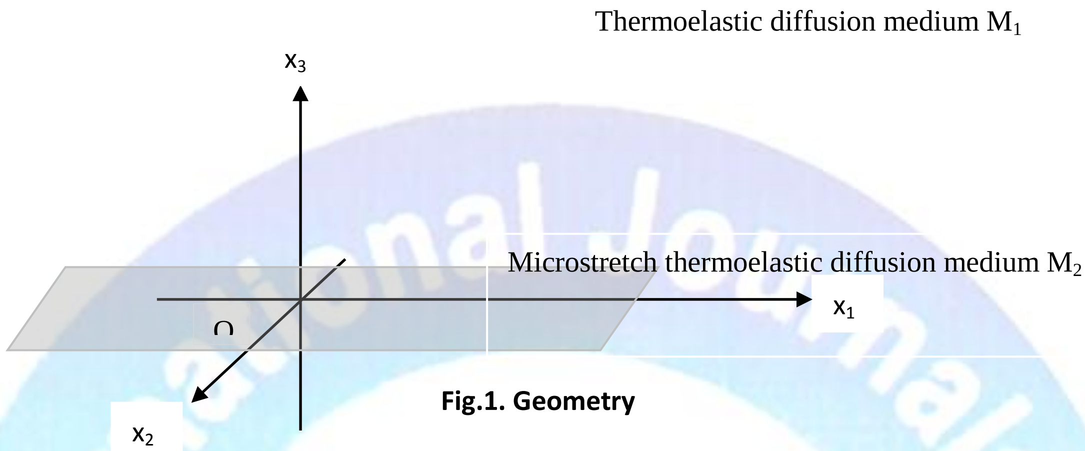

Fig.1. Geometry

Define the dimensionless quantities as

$$x_1' = \frac{\omega^* x_1}{c_1}, x_3' = \frac{\omega^* x_3}{c_1}, u_1' = \frac{\rho c_1 \omega^* u_1}{\beta_1 T_o}, u_3' = \frac{\rho c_1 \omega^* u_3}{\beta_1 T_o}, \varphi_2' = \frac{\rho c_1^2 \varphi_2}{\beta_1 T_o}, \varphi^{*'} = \frac{\rho c_1^2}{\beta_1 T_o}\varphi^*, t_{ij}' = \frac{t_{ij}}{\beta_1 T_o},$$

$$\bar{t}_{ij}' = \frac{\bar{t}_{ij}}{\beta_1 T_o}, m_{ij}' = \frac{\omega^* m_{ij}}{c_1 \beta_1 T_o}, \lambda_i^{*'} = \frac{\lambda_i^* \omega^*}{c_1 \beta_1 T_o}, T' = \frac{T}{T_o}, \bar{T}' = \frac{\bar{T}}{T_o}, C' = \frac{\beta_2 C}{\rho c_1^2}, \bar{C}' = \frac{\beta_2 \bar{C}}{\rho c_1^2}, t' = \omega^* t, \quad (15)$$

$$\left(\tau_o', \tau^{0'}, \tau_1', \tau^{1'}\right) = \omega^*\left(\tau_o, \tau^0, \tau_1, \tau^1\right), \left(\bar{u}_1', \bar{u}_3'\right) = \frac{\rho c_1 \omega^*}{\beta_1 T_o}\left(\bar{u}_1, \bar{u}_3\right).$$

where

$$\omega^* = \frac{\rho C^* c_1^2}{K^*}, \; c_1^2 = \frac{\lambda + 2\mu + K}{\rho},$$

The potential functions $\bar{\phi}, \bar{\psi}, \phi \, and \, \psi$ are introduced as

For medium M_1 $\quad \bar{u}_1 = \bar{\phi}_{,1} - \bar{\psi}_{,3}, u_3 = \bar{\phi}_{,3} + \bar{\psi}_{,1}, \quad$ (16)

and, for medium M_2 $\quad u_1 = \phi_{,1} - \psi_{,3}, u_3 = \phi_{,3} + \psi_{,1}, \quad$ (17)

Putting the values of u_1 and u_3 from (16) in the equations (9)-(11), we obtain

$$\nabla^2\bar{\phi}-\bar{\tau}_t^1\bar{T}-\bar{a}_3\bar{\tau}_c^1\bar{C}=\ddot{\bar{\phi}}, \tag{18}$$

$$\left(1-\bar{\delta}^2\right)\nabla^2\bar{\psi}=\ddot{\bar{\psi}}, \tag{19}$$

$$\nabla^2\bar{T}=\bar{\tau}_e^0\bar{a}_{11}\nabla^2\dot{\bar{\phi}}+\bar{\tau}_t^0\dot{\bar{T}}+\bar{a}_{13}\bar{\tau}_c^0\dot{\bar{C}}, \tag{20}$$

$$\bar{a}_{14}\nabla^4\bar{\phi}+\bar{a}_{15}\bar{\tau}_t^1\nabla^2\bar{T}-\bar{a}_{16}\bar{\tau}_c^1\nabla^2\bar{C}+\bar{\tau}_f^0\dot{\bar{C}}=0 \tag{21}$$

Substituting the values of u_1 and u_3 from (17) in the equations (1)-(5), we obtain

$$\nabla^2\phi+a_2\varphi^*-\tau_t^1T-a_3\tau_c^1C=\ddot{\phi}, \tag{22}$$

$$\left(1-\delta^2\right)\nabla^2\psi+a_1\varphi_2=\ddot{\psi}, \tag{23}$$

$$\left(a_4\nabla^2-a_6\right)\varphi_2-a_5\nabla^2\psi=\ddot{\varphi}_2, \tag{24}$$

$$\left(\delta_1^2\nabla^2-a_7\right)\varphi^*-a_8\nabla^2\phi+a_9\tau_t^1T+a_{10}\tau_c^1C=\ddot{\varphi}^*, \tag{25}$$

$$\nabla^2T=\tau_e^0\left(a_{11}\nabla^2\dot{\phi}+a_{12}\dot{\varphi}^*\right)+\tau_t^0\dot{T}+a_{13}\tau_c^0\dot{C}, \tag{26}$$

$$a_{14}\nabla^4\phi+a_{21}\nabla^2\varphi^*+a_{15}\tau_t^1\nabla^2T-a_{16}\tau_c^1\nabla^2C+\tau_f^0\dot{C}=0, \tag{27}$$

where

$$\left(a_1,a_2\right)=\frac{1}{\rho c_1^2}\left(K,\lambda_0\right),a_3=\frac{\rho c_1^2}{\beta_1T_0},\left(a_4,a_5,a_6\right)=\frac{1}{j\rho}\left(\frac{\gamma}{c_1^2},\frac{K}{\omega^{*2}},\frac{2K}{\omega^{*2}}\right),\delta^2=\frac{\lambda+\mu}{\rho c_1^2},$$

$$\left(a_{11},a_{12},a_{13}\right)=\frac{1}{K^*\omega^*}\left(\frac{T_0\beta_1^2}{\rho},\frac{\beta_1T_0\nu_1}{\rho},\frac{\rho c_1^4a}{\beta_2}\right),\left(a_{14},a_{15},a_{16}\right)=\frac{D\omega^*}{c_1^2}\left(\frac{\beta_1^2}{\rho c_1^2},\frac{\beta_2a}{\beta_1},b\right),$$

$$\left(a_7,a_8,a_9,a_{10}\right)=\frac{2}{j_0\omega^{*2}}\left(\frac{\lambda_1}{\rho},\frac{\lambda_0}{\rho},\frac{\nu_1c_1^2}{\beta_1},\frac{\nu_2\rho c_1^4}{\beta_1\beta_2T_0}\right),\delta_1^2=\frac{c_2^2}{c_1^2},c_2^2=\frac{2\alpha_0}{\rho j_0},a_{21}=\frac{D\nu_2\beta_2\omega^*}{\rho c_1^4},$$

$$\tau_c^1=\tau^1\frac{\partial}{\partial t}+1,\tau_t^1=\tau_1\frac{\partial}{\partial t}+1,\tau_t^0=\tau_0\frac{\partial}{\partial t}+1,\tau_f^0=\varepsilon\tau^0\frac{\partial}{\partial t}+1,$$

$$\tau_c^0=1+\gamma_1\frac{\partial}{\partial t},\tau_e^0=1+\varepsilon\tau_0\frac{\partial}{\partial t},e=\frac{\partial u_1}{\partial x_1}+\frac{\partial u_3}{\partial x_3},\nabla^2=\frac{\partial^2}{\partial x_1^2}+\frac{\partial^2}{\partial x_3^2}.$$

