Chemical reaction and viscous dissipation effects on radiative MHD boundary layer flow of nanofluids over a nonlinear stretching sheet with heat source in porous medium

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Abstract

The present paper aims to find an accurate numerical solution for thermal radiation and chemical reaction effects on MHD boundary layer flow of Nano fluids over a nonlinear stretching sheet in presence of heat source and viscous dissipation. The governing equations are reduced to nonlinear ordinary differential equations by using suitable similarity transformation. Using the Runge-Kutta Fehlberg fourth-fifth order method, numerical calculations are obtained to the desired level of accuracy for different values of dimensionless parameters. The governing parameters effects on dimensionless velocity, dimensionless temperature, dimensionless concentration, local Nusselt number, skin-friction coefficient and local Sherwood number are studied. The results of the local skin friction coefficient and the rate of reduced heat transfer are compared with the published data for a special case and found to be in good agreement.

Keywords: M.H.D.,Nanofluid,Stretching sheet, Heat generation,Viscous dissipation, Chemical reaction, Radiation and Porous medium.

1.Introduction

Nano fluids are attracting the attention of scientists and researchers around the world. This new heat transfer medium category improves the thermal conductivity of a fluid by suspending small solid particles in it and it also offers the possibility of increased heat applications. A lot of studies have been carried out by numerous researchers on thermal conductivity of nanlfluids (Singh [1], Xiang and Wang [2], Kleinstreuer and Feng [3], and Fan and Wang [4], etc.,). Boundary layer flow of a nanofluid past a stretching sheet was investigated by Khan and Pop [5]. Haile and Shankar [6] illustrated the effects of heat and mass transfer in the boundary layer of viscous nanofluid along a vertical stretching sheet. The boundary layer flow of a nanofluid past a stretching sheet with convective boundary conditions was studied by Makinde and Aziz [7]. Rana and Bhargava [8] explained the characteristics of heat transfer of nanofluid flow over a nonlinear stretching sheet. Cortell [9] studied the viscous flow and heat transfer over a nonlinear stretching sheet.Boundary layer flow and heat transfer over a nonlinearly permeable stretching or shrinking sheet in a nanofluid was developed by Zaimi et al. [10]. The presence of porous media in a boundary layer flow can significantly alter the field of flow and, as a consequence, affect the heat transfer rate at the surface.A Porous medium is generally modeled by using the classical Darcy formulation, which implies that the velocity of mean filter is proportional to the summation of the pressure gradient and the gravitational force. The model is empirical and it cannot be derived via a momentum analytically on a small element of porous medium. Sheikholeslami et al. [11] studied the effects of heat transfer in flow of nanofluids over a permeable stretching surface in presence of a porous medium.The flow of Natural convective boundary layer over a horizontal plate embedded in porous medium with a nanofluid was proposed by Gorla and Chamkha [12]. The presence of magnetic field in fluid flow is of great importance because their combination is used in many devices such as electromagnetic propulsion, MHD pump, nuclear reactors and MHD generators. Thus significant number of studies are available in literature which addressed MHD flows. Keshtkar and Amiri [13] derived MHD flow and heat transfer of a nanofluid over a stretching sheet which is permeable. Effects of heat and mass transfer on MHD slip fluid in nanofluids was carried out by Noghrehabadi and Ghalambaz [14]. Pal and Mandal [15] proposed the importance of MHD heat transfer effect of nanofluid over a non-linear stretching or shrinking sheet. Gnaneswara Reddy [16] proposed the Influence of magnetohydrodynamic and thermal radiation boundary layer flow of a nanofluid past a stretching sheet.The effects of radiation on temperature have become very important industrially. Many processes in engineering areas which occur at high temperature and acknowledge radiation heat transfer have become very important for designing pertinent equipment. Nuclear power plants, gas turbines and the various propulsion devices for making air-craft, missiles, satellites and space vehicles are examples of such engineering areas. Rashidi et al. [17] proposed heat and mass transfer effects on MHD fluid flow over a permeable vertical stretching sheet with radiation and buoyancy. Thermal radiation effects on steady MHD free convective boundary layer flow of nanofluid along a nonlinear stretching sheet was studied by Poornima and Bhaskar Reddy [18]. Shateyi and Prakash [19] developed a new numerical approach for MHD laminar boundary layer flow and heat transfer of a nanofluid over a moving surface in the presence of radiation. Radiation effects on MHD boundary layer flow of a nanofluid past a stretching sheet with convective boundary conditions were formulated by Gbadeyan et al. [20]The problem of free convection boundary layer flow of nanofluids over a nonlinear stretching sheet in the presence of MHD and heat source/sink was investigated by Ranga Rao et al. [21]. Chamkha and Aly [22] derived the effects of heat generation on MHD free convection flow of a nanofluid past a vertical plate. MHD boundary layer flow past a wedge moving in a nanofluid in presence of radiation and heat generation was studied by Khan et al. [23]. Gireesha and Rudraswamy [24] proposed chemical reaction and heat source effects on MHD flow of a nanofluid near the stagnation point over a permeable stretching surface. Chemical reaction effect on MHD nanofluid flow due to a stretching or shrinking sheet was studied by Kameswaran et al. [25]. Rosmila Abdul et al. [26] derived the boundary layer flow of a nanofluid past a porous vertical stretching surface with chemical reaction and radiation. The importance of chemical reaction effect in nanofluid flows were studied by many researchers such as Rosca [27], Kasmani et al [28], Venkateswarlu and Satya Narayana [29], and Yohannes and Daniel [30]. Dissipation is the process of converting mechanical energy of downward-flowing water into energy which is thermal and acoustical . Viscous dissipation is of interest for many applications: Rises in significant temperature are observed in flows of polymer processing such as injection modeling or extrusion at high rates. Aerodynamic heating in the boundary layer which is thin around high speed aircraft raises the temperature of the skin. Pal and Mandal [31] found that the effects of MHD boundary layer flow of nanofluids over a nonlinear stretching or shrinking sheet in presence of viscous ohmic dissipation and radiation. Krishnamurthy et al. [32] carried out experiments to study the effect of viscous dissipation on hydromagnetic flow and heat transfer of nanofluids over an exponentially stretching sheet with fluid particle suspension. Heat and mass transfer effects on MHD flow of nanofluids over porous media through a stretching sheet in presence of chemical reaction and viscous dissipation were studied by Yohannes and Shankar [33]. Motsumi and Makinde [34] investigated the effects of radiation and viscous dissipation on boundary layer flow of nanofluids over a permeable moving flat plate. Habibi Matin et al. [35] derived the MHD flow of nanofluid over a nonlinear stretching sheet in presence of viscous dissipation. MHD laminar boundary layer flow with heat and mass transfer of an electrically conducting water based nanofluid over a nonlinear stretching sheet in presence of viscous dissipation was studied numerically by Mabood et al.

