Fuzzy Inventory Model for Slow and Fast Growing Items with Deterioration Constraints in a Single Period

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Abstract:

In this paper, a fuzzy inventory model with random demand rate for two items with deteriorateconstraints have been proposed. The Lower-Upper (L-U) bud with rhombus fuzzy number is defined and its properties are given. The proposed model is formulated in fuzzy environment.ie., the parameters involved in this model are represented by L-Ubud with rhombus fuzzy number. The agreement index of L-Ubud with rhombus fuzzy number is explained and using this technique the total cost is defuzzified. Maximum inventory level per period is determined. A numerical example is given to illustrate both the proposed crisp model and fuzzy model.

Keywords: random demand rate, L- U bud with rhombus fuzzy number, agreement index technique.
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1. Introduction

Inventory system is one of the main streams of the operations research which is essential in business enterprises and Industries. In business organization, inventory management is one of the major core competencies to compete in the global market place. Deterioration cannot be avoided in business scenarios. Deterioration is defined as change, damage, decay, spoilage, obsolescence and loss of utility or loss of original value in a commodity that results in the decreasing usefulness from the original one product. In real life and global market situations demand and various relevant costs are not exactly known. Often uncertainties may be associated with demand, supply and various relevant costs. In conventional inventory models uncertainties are treated as randomness and are handled by appealing to probability theory. However, in certain situations uncertainties are due to fuzziness and in these cases the fuzzy set theory is applicable. Fuzzy set theory is given in [1,5,9& 11]. Zadeh introduced fuzzy concept in decision making in [12]. Fuzzy goals, costs and constraints are explained in [2].

Deterministic and probabilistic models in inventory control is given in [8]. A heuristic for replenishment for deteriorating items with linear trend in demand is explained in [4]. The inventory model for variable demand and production is given in [3]. Integrating maintenance planning and production scheduling are discussed in [7]. A production inventory model with shortages, fuzzy preparation time and variable production and demand is given in [6&10].

Maize, popularly known as corn is one of the most versatile, emerging cash crops having wider adaptability under varied climate conditions. The maize is grown throughout the year in allstates of our country (India) for various purposes including fodder for animals, food grain, sweet corn, baby corn, green cobs and popcorn. The immature roots of the carrot plant are sometimes harvested simply as the result of crop thinning, but are also grown to small size as a specialty crop. They will act as convenient row makers in the land, so consider inter-cropping them with slow- growing vegetables.

In agricultural field, at the time of harvesting and during supply the rate of deterioration is constant. Throughout the period, the demand rate for slow growing and fast growing are taken as a random variables, which follows gamma distribution, pareto distribution respectively. Hence a fuzzy inventory model for these two items with random demand rate and constant deterioration rate is developed. The parameters are represented by L-Ubudwith rhombus fuzzy number. This model is defuzzified by agreement index method. Finally a numerical example is provided to illustrate both the proposed crisp model and fuzzy model.

2. Assumptions and Notations

The following assumptions and notations are used throughout this paper :

Assumptions :

1. The inventory system pertains two items in a single period.
2. The deterioration rates are constant for both items.
3. The demand rate for slow growing (maize) item is a random variable, which follows gamma distribution.
4. The demand rate for fast growing (baby carrot) item is a random variable, which follows pareto distribution.
5. During the cycle the inventory level depleted due to demand and deterioration.

Notations:

t_1- the time at which the fast grow item is harvested.

t_2, t_3, t_4- the time at which thefast growing item is supplied.

t_5 -the time at which the slow growing item is supplied.

I_{m1}, I_{m2}, I_{m3}, I_{m4}- maximum production level at time t_1,t_2,t_3,t_4 respectively.

$I_1(t)$ - inventory level at any instant of time t, $t_1 \le t \le t_2$.

$I_2(t)$ - inventory level at any instant of time t, $t_2 \le t \le t_3$.

$I_3(t)$ - inventory level at any instant of time t, $t_3 \le t \le t_4$.

$I_4(t)$ - inventory level at any instant of time t, $t_4 \le t \le t_5$.

d_i- demand rate forfastgrowing(babycarrot)and slow growing (maize) itemintheith interval, i=1,2,3,4.

θ_i - deterioration rate for bothitems,i=1,2,3,4.

$\tilde{s}_c$ - setup cost per period.

$\tilde{h}_{c1}$ - fuzzy holding cost for slow growing (maize) item per unit per unit time.