5 Solution of the Problem

We assume the solutions of the form

$$\bar{\phi}=\tilde{\phi}e^{i\xi(x_1-ct)},\bar{T}=\tilde{T}e^{i\xi(x_1-ct)},\bar{C}=\tilde{C}e^{i\xi(x_1-ct)},$$
$$\bar{\psi}=\tilde{\psi}e^{i\xi(x_1-ct)},\phi=\hat{\phi}e^{i\xi(x_1-ct)},\psi=\hat{\psi}e^{i\xi(x_1-ct)},$$
$$\phi_2=\hat{\phi}_2e^{i\xi(x_1-ct)},\varphi^*=\hat{\varphi}^*e^{i\xi(x_1-ct)},T=\hat{T}e^{i\xi(x_1-ct)},\quad .$$
$$C=\hat{C}e^{i\xi(x_1-ct)}. \tag{28}$$

where ξ is the wave number, $\omega=\xi c$ is the angular frequency, and c is phase velocity of the wave. Using (28) in equations (18), (20)–(21) and satisfying the radiation condition $\bar{\phi},\bar{T},\bar{C}\to 0$ as $x_3\to-\infty$, we obtain the values of $\bar{\phi},\bar{T},\bar{C}$ for medium M_1,

$$\bar{\phi}=(\bar{A}_1e^{-\bar{m}_1x_3}+\bar{A}_2e^{-\bar{m}_2x_3}+\bar{A}_3e^{-\bar{m}_3x_3})e^{i\xi(x_1-ct)},$$
$$\bar{T}=(\bar{A}_1\bar{n}_{11}e^{-\bar{m}_1x_3}+\bar{A}_2\bar{n}_{12}e^{-\bar{m}_2x_3}+\bar{A}_3\bar{n}_{13}e^{-\bar{m}_3x_3})e^{i\xi(x_1-ct)}, \tag{29}$$
$$\bar{C}=(\bar{A}_1\bar{n}_{21}e^{-\bar{m}_1x_3}+\bar{A}_2\bar{n}_{22}e^{-\bar{m}_2x_3}+\bar{A}_3\bar{n}_{23}e^{-\bar{m}_3x_3})e^{i\xi(x_1-ct)}$$

where

$\bar{m}_1^2,\bar{m}_2^2,\bar{m}_3^2$ are the roots of the equation

$$D^6+\bar{A}_1^*D^4+\bar{B}_1^*D^2+\bar{C}_1^*=0, \tag{30}$$

$$\bar{A}_1^*=\bar{d}_{12}/\bar{d}_{11},\bar{B}_1^*=\bar{d}_{13}/\bar{d}_{11},\bar{C}_1^*=\bar{d}_{14}/\bar{d}_{11},$$
$$\bar{d}_{11}=-\left(\bar{l}_{11}+\bar{b}_{13}l_{18}\right),\bar{d}_{12}=-\bar{l}_{12}+\bar{b}_{11}\bar{l}_{11}-\bar{b}_{12}\bar{l}_{15}+\bar{b}_{13}\left(\xi^2\bar{l}_{18}-\bar{l}_{19}\right)-\bar{a}_2\bar{l}_{22},$$
$$\bar{d}_{13}=-\bar{l}_{13}+\bar{b}_{11}\bar{l}_{12}+\bar{b}_{12}\left(\xi^2\bar{l}_{15}-\bar{l}_{16}\right)+\bar{b}_{13}\left(\xi^2\bar{l}_{19}-\bar{l}_{20}\right)-\bar{a}_2\xi^2\bar{l}_{22}-\bar{a}_2\bar{l}_{23},$$
$$\bar{d}_{14}=-\bar{l}_{14}+\bar{b}_{11}\bar{l}_{13}+\bar{b}_{12}\left(\xi^2\bar{l}_{16}-\bar{l}_{17}\right)+\bar{b}_{13}\left(\xi^2\bar{l}_{20}-\bar{l}_{21}\right)+\bar{a}_2\bar{l}_{23},$$

$\bar{A}_1,\bar{A}_2,\bar{A}_3$ are arbitrary constants and $D=\dfrac{d}{dx_3}$.

The coupling constants $\bar{n}_{11},\bar{n}_{12},\bar{n}_{13},\bar{n}_{21},\bar{n}_{22},\bar{n}_{23}$, are given by

$$\bar{n}_{11} = -\left[-\bar{l}_{22}\bar{m}_1^6 + \left(\xi^2\bar{l}_{22} - \bar{l}_{23}\right)\bar{m}_1^4 + \left(\xi^2\bar{l}_{23} - \bar{l}_{24}\right)\bar{m}_1^2\right] \Big/ \left[\bar{l}_{11}\bar{m}_1^6 + \bar{l}_{12}\bar{m}_1^4 + \bar{l}_{13}\bar{m}_1^2\right],$$
$$\bar{n}_{12} = -\left[-\bar{l}_{22}\bar{m}_2^6 + \left(\xi^2\bar{l}_{22} - \bar{l}_{23}\right)\bar{m}_2^4 + \left(\xi^2\bar{l}_{23} - \bar{l}_{24}\right)\bar{m}_2^2\right] \Big/ \left[\bar{l}_{11}\bar{m}_2^6 + \bar{l}_{12}\bar{m}_2^4 + \bar{l}_{13}\bar{m}_2^2\right],$$
$$\bar{n}_{13} = -\left[-\bar{l}_{22}\bar{m}_3^6 + \left(\xi^2\bar{l}_{22} - \bar{l}_{23}\right)\bar{m}_3^4 + \left(\xi^2\bar{l}_{23} - \bar{l}_{24}\right)\bar{m}_3^2\right] \Big/ \left[\bar{l}_{11}\bar{m}_3^6 + \bar{l}_{12}\bar{m}_3^4 + \bar{l}_{13}\bar{m}_3^2\right],$$
$$\bar{n}_{21} = \left[-\bar{l}_{15}\bar{m}_1^6 + \left(\xi^2\bar{l}_{15} - \bar{l}_{16}\right)\bar{m}_1^4 + \left(\xi^2\bar{l}_{16} - \bar{l}_{17}\right)\bar{m}_1^2 + \xi^2\bar{l}_{17}\right] \Big/ \left[\bar{l}_{11}\bar{m}_1^6 + \bar{l}_{12}\bar{m}_1^4 + \bar{l}_{13}\bar{m}_1^2\right],$$
$$\bar{n}_{22} = \left[-\bar{l}_{15}\bar{m}_2^6 + \left(\xi^2\bar{l}_{15} - \bar{l}_{16}\right)\bar{m}_2^4 + \left(\xi^2\bar{l}_{16} - \bar{l}_{17}\right)\bar{m}_2^2 + \xi^2\bar{l}_{17}\right] \Big/ \left[\bar{l}_{11}\bar{m}_2^6 + \bar{l}_{12}\bar{m}_2^4 + \bar{l}_{13}\bar{m}_2^2\right],$$
$$\bar{n}_{23} = \left[-\bar{l}_{15}\bar{m}_3^6 + \left(\xi^2\bar{l}_{15} - \bar{l}_{16}\right)\bar{m}_3^4 + \left(\xi^2\bar{l}_{16} - \bar{l}_{17}\right)\bar{m}_3^2 + \xi^2\bar{l}_{17}\right] \Big/ \left[\bar{l}_{11}\bar{m}_3^6 + \bar{l}_{12}\bar{m}_3^4 + \bar{l}_{13}\bar{m}_3^2\right],$$