[36]. Ganga et al. [37] illustrated the effects of radiation and heat generation on MHD boundary layer flow of a nanofluid past a vertical plate with viscous and ohmic dissipation. Machireddy Gnaneswara Reddy, Polarapu Padma, Bandari Shankar et al. [38] illustrated the effects of viscous dissipation and heat source on unsteady MHD flow over a stretching sheet.Motivated by the above studies, the present paper aims to numerically analyze the problem of MHD boundary layer flow of nanofluids over a nonlinear stretching sheet in presence of heat generation, viscous dissipation, radiation and chemical reaction. The partial momentum, heat and mass transfer equations are transformed into a set of ordinary differential equations by introducing suitable similarity transformations.By using the Runge-Kutta Fehlberg fourth-fifth order method, numerical calculations to the desired level of accuracy were obtained for different values of dimensionless parameters. The results are presented graphically and in tabular form. The results for special cases were also compared with those by Rana and Bhargava [8], Cortell[9], Zaimi et al. [10]. Plots are presented and discussed for the effects of the physical parameters involved in the developed solutions.

2.Mathematical Formulation

A two-dimensional, steady and incompressible viscous boundary layer flow of an incompressible electrically conducting and radiative past over a nonlinear stretching surface is considered under the assumptions that the external pressure on nonlinear stretching sheet in the x- direction is having diluted nanoparticles and y-axis normal to it. The sheet is extended with velocity $u_w(x) = ax^n$ with fixed origin location, here *n* is a nonlinear stretching parameter, *a* is a constant. The fluid is considered to be a gray, absorbing and emitting radiation but nonscattering medium. A uniform magnetic field was applied in the transverse direction to the flow. We assumed that the variable magnetic field B(x) is of the form $B(x) = B_0 x^{\frac{(n-1)}{2}}$. The fluid is assumed to be slightly conducting, so that the magnetic Reynolds number is much less than unity and hence the magnetic field which is induced is negligible in comparison with the applied magnetic field.By using the Oberbeck-Boussinesq approximation, the governing equations of the flow field can be written in the dimensional form as

$$\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 0 \qquad (1)$$

$$\rho_f\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)=-\frac{\partial p}{\partial x}+\mu\frac{\partial^2 u}{\partial y^2}+\left(1-C_\infty\right)\rho_{f_\infty}\beta_T g\left(T-T_\infty\right)-\left(\rho_p-\rho_{f_\infty}\right)\beta_C g\left(C-C_\infty\right)-\sigma B_0^2 u-\frac{\mu}{Kp}u \quad (2)$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2}-\frac{1}{\left(\rho c_p\right)_f}\frac{\partial q_r}{\partial y}+\tau\left[D_B\left(\frac{\partial T}{\partial y}\frac{CT}{\partial y}\right)+\frac{D_T}{T_\infty}\left(\frac{\partial T}{\partial y}\right)^2\right]+\frac{\upsilon}{c_p}\left(\frac{\partial u}{\partial y}\right)^2+\frac{Q'}{\left(\rho c_p\right)_f}\left(T-T_\infty\right) \quad (3)$$

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D_B\frac{\partial^2 C}{\partial y^2}+\frac{D_T}{T_\infty}\frac{\partial^2 T}{\partial y^2}-k_1\left(C-C_\infty\right) \quad (4)$$

Here, *u* and *v* are the velocity components in the x and y directions, respectively, *g* is the acceleration due to gravity, μ is the viscosity, σ is the electrical conductivity, p is the pressure, ρ_f is the density of the base fluid, ρ_f is the density of nanoparticle, β_T is the coefficient of volumetric thermal expansion, β_C the coefficient of volumetric concentration expansion, *Kp* is permeability of the porous medium, *T* - the temperature of the nanofluid, *C* - the concentration of the nanofluid, T_w and C_w are the temperature and concentration along the stretching sheet, T_∞ and C_∞ are the ambient temperature and concentration, D_B is the Brownian diffusion coefficient, D_T is the thermophoresis coefficient, B_0 is the magnetic induction, q_r is the radiative heat flux, *k* - the thermal conductivity, c_p is specific heat at constant pressure, $\left(\rho c_p\right)_p$ is the heat capacitance of the nanoparticles, $\left(\rho c_p\right)_f$ is the capacitance of heat of the base fluid, $\alpha = k/\left(\rho c_p\right)_f$ is the thermal diffusivity parameter, $\tau = \left(\rho c_p\right)_p/\left(\rho c_p\right)_f$ is the ratio between the effective heat capacity of the material of nanoparticles and heat capacity of the fluid, Q' is the dimensional heat generation co-efficient and k_1 is the rate chemical reaction parameter.

The corresponding boundary conditions are

$$u=u_w(x)=ax^n,\quad v=0,\quad T=T_w,\quad C=C_w \quad \text{at } y=0 \quad (5)$$
$$u\to 0,\quad v\to 0,\; T\to T_\infty,\; C\to C_\infty \quad \text{as } y\to\infty$$

By using the Rosseland approximation, the radiative heat flux q_r is given by

$$q_r = -\frac{4\sigma_s}{3k_e}\frac{\partial T^4}{\partial y} \tag{6}$$

where σ_s is the Stephen Boltzmann constant where as k_e is the mean absorption coefficient.