$\tilde{h}_{c2}$ -fuzzy holding cost for fast growing (baby carrot) item per unit per unit time.

$\tilde{d}_{c1}$ - fuzzy deterioration cost for slow growing (maize) item per unit per unit time.

$\tilde{d}_{c2}$ - fuzzy deterioration cost for fast growing (baby carrot) item per unit per unit time.

3. Mathematical Model in Crisp Environment

The proposed inventory model is formulated to minimize the average total cost, which includes setup cost, holding cost, deterioration cost.

The rate of change of the inventory during the following periods are governed by the following differential equations

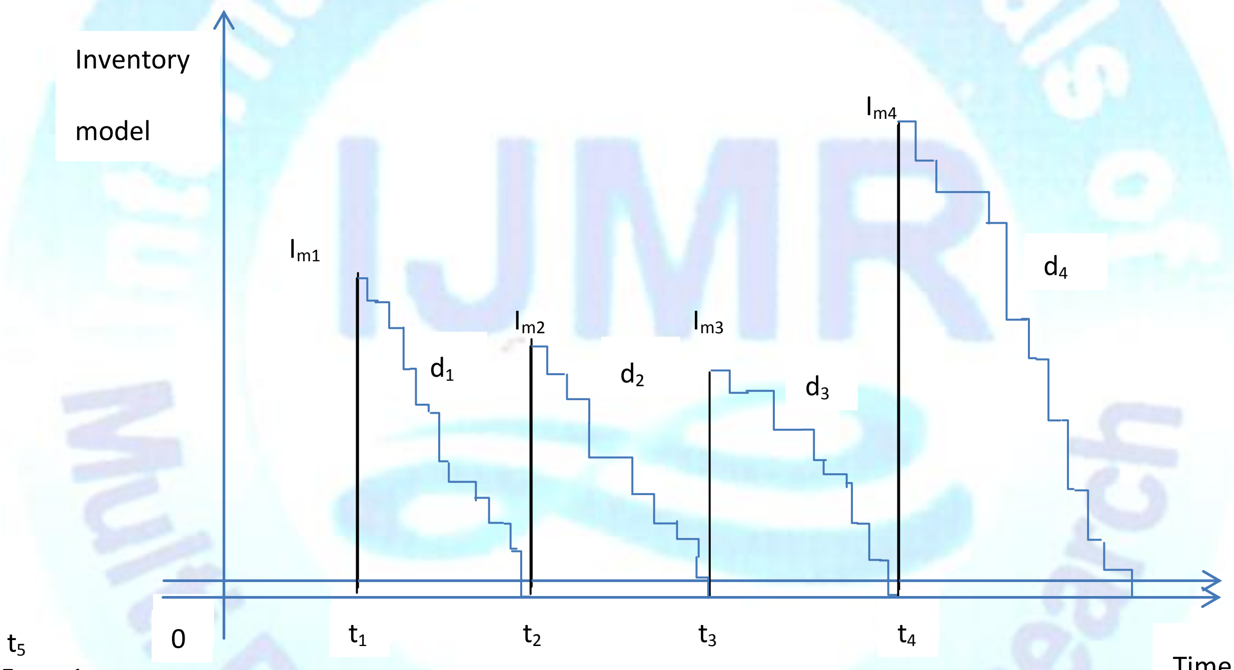
$$\frac{dI_1(t)}{dt}+\theta_1 I_1(t)=-d_1, \qquad t_1 \le t \le t_2 \qquad ----(1)$$

$$\frac{dI_2(t)}{dt}+\theta_2 I_2(t)=-d_2, \qquad t_2 \le t \le t_3 \qquad ----(2)$$

$$\frac{dI_3(t)}{dt}+\theta_3 I_3(t)=-d_3, \qquad t_3 \le t \le t_4 \qquad ----(3)$$

$$\frac{dI_4(t)}{dt}+\theta_4 I_4(t)=-d_4, \qquad t_4 \le t \le t_5 \qquad ----(4)$$

with the boundary conditions $I_1(t_1)= I_{m1}$; $I_2(t_2) = I_{m2}$; $I_3(t_3)=I_{m3}$; $I_4(t_4) = I_{m4}$; $I_1(t_2) =0$; $I_2(t_3) =0$;$I_3(t_4) = 0$; $I_4(t_5) =0$.