and

$$\bar{l}_{11} = \bar{b}_{23}\bar{\delta}_1^2,\ \bar{l}_{12} = \bar{\delta}_1^2(\bar{b}_{19}\bar{b}_{22} - \bar{g}_{12}) + \bar{b}_{23}\bar{g}_{14},\ \bar{l}_{13} = -\bar{b}_{22}\xi^2(\bar{b}_{19}\bar{\delta}_1^2 + \bar{g}_{11}) - \bar{g}_{12}\bar{g}_{14} + \bar{b}_{23}\bar{g}_{13} + \bar{a}_{21}\left(\bar{g}_{15} - \bar{b}_{15}\right),$$
$$\bar{l}_{14} = \xi^2\left(\bar{b}_{22}\bar{g}_{11} - \bar{a}_{21}\bar{b}_{15}\right) - \bar{g}_{12}\bar{g}_{13} - \bar{a}_2\xi^2\left(\bar{g}_{15} + \bar{b}_{15}\right),\ \bar{l}_{15} = \bar{\delta}_1^2(\bar{a}_{14}\bar{b}_{19} + \bar{b}_{17}\bar{b}_{23}),$$
$$\bar{l}_{16} = \bar{\delta}_1^2(\bar{g}_{12}\bar{b}_{17} - \bar{a}_{14}\bar{b}_{19}\xi^2) - \bar{a}_{14}\bar{g}_{16} + \bar{g}_{18}\bar{b}_{23} + \bar{a}_{21}\bar{g}_{19},\ \bar{l}_{17} = \xi^2\left(\bar{a}_{14}\bar{g}_{16} - \bar{a}_{21}\bar{g}_{19}\right) - \bar{g}_{12}\bar{g}_{18},$$
$$\bar{l}_{18} = -\bar{a}_{14}\delta_1^2,\ \bar{l}_{19} = \bar{\delta}_1^2(\bar{a}_{14}\xi^2 - \bar{b}_{17}\bar{b}_{22}) - \bar{a}_{14}\bar{g}_{14},\ \bar{l}_{20} = \xi^2(\bar{b}_{17}\bar{b}_{22}\bar{\delta}_1^2 + \bar{a}_{14}\bar{g}_{14}) - \bar{a}_{14}\bar{g}_{13} + \bar{b}_{22}\bar{g}_{18} + \bar{a}_{21}\bar{g}_{20},$$
$$\bar{l}_{21} = \xi^2(\bar{a}_{14}\bar{g}_{13} - \bar{b}_{22}\bar{g}_{18} - \bar{a}_{21}\bar{g}_{20}),\ \bar{l}_{22} = \bar{a}_8\bar{b}_{22} - \bar{a}_{14}\bar{b}_{15},\ \bar{l}_{23} = \bar{a}_{14}(\xi^2\bar{b}_{15} - \bar{g}_{21}) + \bar{b}_{22}\bar{g}_{22} - \bar{a}_8\bar{g}_{23} - \bar{b}_{22}\bar{g}_{24},$$

$$\bar{g}_{11} = \bar{b}_{15}\bar{b}_{20} - \bar{b}_{16}\bar{b}_{19},\ \bar{g}_{12} = \bar{b}_{23}\xi^2 - \bar{b}_{24},\ \bar{g}_{13} = \bar{b}_{14}\bar{b}_{20} - \bar{b}_{16}\bar{b}_{18},\ \bar{g}_{14} = \bar{b}_{16} - \bar{b}_{18}\bar{\delta}_1^2,\ \bar{g}_{15} = \bar{b}_{14}\bar{b}_{19} - \bar{b}_{15}\bar{b}_{18},$$
$$\bar{g}_{16} = \bar{b}_{15}\bar{b}_{20} - \bar{b}_{16}\bar{b}_{19},\ \bar{g}_{18} = \bar{a}_8\bar{b}_{20} - \bar{b}_{16}\bar{b}_{17},\ \bar{g}_{19} = \bar{a}_8\bar{b}_{19} - \bar{b}_{15}\bar{b}_{17},\ \bar{g}_{20} = \bar{a}_8\bar{b}_{18} - \bar{b}_{14}\bar{b}_{17},\ \bar{g}_{21} = \bar{b}_{14}\bar{b}_{19} - \bar{b}_{15}\bar{b}_{18},$$
$$\bar{g}_{22} = \bar{a}_8\bar{b}_{19} - \bar{b}_{15}\bar{b}_{17},\ \bar{g}_{23} = (\bar{b}_{22}\xi^2 - \bar{b}_{24}),\ \bar{g}_{24} = \bar{a}_8\bar{b}_{18} - \bar{b}_{14}\bar{b}_{17},$$

$$\bar{b}_{11} = \xi^2\left(1 - c^2\right),\ \bar{b}_{12} = -i\xi c\bar{\tau}_1 + 1,\ \bar{b}_{13} = \bar{a}_3\left(-ic\bar{\tau}^1\xi + 1\right),\ \bar{b}_{14} = \bar{a}_9\left(1 - ic\bar{\tau}_1\xi\right),\ \bar{b}_{15} = \bar{a}_{10}\left(-ic\bar{\tau}^1\xi + 1\right),$$
$$\bar{b}_{16} = \xi^2\left(c^2 - \bar{\delta}_1^2\right) - \bar{a}_7,\ \bar{b}_{17} = \bar{a}_{11}(i\xi c + \varepsilon\bar{\tau}_0\xi^2c^2),\ \bar{b}_{18} = \xi^2\left(1 - c^2\bar{\tau}_0\right) - i\xi c,\ \bar{b}_{19} = -\bar{a}_{13}(i\xi c + \gamma_1\xi^2c^2),$$
$$\bar{b}_{20} = -\bar{a}_{12}(i\xi c + \varepsilon\bar{\tau}_0\xi^2c^2),\ \bar{b}_{21} = 2\xi^2\bar{a}_{14},\ \bar{b}_{22} = -\bar{a}_{15}\left(-ic\bar{\tau}_1\xi + 1\right),\ \bar{b}_{23} = \bar{a}_{16}\left(-ic\bar{\tau}^1\xi + 1\right),\ \bar{b}_{24} = i\xi c(1 - i\varepsilon\bar{\tau}^0\xi c).$$