It should be noted that by using the Rosseland approximation, the present analysis is limited to fluids which are optically thick . If the temperature differences within the flow are sufficiently small, equation (6) can be linearized by expanding T^4 into the Taylor series about T_∞, which after neglecting higher order terms takes the form

$$T^4 \cong 4T_\infty^3 T - 3T_\infty^4 \tag{7}$$

Then the radiation term in equation (3) takes the form

$$\frac{\partial q_r}{\partial y} = \frac{16\sigma_s T_\infty^3}{3k_e}\frac{\partial^2 T}{\partial y^2} \tag{8}$$

Invoking equation (8), equation (3) gets modified as

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{k}{\left(\rho c_p\right)_f}\frac{\partial^2 T}{\partial y^2} + \frac{16\sigma_s}{3k_e}\frac{T_\infty^3}{\left(\rho c_p\right)_f}\frac{\partial^2 T}{\partial y^2} + \tau\left[D_B\left(\frac{\partial T}{\partial y}\frac{CT}{\partial y}\right) + \frac{D_T}{T_\infty}\left(\frac{\partial T}{\partial y}\right)^2\right] + \frac{\upsilon}{c_p}\left(\frac{\partial u}{\partial y}\right)^2 + \frac{Q'}{\left(\rho c_p\right)_f}\left(T - T_\infty\right) \tag{9}$$

Using the steam function $\psi = \psi(x, y)$, the velocity components *u* and *v* are defined as

$$u = \frac{\partial \psi}{\partial y} \quad \text{and} \quad v = -\frac{\partial \psi}{\partial x}$$

Assuming that the external pressure on the plate, in the direction having diluted nanoparticles, to be constant, the similarity transformations are taken as (Rana and Bhargava [8])

$$\eta = y\sqrt{\frac{(n+1)ax^{n-1}}{2\upsilon}} \qquad \psi = \sqrt{\frac{2\upsilon ax^{n+1}}{n+1}}\, f(\eta), \qquad u = ax^n f'(\eta),$$

$$v = -\sqrt{\frac{av(n+1)}{2}}\, x^{(n-1)/2}\left(f(\eta) + \frac{n-1}{n+1}\eta f'(\eta)\right), \quad \theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty}, \quad \phi(\eta) = \frac{C - C_\infty}{C_w - C_\infty},$$

$$Nr = \frac{kk_e}{4\sigma_s T_\infty^3}, \quad R = \frac{4}{3Nr}, \quad \lambda = \frac{Gr}{\mathrm{Re}_x^{\frac{3}{2}}}, \quad \delta = \frac{Gm}{\mathrm{Re}_x^{\frac{3}{2}}}, \quad M = \frac{2\sigma B_0^2}{\rho_f a(n+1)x^{n-1}}$$

$$K = \frac{Kpa(n+1)}{2\upsilon}x^{n-1} \qquad \Pr = \frac{\upsilon}{\alpha}, \quad \upsilon = \frac{\mu}{\rho_f} \quad Le = \frac{\upsilon}{D_B}, \tag{10}$$

$$Nb=\frac{\tau D_B}{\upsilon}\left(C_w-C_\infty\right) \quad , \quad Q=\frac{2Q'}{(n+1)\left[\rho c_p\right]_f ax^{n-1}} \quad , \quad Ec=\frac{a^2}{c_p(T_w-T_\infty)} \quad Nt=\frac{\tau D_T}{\upsilon T_\infty}\left(T_w-T_\infty\right) \quad ,$$

$$Gr=\frac{(1-C_\infty)\left(\frac{\rho_{f_\infty}}{\rho_f}\right)gn\left(T_w-T_\infty\right)}{\upsilon^2 \operatorname{Re}_x^{\frac{1}{2}}}, Gm=\frac{\left(\frac{\rho_P-\rho_{f_\infty}}{\rho_f}\right)gn_1\left(C_w-C_\infty\right)}{\upsilon^2 \operatorname{Re}_x^{\frac{1}{2}}}, \quad \gamma=\frac{2k_l}{(n+1)ax^{n-1}},$$

$$\operatorname{Re}_x=\frac{u_w(x)x}{\upsilon},$$

In view of the above similarity transformations, the equations (2), (9) and (4) reduce to

$$f'''(\eta)+f(\eta)f''(\eta)-\frac{2n}{n+1}\left(f'(\eta)\right)^2+\frac{2}{n+1}(\lambda\theta-\delta\phi)-(M+K)f'=0 \qquad (11)$$

$$(1+R)\theta''+\Pr f(\eta)\theta'(\eta)+\Pr Nb\theta'(\eta)\phi'(\eta)+\Pr Nt\left(\theta'(\eta)\right)^2+\Pr Q\theta(\eta)+E_c \Pr f''^2=0 \qquad (12)$$

$$\phi''(\eta)+Lef(\eta)\phi'(\eta)+\frac{Nt}{Nb}\theta''(\eta)-Le\gamma\phi(\eta)=0 \qquad (13)$$

where λ is the buoyancy parameter, δ is the solutal buoyancy parameter, *n* is nonlinear stretching parameter, *Pr* is the Prandtl number, *Le* is the Lewis number, υ is the kinematic viscosity of the nanofluid, *Nb* is the Brownian motion parameter, *Nt* is the thermophoresis parameter, Re_x is the local Reynolds number which is based on the stretching velocity, *Gr* is the local thermal Grashof number, *Gm* is the local concentration Grashof number, *M* is magnetic field parameter, *K* is the permeability of porous medium, *R* is the radiation parameter, *Q* is the heat generation parameter, γ is the chemical reaction parameter and f,θ,ϕ are the dimensionless stream functions, temperature, concentration respectively. This boundary value problem is reduced to classical problem of flow and heat and mass transfer owing to a stretching surface in a viscous fluid when *n = 1* and *Nb = Nt = 0* in equations (12) and (13)

Here, β_T and β_C are proportional to x^{-3}, that is $\beta_T=nx^{-3}$ and $\beta_C=n_1x^{-3}$, where n and n_1 are constants of proportionality

The corresponding boundary conditions are

$$f(\eta)=0,\quad f'(\eta)=1,\quad \theta(\eta)=1,\quad \phi(\eta)=1 \quad \text{at } \eta=0$$

$$f'(\eta)\to 0,\quad \theta(\eta)\to 0,\quad \phi(\eta)\to 0 \quad \text{as } \eta\to\infty \qquad (14)$$

For the type of boundary layer flow under consideration, the skin-friction coefficient, both Nusselt number and Sherwood number are main physical parameters.

Knowing the velocity field, the shearing stress at the plate can be got, which in the non-dimensional form (skin-friction coefficient) is given by

$$C_f = \frac{\mu}{\rho u_w^2}\left(\frac{\partial u}{\partial y}\right)_{y=0} = \sqrt{\frac{(n+1)}{2}}\,\mathrm{Re}_x^{-\frac{1}{2}}\, f''(0)$$

Knowing the temperature field, the heat transfer coefficient at the plate can be obtained, which in the non-dimensional form, according to Nusselt number is given by

$$Nu = \frac{q_w x}{k\left(T_w - T_\infty\right)} = -\frac{x}{\left(T_w - T_\infty\right)}\left(\frac{\partial T}{\partial y}\right)_{y=0} = -\sqrt{\frac{(n+1)}{2}}\,\mathrm{Re}_x^{\frac{1}{2}}\,\theta'(0)$$

Knowing the concentration field, the mass transfer coefficient at the plate can be obtained, shich in the non-dimensional form, in terms of the Sherwood number is given by

$$Sh = \frac{q_m x}{k\left(C_w - C_\infty\right)} = -\frac{x}{\left(C_w - C_\infty\right)}\left(\frac{\partial C}{\partial y}\right)_{y=0} = -\sqrt{\frac{(n+1)}{2}}\,\mathrm{Re}_x^{\frac{1}{2}}\,\phi'(0)$$

Where $\mathrm{Re}_x = \frac{u_w x}{\upsilon}$ is the local Reynolds number.