We write the appropriate values of ϕ, φ^*, T, C for $M_2(x_3 < 0)$ satisfying the radiation conditions as

$$\phi = (A_1 e^{m_1 x_3} + A_2 e^{m_2 x_3} + A_3 e^{m_3 x_3} + A_4 e^{m_4 x_3}) e^{i\xi(x_1 - ct)},$$
$$\varphi^* = (A_1 n_{11} e^{m_1 x_3} + A_2 n_{12} e^{m_2 x_3} + A_3 n_{13} e^{m_3 x_3} + A_4 n_{14} e^{m_4 x_3}) e^{i\xi(x_1 - ct)},$$
$$T = (A_1 n_{21} e^{m_1 x_3} + A_2 n_{22} e^{m_2 x_3} + A_3 n_{23} e^{m_3 x_3} + A_4 n_{24} e^{m_4 x_3}) e^{i\xi(x_1 - ct)}, \quad (31)$$
$$C = (A_1 n_{31} e^{m_1 x_3} + A_2 n_{32} e^{m_2 x_3} + A_3 n_{33} e^{m_3 x_3} + A_4 n_{34} e^{m_4 x_3}) e^{i\xi(x_1 - ct)}.$$

where

$m_1^2, m_2^2, m_3^2, m_4^2$, are the roots of the equation

$$D^8 + A_1^* D^6 + B_1^* D^4 + C_1^* D^2 + D_1^* = 0, \quad (32)$$

where A_1^*, B_1^*, C_1^* and D_1^* are given by,

$$A_1^* = d_{12}/d_{11},\ B_1^* = d_{13}/d_{11},\ C_1^* = d_{14}/d_{11},\ D_1^* = d_{15}/d_{11},$$

also

$$d_{11} = -(l_{11} + b_{13} l_{18}),\ d_{12} = -l_{12} + b_{11} l_{11} - b_{12} l_{15} + b_{13}\left(\xi^2 l_{18} - l_{19}\right) - a_2 l_{22},$$
$$d_{13} = -l_{13} + b_{11} l_{12} + b_{12}\left(\xi^2 l_{15} - l_{16}\right) + b_{13}\left(\xi^2 l_{19} - l_{20}\right) - a_2 \xi^2 l_{22} - a_2 l_{23},$$
$$d_{14} = -l_{14} + b_{11} l_{13} + b_{12}\left(\xi^2 l_{16} - l_{17}\right) + b_{13}\left(\xi^2 l_{20} - l_{21}\right) + a_2 l_{23},$$
$$d_{15} = b_{11} l_{14} + b_{12} \xi^2 l_{17} + b_{13} \xi^2 l_{21} + a_2 \xi^2 l_{24} - a_2 l_{24},$$

$l_{11} = b_{23}\delta_1^2, \, l_{12} = \delta_1^2(b_{19}b_{22} - g_{12}) + b_{23}g_{14},$

$l_{13} = -b_{22}\xi^2(b_{19}\delta_1^2 + g_{11}) - g_{12}g_{14} + b_{23}g_{13} + a_{21}\left(g_{15} - b_{15}\right),$

$l_{14} = \xi^2\left(b_{22}g_{11} - a_2 b_{15}\right) - g_{12}g_{13} - a_{21}\xi^2\left(g_{15} + b_{15}\right),$

$l_{15} = \delta_1^2(a_{14}b_{19} + b_{17}b_{23}),$

$l_{16} = \delta_1^2(g_{12}b_{17} - a_{14}b_{19}\xi^2) - a_{14}g_{16} + g_{18}b_{23} + a_{21}g_{19},$

$l_{17} = \xi^2\left(a_{14}g_{16} - a_{21}g_{19}\right) - g_{12}g_{18},$

$l_{18} = -a_{14}\delta_1^2,$

$l_{19} = \delta_1^2(a_{14}\xi^2 - b_{17}b_{22}) - a_{14}g_{14},$

$l_{20} = \xi^2(b_{17}b_{22}\delta_1^2 + a_{14}g_{14}) - a_{14}g_{13} + b_{22}g_{18} + a_{21}g_{20},$

$l_{21} = \xi^2(a_{14}g_{13} - b_{22}g_{18} - a_{21}g_{20}),$

$l_{22} = a_8 b_{22} - a_{14}b_{15},$

$l_{23} = a_{14}(\xi^2 b_{15} - g_{21}) + b_{22}g_{22} - a_8 g_{23} - b_{22}g_{24},$

$l_{24} = \xi^2(a_{14}g_{21} - b_{22}g_{22}) + g_{23}g_{24},$

$g_{11} = b_{15}b_{20} - b_{16}b_{19}, g_{12} = b_{23}\xi^2 - b_{24}, g_{13} = b_{14}b_{20} - b_{16}b_{18}, g_{14} = b_{16} - b_{18}\delta_1^2, g_{15} = b_{14}b_{19} - b_{15}b_{18},$
$g_{16} = b_{15}b_{20} - b_{16}b_{19}, g_{18} = a_8 b_{20} - b_{16}b_{17}, g_{19} = a_8 b_{19} - b_{15}b_{17}, g_{20} = a_8 b_{18} - b_{14}b_{17}, g_{21} = b_{14}b_{19} - b_{15}b_{18},$ The
$g_{22} = a_8 b_{19} - b_{15}b_{17}, g_{23} = (b_{22}\xi^2 - b_{24}), g_{24} = a_8 b_{18} - b_{14}b_{17},$

values of $\bar{\psi}$ for medium $M_1(x_3 < 0)$ and ψ, φ_2 for $M_2(x_3 > 0)$ satisfying the radiation conditions are

$$\bar{\psi} = \bar{A}_4 e^{-\bar{m}_4 x_3} e^{i\xi(x_1 - ct)}, \tag{33}$$

$$\psi = (A_5 e^{m_5 x_3} + A_6 e^{m_6 x_3}) e^{i\xi(x_1 - ct)}, \tag{34}$$

$$\varphi_2 = (A_5 n_{45} e^{m_5 x_3} + A_6 n_{46} e^{m_6 x_3}) e^{i\xi(x_1 - ct)} \tag{35}$$

.

where

$\bar{m}_4^2 = \xi^2 \left(1 - \dfrac{c^2}{1 - \delta^2}\right)$ and m_5^2, m_6^2 are the roots of the equation

$$D^4 + A_2^* D^2 + B_2^* = 0, \tag{36}$$

where

$A_2^* = \left(\bar{a}_4\bar{b}_{24} + \bar{b}_{25}(1-\bar{\delta}^2) + \bar{a}_1\bar{a}_5\right)\big/\bar{a}_4(1-\bar{\delta}^2), B_2^* = \left(\bar{b}_{24}\bar{b}_{25} - \bar{a}_1\bar{a}_5\xi^2\right)\big/\bar{a}_4(1-\bar{\delta}^2),$

The roots of equation (31) in the descending order corresponds to the velocities of propagation of three waves viz. longitudinal wave (LW1), thermal wave (TW1) and mass diffusion wave (MW1), respectively.Also, the roots of equation (32) in the descending order corresponds to the velocities of propagation of four possible waves, namely longitudinal displacement wave (LDW2), thermal wave (TW2), mass diffusion wave (MDW2), and longitudinal microstretch wave (LMW2), respectively. Similarly, two roots of the equation (36) corresponds to the coupled transverse displacement and transverse microrotational waves (CDW2- I, CDW2- II) respectively.

(i) Neglecting diffusion effect, equation (32) leads to sixth order differential equation

$$D^6 + A_3^* D^4 + B_3^* D^2 + C_3^* = 0\,, \qquad (37)$$

$$A_3^* = d_{32}/d_{31}\,, B_3^* = d_{33}/d_{31}\,, C_3^* = d_{34}/d_{31}\,,$$

$$d_{31} = -\delta_1^2, d_{32} = \left(a_2 a_8 - b_{16}\right) + \delta_1^2\left(b_{18} + b_{11} - b_{12}b_{17}\right),$$
$$d_{33} = \delta_1^2\left(-b_{11}b_{18} + b_{12}b_{17}\xi^2\right) + b_{16}\left(b_{18} + b_{11} - b_{12}b_{17}\right) + b_{20}(a_8 b_{12} - b_{14}) + a_2(b_{14}b_{17} - a_8 b_{18} - a_8\xi^2),$$
$$d_{34} = b_{11}\left(b_{14}b_{20} - b_{16}b_{18}\right) + \xi^2\left[a_2(a_8 b_{18} - b_{14}b_{17}) - b_{12}(a_8 b_{20} - b_{16}b_{17})\right],$$

and

$$b_{11} = \xi^2\left(1-c^2\right), b_{12} = -ic\tau_1\xi + 1, b_{13} = a_3\left(-ic\tau^1\xi + 1\right), b_{14} = a_9\left(-ic\tau_1\xi 1\right), b_{15} = a_{10}\left(-ic\tau^1\xi + 1\right),$$
$$b_{16} = \xi^2\left(c^2 - \delta_1^2\right) - a_7, b_{17} = a_{11}(i\xi c + \varepsilon\tau_0\xi^2 c^2), b_{18} = \xi^2\left(1 - c^2\tau_0\right) - i\xi c, b_{19} = -a_{13}(i\xi c + \gamma_1\xi^2 c^2),$$
$$b_{20} = -a_{12}(i\xi c + \varepsilon\tau_0\xi^2 c^2), b_{21} = 2\xi^2 a_{14}, b_{22} = -a_{15}\left(-ic\tau_1\xi + 1\right), b_{23} = a_{16}\left(-ic\tau^1\xi + 1\right), b_{24} = i\xi c(1 - i\varepsilon\tau^0 c\xi),$$

The roots of the equation (37) m_p^2 $(p = 1,2,3)$ correspond to the LDW2, TW2 and LMW2 waves, respectively.

Clearly on neglecting the diffusion effect, the wave corresponding to this parameter namely mass diffusion wave (MDW2) become deceased.Therefore, it is observed from the equation (32) and (37), that there exist a new type of wave namely MDW2 .

(ii) On neglecting the diffusion, micropolarity and microstretch effects, equation (32) and (36) simultaneously leads to the forth and second order differential equations as

$$D^4 + A_4^* D^2 + B_4^* = 0, \tag{38}$$

$$D^2 + \frac{b_{26}}{\left(1-\delta^2\right)} = 0, \tag{39}$$

where A_4^* and B_4^* are given by

$$A_4^* = d_{42}/d_{41}, B_4^* = d_{43}/d_{41},$$

$$d_{41} = 1, d_{42} = \left(-b_{18} - b_{11} + b_{12}b_{17}\right), d_{43} = \left(b_{11}b_{18} - b_{12}b_{17}\xi^2\right),$$

The roots of the equation (38) correspond to the Longitudinal wave (P-wave), and T waves, and (39) relate to the SV- wave, respectively.

Therefore, it is again observed that there exist new type of wave in (32) namely Longitudinal microstretch wave (LMW2) and transverse microrotational waves (CDW2- II) in (36) which become decoupled in this case .

Substituting the values of $\phi, \psi, \bar{\phi}$ and $\bar{\psi}$ from equations (29), (33) and (30), (34) in equations (16) and (17), we obtained displacement components

For medium M_1

$$\begin{aligned} \bar{u}_1 &= \left[i\xi\left(\bar{A}_1 e^{-\bar{m}_1 x_3} + \bar{A}_2 e^{-\bar{m}_2 x_3} + \bar{A}_3 e^{-\bar{m}_3 x_3}\right) + \bar{m}_4 \bar{A}_4 e^{-\bar{m}_4 x_3} \right] e^{i\xi(x_1 - ct)}, \\ \bar{u}_3 &= \left[-\left(\bar{m}_1 \bar{A}_1 e^{-\bar{m}_1 x_3} + \bar{m}_2 \bar{A}_2 e^{-\bar{m}_2 x_3} + \bar{m}_3 \bar{A}_3 e^{-\bar{m}_3 x_3}\right) + i\xi \bar{A}_4 e^{-\bar{m}_4 x_3} \right] e^{i\xi(x_1 - ct)}, \end{aligned} \tag{40}$$

For medium M_2

$$\begin{aligned} u_1 &= \left[i\xi\left(A_1 e^{m_1 x_3} + A_2 e^{m_2 x_3} + A_3 e^{m_3 x_3} + A_4 e^{m_4 x_3}\right) - \left(m_5 A_5 e^{m_5 x_3} + m_6 A_6 e^{m_6 x_3}\right) \right] e^{i\xi(x_1 - ct)}, \\ u_3 &= \left[\left(m_1 A_1 e^{m_1 x_3} + m_2 A_2 e^{m_2 x_3} + m_3 A_3 e^{m_3 x_3} + m_4 A_4 e^{m_4 x_3}\right) + i\xi\left(A_5 e^{m_5 x_3} + A_6 e^{m_6 x_3}\right) \right] e^{i\xi(x_1 - ct)}. \end{aligned} \tag{41}$$

6.Boundary Conditions

The boundary conditions are taken as the continuity of stress components, displacement components, temperature change,mass concentration, normal heat flux vector, normal mass diffusion flux vector and vanishing of tangential couple stress component, microstretch component, microrotation component. Mathematically, these can be written (at the boundary surface $x_3 = 0$) as

(i). $t_{33} = \bar{t}_{33}$, (42)

(ii). $t_{31} = \bar{t}_{31}$, (43)

(iii). $\bar{m}_{32} = 0,$ (44)

(iv). $\bar{\lambda}_3^* = 0,$ (45)

(v). $u_3 = \bar{u}_3$, (46)

(vi). $u_1 = \bar{u}_1$, (47)

(vii). $T = \bar{T}$ (48)

(viii). $C = \bar{C}$ (49)

(ix). $K^*\left(\frac{\partial T}{\partial x_3}\right) = \bar{K}^* \frac{\partial \bar{T}}{\partial x_3},$ (50)

(x). $D^* \frac{\partial C}{\partial x_3} = \bar{D}^* \frac{\partial \bar{C}}{\partial x_3},$ (51)