3.Solution of the Problem

The reduced equations (11) – (13) are nonlinear and coupled, and thus their exact analytical solutions are not possible. They can be solved numberically using Runge-Kutta Fehlberg fourth fifth order method for different values of parameters such as porous parameter, magnetic field parameter, Prandtl number, radiation parameter, heat source, Eckert number, Lewis number, the Brownian motion parameter, the thermpohoresis parameter and chemical reaction parameter. The effects of the emerging parameters on the dimensional velocity, concentration,temperature skin friction coefficient, Nusselt number and Sherwood number are studied. The step size and convergence criteria were chosen to be 0.001 and 10^{-6}, respectively. The asymptotic boundary conditions in (14) were approximated by using a value of 8 for $\eta_{\max}$. This ensures that all numerical solutions approached the asymptotic values correctly.

4. Results and Discussion

Computations were carried out with Runge-Kutta Fehlberg fourth fifth order method for several non-dimensional parameters. We have compared our numerical computation results of Nusselt and Sherwood numbers with those which are available in open literature in tables (1) – (3), which show an excellent agreement. The values of skin friction, Nusselt and Sherwood numbers for different values of parameters are displayed in table 3.

To analyze the results, numerical computation are carried out for variation in physical parameters. In the present case, the following default parameter values are taken for computations: $\lambda = 0.5$, δ =0.5, *M* = 0.5, *K* = 0.5, *n* = 2.0, *Nb* = 0.1, *Nt* = 0.1, *Pr* = 0.71, *R* = 1.0, *Q* = 0.1, *Ec* = 0.01, *Le* = 0.01 and γ = 0.5. All graphs therefore correspond to these values unless specifically indicated in the appropriate graph.

The influence of the thermal buoyancy parameter λ and magnetic field parameter *M* on dimensionless velocity is plotted in Fig. 1. Here, the positive buoyancy force acts like a favorable pressure gradient and therefore it accelerates the fluid in the boundary layer. This results in higher velocity as buoyancy parameter increases. And it is found that the dimensionless velocity profile of the nanofluid is insignificantly reduced with increasing value of magnetic field parameter. This is due to the fact that magnetic field introduces a retarding body force which acts transversely to the direction of the applied magnetic field. This body force, known as the Lorentz force, decelerates the boundary layer flow and thickens the momentum boundary layer.

Fig. 2 exhibits the impact of the solutal buoyancy parameter δ and permeability parameter *K* on the dimensionless velocity. It is seen that the velocity profile of nanofluids is insignificantly reduced with increasing values of solutal buoyancy parameter and permeability parameter. The effect of Prandtl number *Pr* and heat source *Q* on velocity is presented in Fig. 3. It seen that the dimensionless velocity accelerates with an increase in heat source and decelerates with an increase in the Prandtl number. Fig.4 shows the effects of radiation parameter *R* and nonlinear stretching parameter *n* on the dimensionless velocity. It is observed that an increase in the radiation parameter leads to a rise in the velocity, whereas the opposite effect has been found for nonlinear stretching parameter. The effect of viscous dissipation parameter *Ec* on dimensionless velocity profile is shown in Fig. 5. It is clear that the velocity profile increases with an increase in the viscous dissipation parameter. The effect of Lewis number *Le* on dimensionless velocity profile is presented in Fig. 6. It is observed that increasing Lewis number significantly increases the velocity of nanofluids. Lewis number is a dimensionless number which is defined as the ratio between thermal diffusivity and mass diffusivity.

Figs. 7 and 8 are prepared to show the influence of radiation parameter *R*, Prandtl number *Pr*, buoyancy parameter λ, and Brownian motion parameter *Nb* on the dimensionless temperature for the fixed values of other parameters. It is found that the dimensionless temperature slightly increases and decreases with radiation parameter or buoyancy parameter and Prandtl number or Brownian motion parameter, respectively. Fig. 9. displays the result of the dimensionless temperature for various values of chemical reaction parameter γ and heat generation parameter *Q*. It is obvious from the figure that the temperature profile increases with an increase in heat source parameter, whereas it shows reverse effect in the case of chemical reaction parameter. Figs. (10) – (12) show the effect of thermpohoresis parameter *Nt*, Eckert number *Ec*, magnetic field parameter *M*, solutal buoyancy parameter δ, permeability parameter *K* and nonlinear stretching parameter *n* on the dimensionless temperature. It is clear from these figures that the dimensionless temperature increases with an increase in thermpohoresis parameter, Eckert number, magnetic field parameter, solutal buoyancy parameter, permeability parameter and nonlinear stretching parameter.

The influence of the Lewis number *Le*, Brownian motion parameter *Nb*, Eckert number *Ec* and heat source parameter *Q* on the dimensionless concentration field are displayed in Figs. 13 and 14, respectively. It is noticed that the concentration decreases as the Lewis number or Brownian motion parameter or Eckert number or heat source parameter increases.

Fig. 15 illustrates the effects of chemical reaction γ and thermopherosis *Nt* on the dimensionless concentration. It is noticed that the dimensionless concentration slightly decreases or increases with chemical reaction and thermopherosis, respectively.

The effects of solutal buoyancy parameter δ and magnetic field parameter *M* on the dimensionless concentration profile are presented in Figs. 16 and 17. It is noted that, by increasing either value of solutal buoyancy parameter or magnetic field parameter, the dimensionless concentration increases.

The variation in skin-friction coefficient, Nusselt number and Sherwood number for various parameters are investigated through tables 3 - 4. The behavior of these physical parameters is self evident from tables 1 and 2 and hence they are not discussed any further for the sake of brevity.