7 Derivations of the secular equations

Making use of equations (29),(30),(33),(34),(40) and (41) in the equations (42)-(51) with the aid of (6)-(8),(12) and (15), we obtain

$$\sum_{p=1}^{4} k_{qp} A_p + \sum_{p=1}^{6} k_{q,p+4} \bar{A}_p = 0, \quad \text{for } 1 \le q \le 10 \tag{52}$$

where the values of k_{ij} , $for\ i, j = (1,2,3,.......,10)$ are

$$k_{1p}=\begin{cases}\bar{m}_p^2-\bar{b}_1\xi^2+\bar{a}_2\bar{n}_{1p}+\bar{n}_{2p}\left(-1+ic\bar{\tau}_1\xi-1\right)+\bar{n}_{3p}\left(-1+ic\bar{\tau}^1\xi\right),1\le p\le 3,\\ i\bar{m}_p\xi\left(\bar{b}_1-1\right), \qquad p=4,\\ m_{p-4}^2-b_1\xi^2+a_2n_{1,p-4}+n_{2,p-4}\left(-1+ic\tau_1\xi\right)+n_{3,p-4}\left(-1+ic\tau^1\xi\right),5\le p\le 8,\\ im_{p-4}\xi\left(1-b_1\right), \qquad 9\le p\le 10.\end{cases},$$

$$k_{2p}=\begin{cases}-i\xi\bar{m}_p\left(\bar{b}_2+\bar{b}_3\right),\ 1\le p\le 3,\\ -\left(\bar{b}_2\bar{m}_p^2+\bar{b}_3\xi^2\right),\ p=4,\\ i\xi m_{p-4}\left(b_2+b_3\right),\ 5\le p\le 8,\\ -\left(b_2m_{p-4}^2+b_3\xi^2+a_1n_{4,p-4}\right),9\le p\le 10.\end{cases},$$

$$k_{3p}=\begin{cases}0,\ 1\le p\le 3,\\ 0,\ p=4,\\ -i\xi b_5n_{1,p-4},\ 5\le p\le 8,\\ -b_4n_{4,p-6}m_{p-4},9\le p\le 10.\end{cases},$$

$$k_{4p}=\begin{cases}0,\ 1\le p\le 3,\\ 0,\ p=4,\\ b_6m_{p-4}n_{1,p-4},\ 5\le p\le 8,\\ -i\xi b_5n_{4,p-4},\ p\ge 9,\end{cases},$$

$$k_{5p}=\begin{cases}0,\ 1\le p\le 3,\\ 0,\quad p=4,\\ m_{p-4},\ 5\le p\le 8,\\ i\xi,\quad p\ge 9\end{cases},$$

$$k_{6p}=\begin{cases}i\xi,\ 1\le p\le 3,\\ \bar{m}_p,\quad p=4,\\ -i\xi,\ 5\le p\le 8,\\ m_{p-4},\quad p\ge 9.\end{cases},$$

$$k_{7p}=\begin{cases}\bar{n}_{2p}, \qquad 1\le p\le 3\\ 0, \qquad p=4,\\ -n_{2,p-4}, \qquad 5\le p\le 8\\ 0, \quad p\ge 9,\end{cases},$$

$$k_{8p} = \begin{cases} \bar{n}_{3p}, & 1 \le p \le 3, \\ 0, & p = 4, \\ -n_{3,p-4}, & 5 \le p \le 8, \\ 0, & 9 \le p \le 10, \end{cases},$$

$$k_{9p} = \begin{cases} \bar{K}^* \bar{m}_p \bar{n}_{2p}, & 1 \le p \le 3, \\ 0, & p = 4, \\ K^* m_{p-4} n_{2,p-4}, & 5 \le p \le 8, \\ 0, & 9 \le p \le 10. \end{cases},$$

$$k_{10p} = \begin{cases} \bar{D}^* \bar{m}_p \bar{n}_{3p}, & 1 \le p \le 3, \\ 0, & p = 4, \\ D^* m_{p-4} n_{3,p-4}, & 5 \le p \le 8, \\ 0, & 9 \le p \le 10, \end{cases},$$

$$b_1 = \frac{\lambda}{\rho c_1^2}, b_2 = \frac{\mu + K}{\rho c_1^2}, b_3 = \frac{\mu}{\rho c_1^2}, b_4 = \frac{\omega^2 \gamma}{\rho c_1^2}, b_5 = \frac{\omega^2 b_0}{\rho c_1^2}, b_6 = \frac{\omega^2 \alpha_0}{\rho c_1^2}.$$

The system of equations (52) has a non-trivial solution if the determinant of amplitudes $A_p, \bar{A}_p$ vanishes leading to the secular equation

$$\left| k_{ij} \right|_{10\times 10} = 0 \quad , \qquad for\, i, j = (1,2,3,.......,10) \tag{53}$$

Equation (53) is the dispersion equation for the Stoneley wave propagation at an interface between thermoelastic diffusion and microstretch thermoelastic diffusion solid half spaces.

8 Particular Cases

(i) Ignoring the diffusion effect, the dispersion equation for the propagation of Stoneley waves at an interface between thermoelastic and microstretch thermoelastic solid half spaces is obtained as

(ii) $$\left| k_{ij} \right|_{8\times 8} = 0 \qquad for\, i, j = (1,2,3,.......,8) \tag{54}$$

with the values of k_{ij} as

$$k_{1p}=\begin{cases}\bar{m}_p^2-\bar{b}_1\xi^2+\bar{a}_2\bar{n}_{1p}, & p=1,2\\ i\xi\bar{m}_p\left(\bar{b}_1-1\right), & p=3\\ m_{p-3}^2-b_1\xi^2+a_2n_{1,p-3}+n_{2,p-3}\left(i\xi c\tau_1-1\right), & p=4,5,6\\ i\xi m_{p-3}\left(1-b_1\right), & p=7,8\end{cases},$$

$$k_{2p}=\begin{cases}-i\xi\bar{m}_p\left(\bar{b}_2+\bar{b}_3\right), & p=1,2\\ -\left(\bar{b}_2\bar{m}_p^2+\bar{b}_3\xi^2\right), & p=3\\ i\xi m_{p-3}\left(b_2+b_3\right), & p=4,5,6\\ -\left(b_2m_{p-3}^2+b_3\xi^2+a_1n_{4,p-3}\right), & p=7,8\end{cases},$$

$$k_{3p}=\begin{cases}0, & 1\le p\le 2\\ 0, & p=3\\ m_{p-3}, & 4\le p\le 6\\ i\xi, & 7\le p\le 8\end{cases},\quad k_{4p}=\begin{cases}i\xi, & 1\le p\le 2)\\ \bar{m}_p, & p=3\\ -i\xi, & 4\le p\le 6\\ m_{p-3}, & 7\le p\le 8\end{cases},\quad k_{5p}=\begin{cases}0, & 1\le p\le 2)\\ 0, & p=3\\ -i\xi b_5n_{1,p-3}, & 4\le p\le 6\\ -b_4n_{4,p-3}m_{p-3}, & 7\le p\le 8\end{cases},$$

$$k_{6p}=\begin{cases}0, & 1\le p\le 2\\ 0, & p=3\\ b_6m_{p-5}n_{1,p-3}, & 4\le p\le 6\\ -i\xi b_5n_{4,p-3}, & 7\le p\le 8\end{cases},\quad k_{7p}=\begin{cases}0, & 1\le p\le 2\\ 0, & p=3\\ 0, & 4\le p\le 6\\ -n_{4,p-3}, & 7\le p\le 8\end{cases},\quad k_{8p}=\begin{cases}0, & 1\le p\le 2\\ 0, & p=3\\ K^*m_{p-3}n_{2,p-3}, & 4\le p\le 6\\ 0, & 7\le p\le 8\end{cases},$$

(ii) Neglecting thermal and diffusion effects, the dispersion equation (54) reduced to theStoneley wave propagation at an interface of elastic/microstretch elastic solid half spaces.

(iii) Take $\tau^0>0,\ \varepsilon=0$ and $\gamma_1=\tau^0$ in equation (54), **y**ield the corresponding expressions with two relaxation times.

(iv) Using $\tau^1=\tau_1=0,\tau_0=\gamma_1,$ and $\varepsilon=1$ in equations (54),gives the corresponding results with one relaxation time.

(v) On taking $\tau_0=\tau_1=\tau^0=\tau^1=0=\gamma_1$ in equations (54), provide the corresponding expression of secular equation for the desired expressions with coupled thermoelastic (CT) theory.