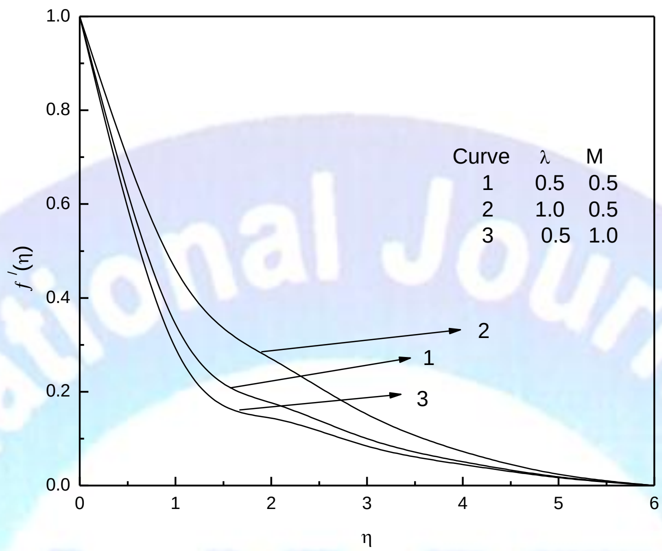

Fig.1 Velocity profiles for different values of λ and M

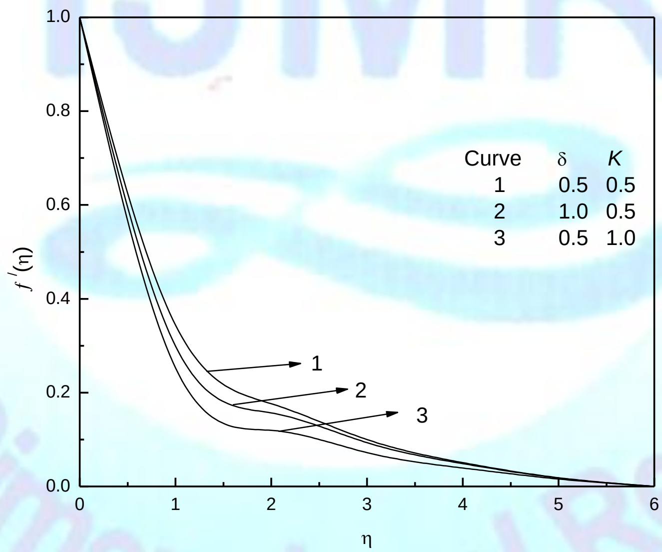

Fig.2 Velocity profiles for different values of δ and K

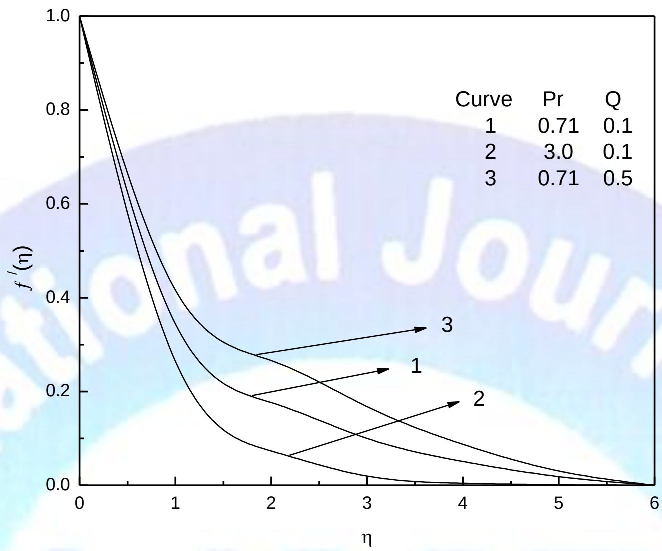

Fig.3 Velocity profiles for different values of *Pr* and *Q*

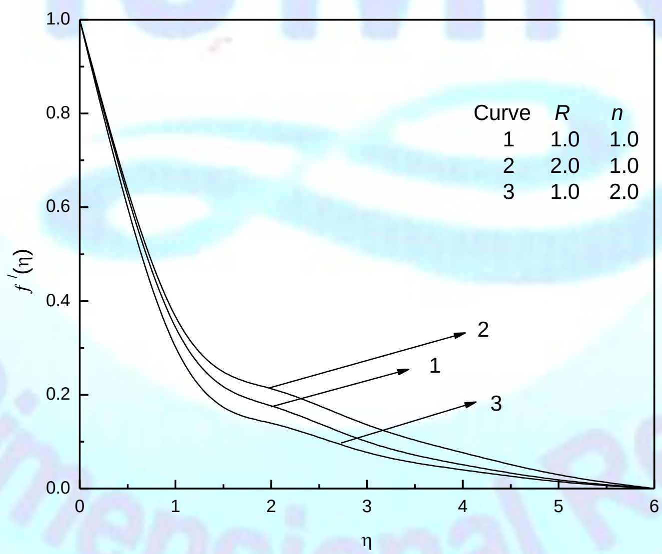

Fig.4 Velocity profiles for different values of *R* and *n*

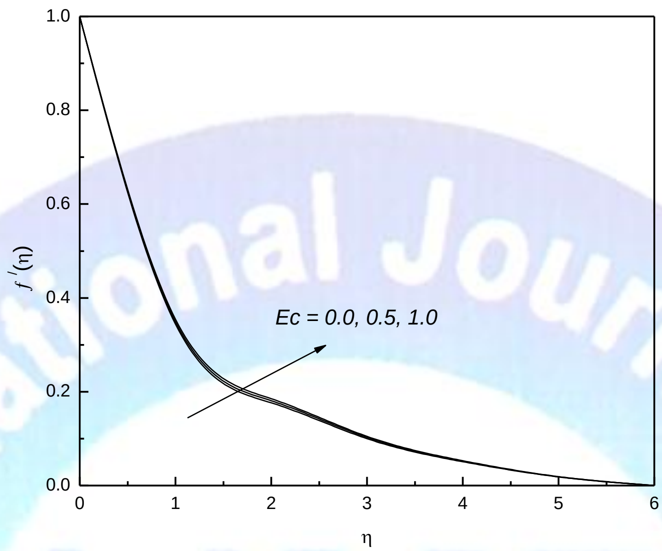

Fig.5 Velocity profiles for different values of *Ec*

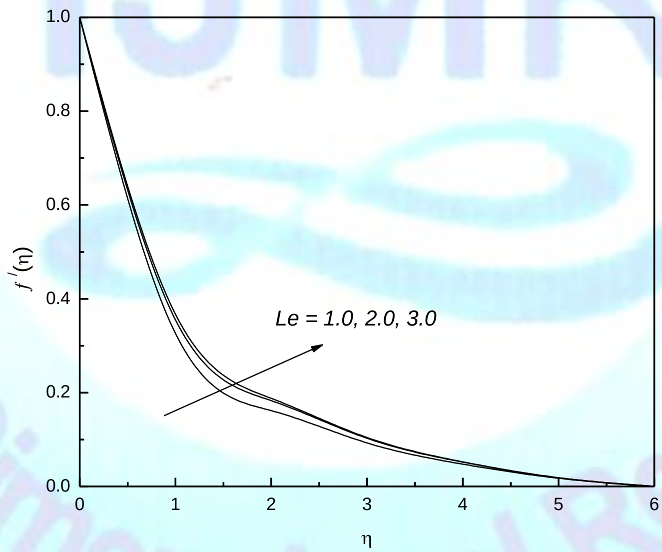

Fig.6 Velocity profiles for different values of *Le*

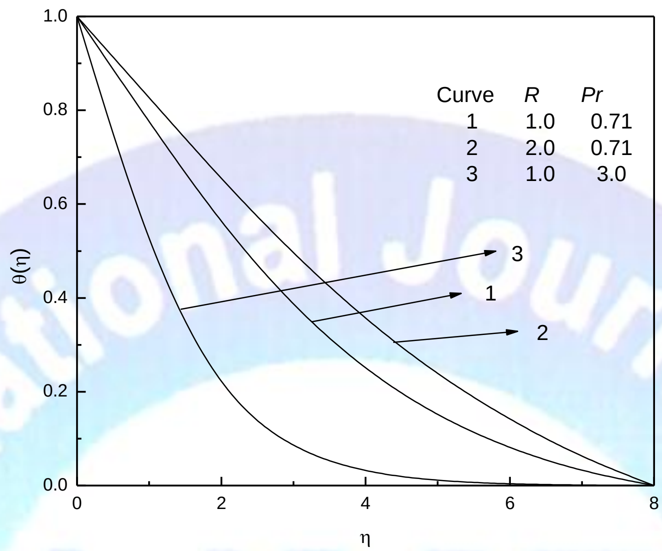

Fig.7 Temperature profiles for different values of *Pr* and *R*

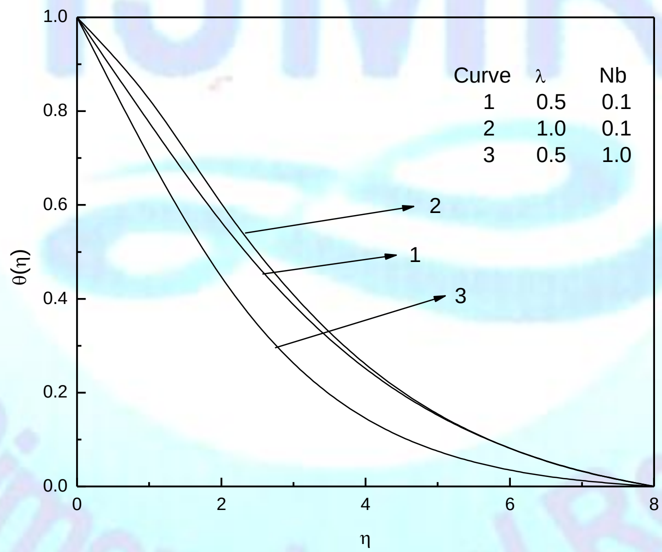

Fig.8 Temperature profiles for different values of λ and *Nb*

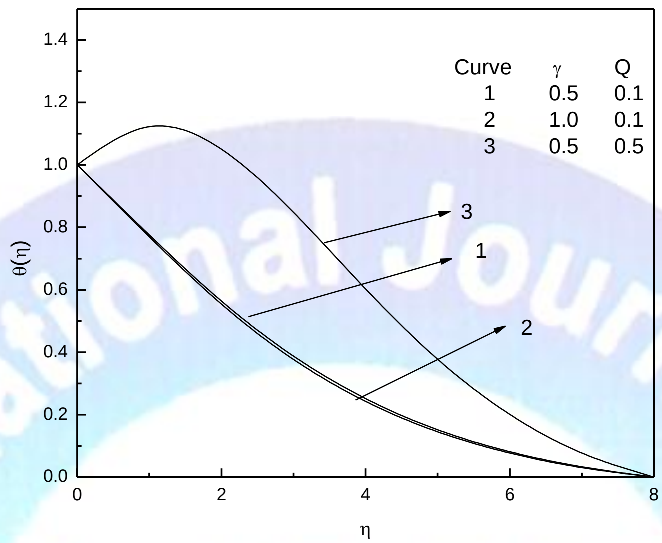

Fig.9 Temperature profiles for different values of γ and *Q*

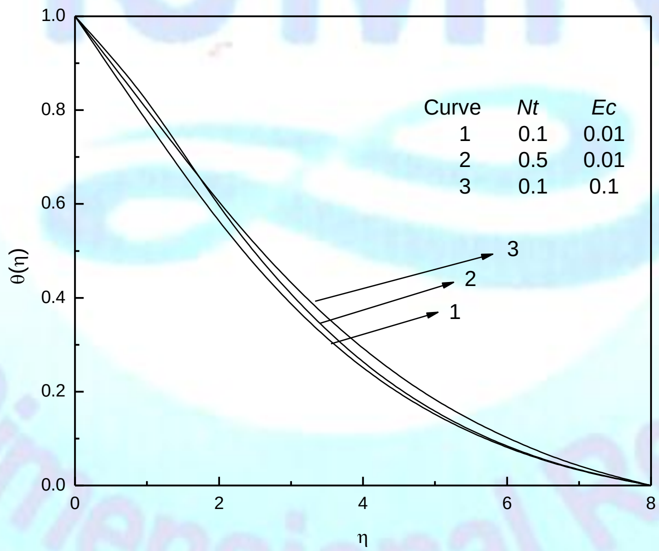

Fig.10 Temperature profiles for different values of *Nt* and *Ec*

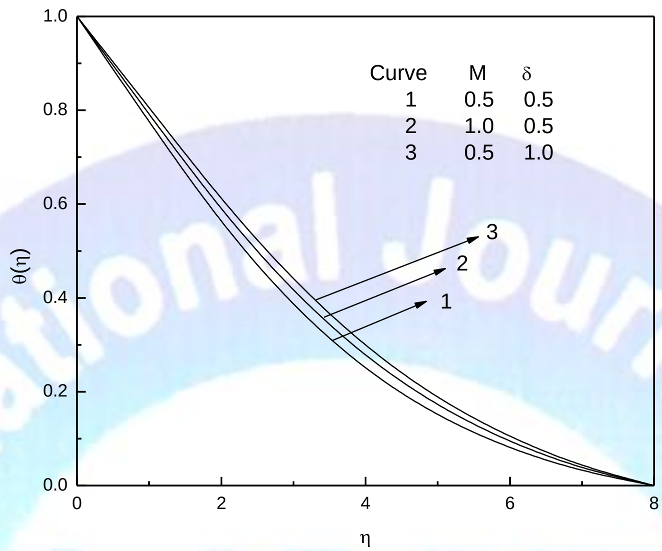

Fig.11 Temperature profiles for different values of M and δ

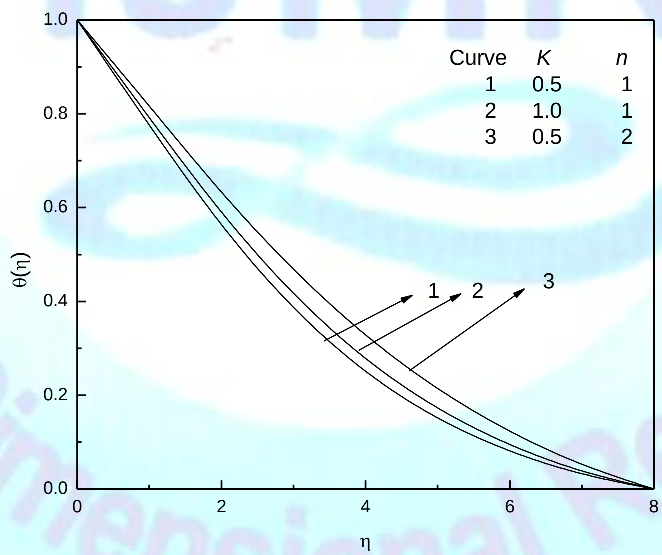

Fig.12 Temperature profiles for different values of K and n

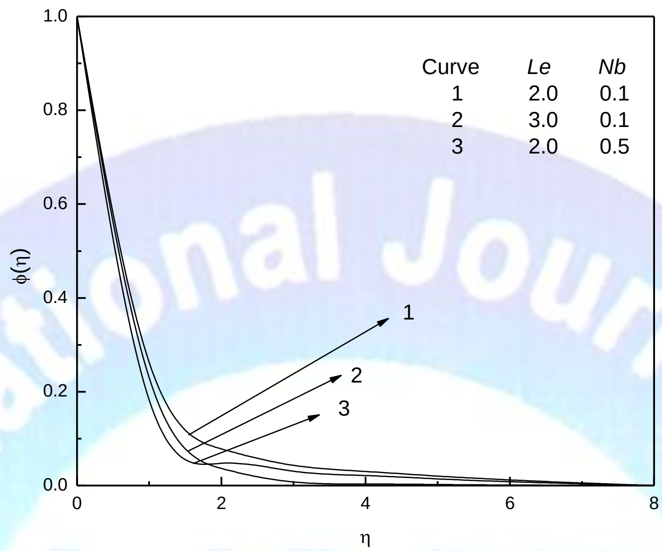

Fig.13 Concentration profiles for different values of *Le* and *Nb*

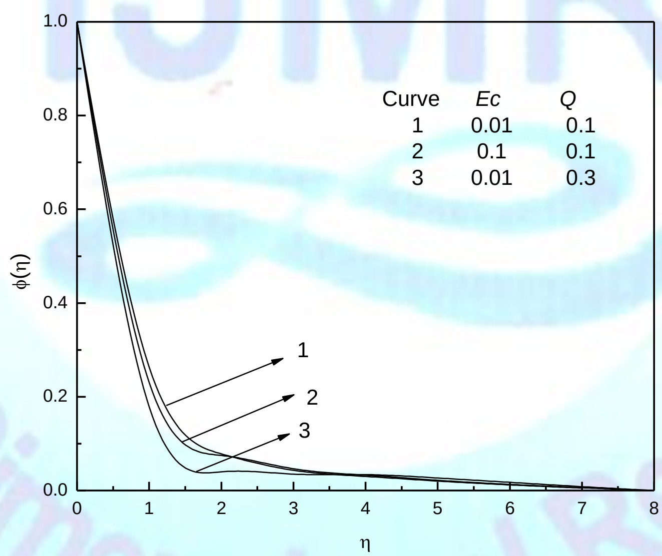

Fig.14 Concentration profiles for different values of Ec and *Q*

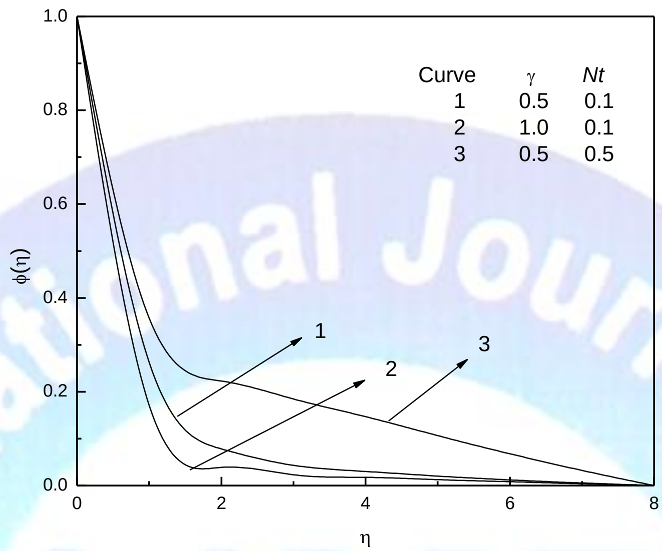

Fig.15 Concentration profiles for different values of γ and *Nt*

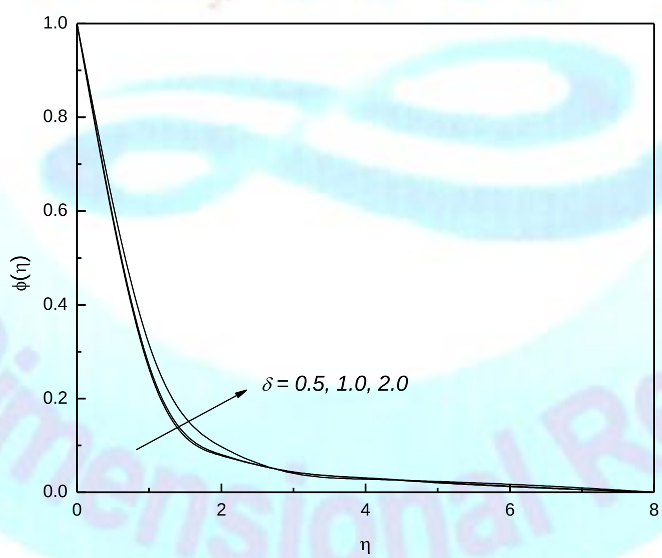

Fig.16 Concentration profiles for different values of δ

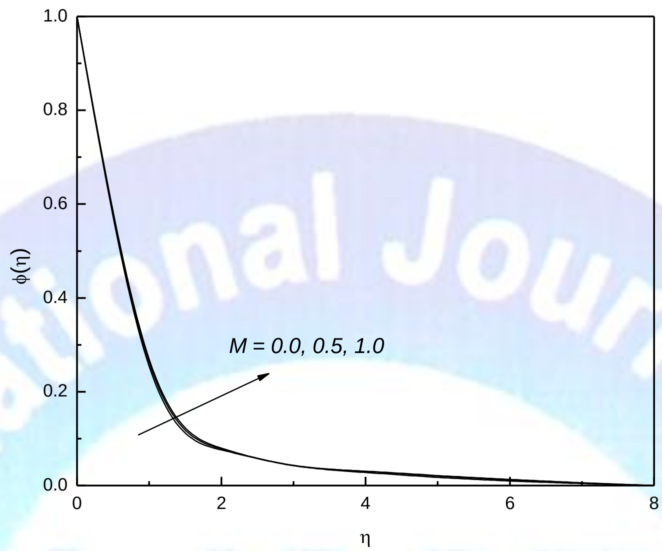

Fig.17 Concentration profiles for different values of *M*

Table 1 Comparison of $-\theta'(0)$ for Pr and n values when λ = δ = Nb = Nt = M = K = Q = Ec = γ = 0.

Pr	*n*	*Ec*	Rana and Bhargava [8]	Cortell [9]	Zaimi et al.[10]	Mabood et al.[36]	Present
1	0.2	0.0	0.6113	0.610262	0.61131	0.61131	0.61121
5	0.2	0.0	1.5910	1.607175	1.60757	1.60757	1.60748
1	0.2	0.1	-	0.574985		0.57528	0.57523
1	0.5	0.1	-	0.556623	-	0.55679	0.55639
5	10	0.1	-	1.324772	-	1.32538	1.32514

Table 2 Comparison of Nusselt number and Sherwood number when Pr = Le = 2 and M = Ec = Q = λ = δ = γ = 0.