9 Numerical results and discussion

Persuing [9], the values of micropolar parameters for medium M_1(magnesium)are taken as

$$\bar{\lambda} = 94\times10^9\mathrm{Nm}^{-2}, \bar{\mu} = 40\times10^9\mathrm{Nm}^{-2}, \bar{\rho} = 17.4\times10^2 Kgm^{-3},$$

$$\bar{a} = 29\times10^3\mathrm{m}^2\mathrm{s}^{-2}\mathrm{K}^{-1}, \bar{C}^* = 1.04\times10^3 JKg^{-1}K^{-1}, \bar{\alpha}_{t1} = 23.3\times10^{-6}\mathrm{K}^{-1}, \bar{\tau}_0 = 0.02,$$

$$\bar{\tau}_1 = 0.01, \bar{\alpha}_{c1} = 26.5\times10^{-5}\mathrm{m}^3\mathrm{Kg}^{-1}, \bar{\alpha}_{c2} = 28.3\times10^{-5}\mathrm{m}^3\mathrm{Kg}^{-1},$$

$$\bar{b} = 32\times10^5\mathrm{Kg}^{-1}\mathrm{m}^5\mathrm{s}^{-2}, \bar{\tau}^1 = 0.04, \bar{D} = 0.85\times10^{-8} Kgm^{-3}s, \bar{\mathrm{K}}^* = 17\times10^5 Jm^{-1}s^{-1}K^{-1},$$

$$\bar{\eta}^* = 1.5, \bar{\tau}^0 = 0.03, \bar{\alpha}_{t2} = 24.8\times10^{-6}\mathrm{K}^{-1}.$$

and for medium M_2, data are given by

$$\lambda = 0.759\times10^{10}\mathrm{Nm}^{-2}, \mu = 0.189\times10^{10}\mathrm{Nm}^{-2},\ K = 1.49\times10^{10} Nm^{-2},$$

$$\rho = 2.190\times10^3 Kgm^{-3},\ j = 0.196\times10^{-19} m^2, \gamma = 0.268\times10^{-9} N$$

Thermal and diffusion parameters are given by

$$C^* = 11.8\times10^2 JKg^{-1}K^{-1}, \mathrm{K}^* = 15\times10^5 Jm^{-1}s^{-1}K^{-1}, \alpha_{t1} = 22.2\times10^{-6}\mathrm{K}^{-1}, \alpha_{t2} = 23.8\times10^{-6}\mathrm{K}^{-1},$$

$$T_0 = 19.8\times10\mathrm{K}, \tau_1 = 0.9\times10^{-2}, \tau_0 = 1\times10^{-2}, \mathrm{K}^* = 15\times10^5 Jm^{-1}s^{-1}K^{-1}, \alpha_{c1} = 23.4\times10^{-5}\mathrm{m}^3\mathrm{Kg}^{-1}, \alpha_{c2} = 26.1\times10^{-5}\mathrm{m}^3\mathrm{Kg}^{-1},$$

$$a = 2.32\times10^4\mathrm{m}^2\mathrm{s}^{-2}\mathrm{K}^{-1}, b = 30.61\times10^5\mathrm{Kg}^{-1}\mathrm{m}^5\mathrm{s}^{-2}, \tau^1 = 0.03, \eta^* = 14.8\times10^{-1}, \tau^0 = 2\times10^{-2}.$$

$$D = 63\times10^{-10} Kgm^{-3}s,$$

and the microstretch parameters are taken as

$$j_o = 16.5\times10^{-21} m^2, \lambda_o = 0.37\times10^{10} Nm^{-2}, \alpha_o = 6.1\times10^{-10} N,, \lambda_1 = 3.7\times10^9 Nm^{-2}, b_o = 2.5\times10^{-10} N.$$

Above data has been used for numerical computation of the resulting quantities.

Fig.2 shows that phase velocity initially decreases sharply, attains minima and then shows a stationary behavior. Also the values of phase velocity decreases under the effect of diffusion.

Fig.3 shows that in absence of diffusion effect, attenuation coefficient increases smoothly with increase in wave number for LS and GL theories. While under the effect of diffusion, minimum variation is observed in the magnitude values of attenuation coefficient which appears to be stationary.

Fig.4 shows that initially small variation is observed in the magnitude values of normal displacement u_3 with wave number. But amplitude of variation increases with increase in wave number. Maximum value is observed under the effect of diffusion.

Fig.5 depicts that normal stress t_{33} fluctuates with wave number. This fluctuation increases with increase in the wave number. Values are maximum when diffusion is absent. Initially under the effect of diffusion the values of displacement starts with an initial decrease.

Fig.6 depicts that magnitudes of temperature for LS and GL theories under the effect of diffusion initially decreases and then shows a stationary behavior. But if there is no diffusion, a sharp increase and smooth decrease in values is observed till it becomes stationary.

10 Conclusion

In the present Chapter, frequency equation for the Stoneley waves at bounded interface is derived in the compact form by using appropriate boundary conditions. It is found that Stoneley waves in the considered model are dispersive. Computer algorithm has been developed for a particular model for numerical. Numerically simulated results are depicted with the aid of graphs to study the variation of phase velocity and attenuation coefficients with respect to wavenumbers. It is seen that for small values of non-dimensional wavenumber, the effect of different relaxation times have a significant effect on dispersion curve and less effect is seen for higher value.