n	Nt	Nb	Rana and Bhargava [8]		Mabood et al. [36]		Prsent	
			$-\theta'(0)$	$-\phi'(0)$	$-\theta'(0)$	$-\phi'(0)$	$-\theta'(0)$	$-\phi'(0)$

0.2	0.1	0.5	0.5160	0.9012	0.5148	0.9014	0.5128	0.9007
3.0	0.1	0.5	0.4864	0.8445	0.4852	0.8447	0.4842	0.8432
10.0	0.1	0.5	0.4799	0.8323	0.4788	0.8325	0.4710	0.8301
10.0	0.5	0.5	0.3739	0.7238	0.3728	0.7248	0.3705	0.7215

Table 3 Comparison of Nusselt number ($-\theta'(0)$) when
Nb = Nt = M = Ec = λ = δ = Q = $\lambda = \delta = \gamma$ = 0.

Pr	Khan and Pop [5]	Wang [39]	Gorla and Sidawi [40]	Mabood et al. [36]	Present
0.07	0.0663	0.0656	0.0656	0.0665	0.0661
0.20	0.1691	0.1691	0.1691	0.1691	0.1690
0.70	0.4539	0.4539	0.4539	0.4539	0.4537
7.00	1.8954	1.8954	1.8905	1.8954	1.8952
20.00	3.3539	3.3539	3.3539	3.3539	3.3538
70.00	6.4621	6.4622	6.4622	6.4622	6.4621

Table 4 Calculation of skin-friction coefficient, Nusselt number and Sherwood number when
Pr = 0.71, R = 1.0, Q = 0.1, Ec = 0.01, Le = 2.0 and γ = 0.5.

λ	δ	*M*	*K*	*n*	*Nb*	*Nt*	$f''(0)$	$-\theta'(0)$	$-\phi'(0)$
0.5	0.5	0.5	0.5	2.0	0.1	0.1	-1.29678	0.260038	1.10257
1.0							-1.01555	0.288606	1.13064
2.0							-0.505073	0.33044	1.17554
	1.0						-1.48407	0.24861	1.08722
	2.0						-1.88186	0.221503	1.05096
		0.7					-1.37262	0.254558	1.09611
		1.0					-1.47961	0.247261	1.08717
			0.7				-1.37262	0.254558	1.09611
			1.0				-1.47961	0.247261	1.08717
				3.0			-1.41444	0.249425	1.09201
				4.0			-1.47765	0.288856	1.08244
					0.5		-1.30012	0.267206	1.13051
					1.0		-1.29476	0.229841	1.13522
						0.5	-1.32917	0.274569	1.05287
						1.0	-1.32971	0.248691	1.16142

Table 5 Calculation of skin-friction coefficient, Nusselt number and Sherwood number when *λ =0.5, δ =0.5, M = 0.5, K = 0.5, n = 2.0, Nb = 0.1 and Nt = 0.1.*

Pr	R	Q	Ec	Le	γ	$f''(0)$	$-\theta'(0)$	$-\phi'(0)$
0.71	1.0	0.1	0.01	2.0	0.5	-1.29678	0.260038	1.10257
1.0						-1.30907	0.286449	1.08545
3.0						-1.37851	0.456041	0.972252
	2.0					-1.28648	0.238922	1.11627
	3.0					-1.28128	0.228665	1.12296
		0.5				-1.2304	0.002851	1.28091
		1.0				-1.06009	-0.666422	1.68288
			0.1			-1.29507	0.243829	1.11632
			0.3			-1.2913	0.208124	1.14656
				3.0		-1.27789	0.262299	1.30159
				5.0		-1.25288	0.264601	1.65353
					1.0	-1.28234	0.261662	1.32566
					2.0	-1.26388	0.26348	1.6765

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