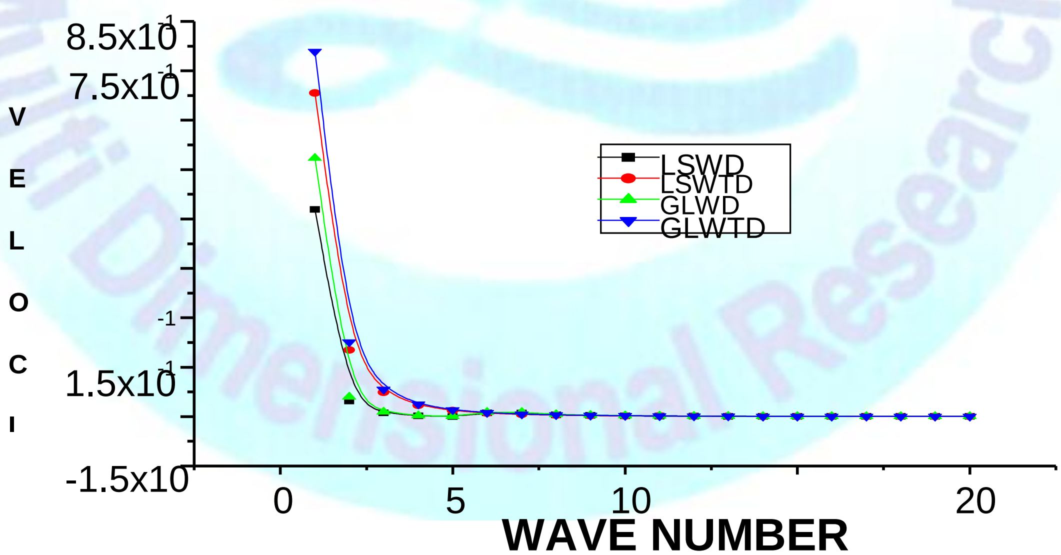

Fig.2 : Phase velocity with wavenumber

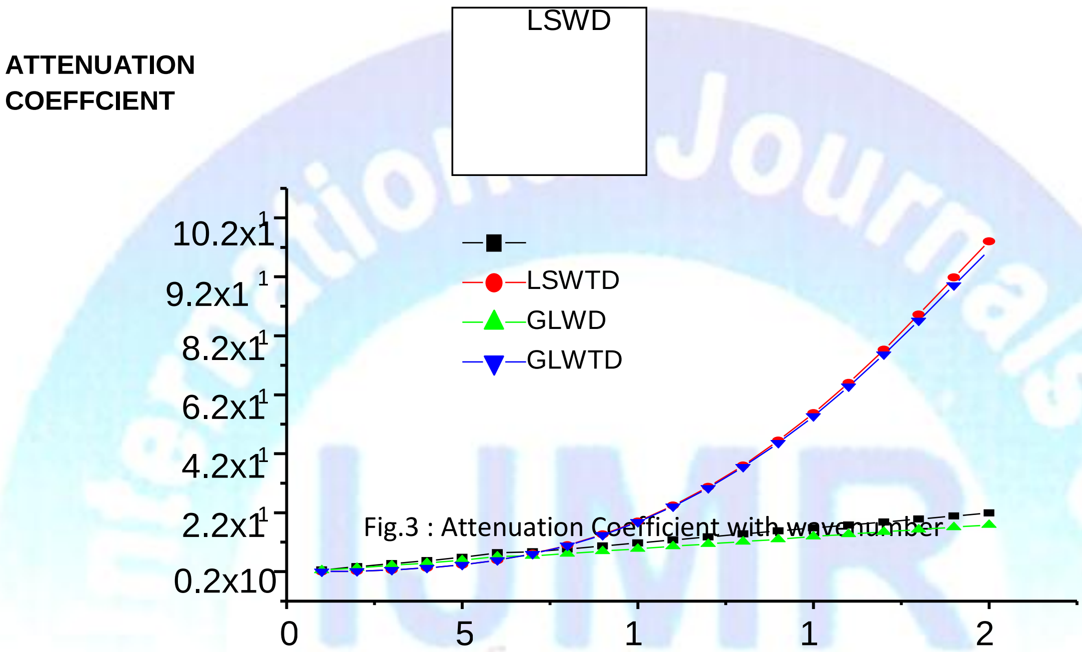

Fig.3 : Attenuation Coefficient with wave number

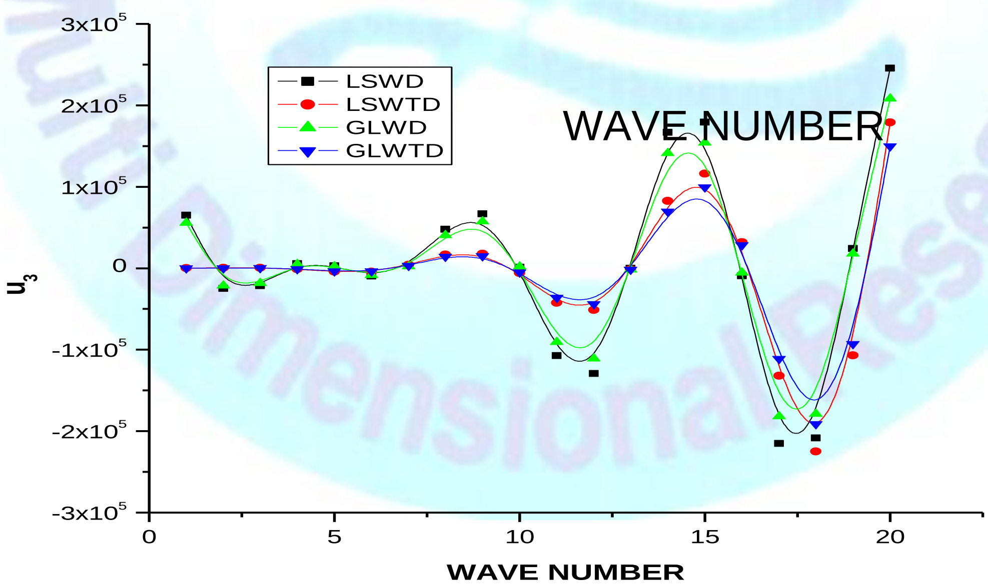

Fig. 4: u_3 with wavenumber

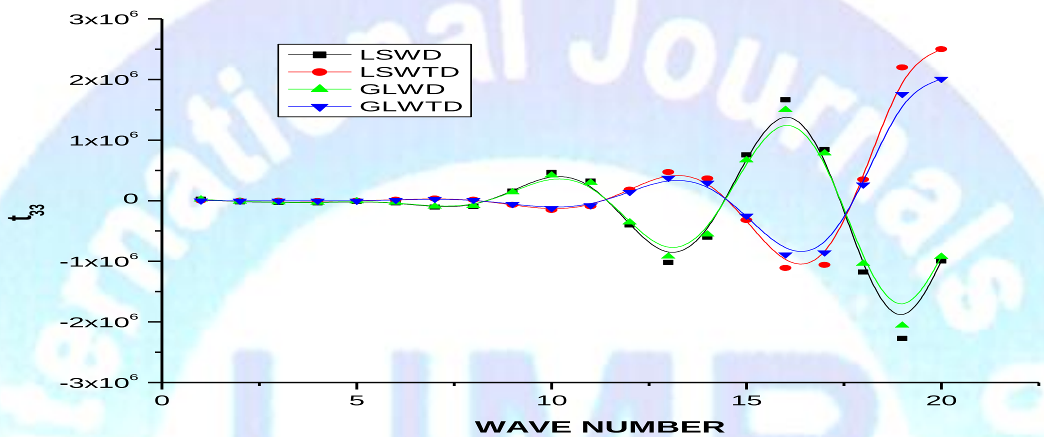

Fig.5 : Normal stress t_{33} with wavenumber

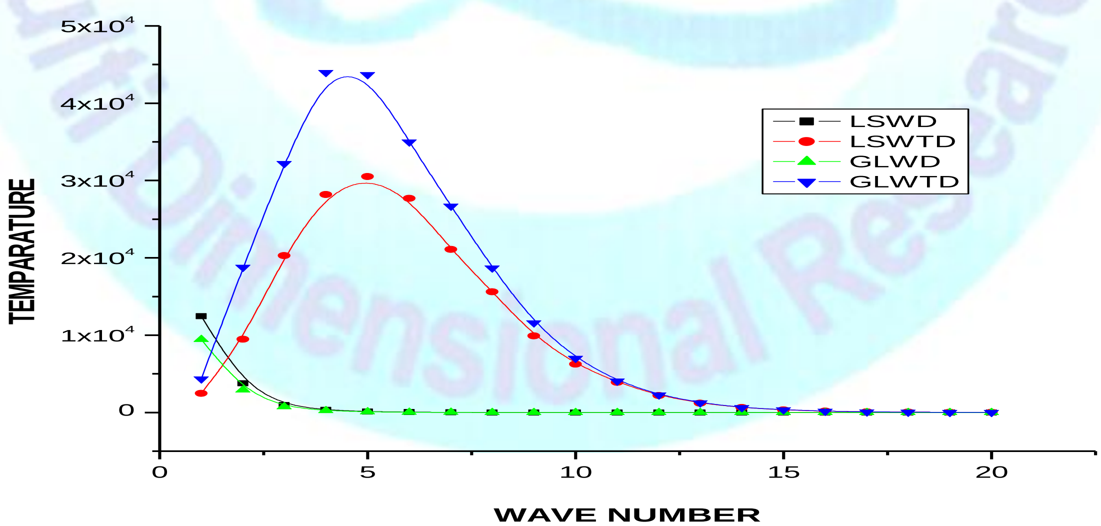

Fig.6.6: Temperature distribution with wavenumber.

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