From 1,

$$I_1(t) = -\frac{d_1}{\theta_1} + c_1\, e^{-\theta_1 t}; \qquad t_1 \le t \le t_2$$

using boundary condition $I_1(t_2) =0$,

$$I_1(t)=\frac{d_1}{\theta_1}\left(e^{\theta_1(t_2-t)}-1\right) \qquad --------(5)$$

at $t=t_1$,

$$I_{m1} = \frac{d_1}{\theta_1}\left(e^{\theta_1(t_2-t_1)}-1\right) \qquad --------(6)$$

From 2,

$$I_2(t) = -\frac{d_2}{\theta_2} + c_2\, e^{-\theta_2 t}; \qquad t_2 \le t \le t_3$$

using boundary condition $I_2(t_3)=0$,

$$I_2(t)=\frac{d_2}{\theta_2}\left(e^{\theta_2(t_3-t)}-1\right) \qquad --------(7)$$

at $t=t_2$,

$$I_{m2}=\frac{d_2}{\theta_2}\left(e^{\theta_2(t_3-t_2)}-1\right) \qquad --------(8)$$

From 3,

$$I_3(t)=-\frac{d_3}{\theta_3}+c_3\,e^{-\theta_3 t}; \qquad t_3\le t\le t_4$$

using boundary condition $I_3(t_4)=0$,

$$I_3(t)=\frac{d_3}{\theta_3}\left(e^{\theta_3(t_4-t)}-1\right) \qquad --------(9)$$

at $t=t_3$,

$$I_{m3}=\frac{d_3}{\theta_3}\left(e^{\theta_3(t_4-t_3)}-1\right) \qquad --------(10)$$

From 4,

$$I_4(t)=-\frac{d_4}{\theta_4}+c_4\,e^{-\theta_4 t}; \qquad t_4\le t\le t_5$$

using boundary condition $I_1(t_5)=0$,

$$I_4(t)=\frac{d_4}{\theta_4}\left(e^{\theta_4(t_5-t)}-1\right) \qquad --------(11)$$

at $t=t_4$,

$$I_{m4}=\frac{d_4}{\theta_4}\left(e^{\theta_4(t_5-t_4)}-1\right) \qquad --------(12)$$

Expected value of demand rate for slow growing (maize) item

$$E(d)=\frac{\alpha}{\beta},\alpha>0,\beta>0.$$

Expected value of demand for fast growing (baby carrot) item

$$E(d_i)=\frac{a_i m_i}{m_i-1},m_i>1,a_i>0,i=1,2,3.$$

Holding cost

$$=h_{c1}\left[\int_{t_1}^{t_2}I_1(t)\,dt+\int_{t_2}^{t_3}I_2(t)\,dt+\int_{t_3}^{t_4}I_3(t)\,dt\right]+h_{c2}\left[\int_{t_4}^{t_5}I_4(t)\,dt\right]$$

$$=h_{c1}\left[\frac{d_1}{\theta_1}\left(e^{\theta_1(t_2-t_1)}-1\right)-\frac{d_1}{\theta_1}(t_2-t_1)+\frac{d_2}{\theta_2}\left(e^{\theta_2(t_3-t_2)}-1\right)-\frac{d_2}{\theta_2}(t_3-t_2)+\frac{d_3}{\theta_3}\left(e^{\theta_3(t_4-t_3)}-1\right)-\frac{d_3}{\theta_3}(t_4-t_3)\right]+$$

$$h_{c2}\left[\frac{d_4}{\theta_4}\left(e^{\theta_4(t_5-t_4)}-1\right)-\frac{d_4}{\theta_4}(t_5-t_4)\right]$$

Deteriorating cost

$$= d_{c1}\left[\left(I_{m1}-\int_{t_1}^{t_2} d_1\,dt\right)+\left(I_{m2}-\int_{t_2}^{t_3} d_2 dt\right)+\left(I_{m3}-\int_{t_3}^{t_4} d_3 dt\right)\right]+d_{c2}\left[I_{m4}-\int_{t_4}^{t_5} d\,dt\right]$$

$$= d_{c1}\left[\frac{d_1}{\theta_1}\left(e^{\theta_1(t_2-t_1)}-1\right)-d_1\left(t_2-t_1\right)+\frac{d_2}{\theta_2}\left(e^{\theta_2(t_3-t_2)}-1\right)-d_2\left(t_3-t_2\right)+\frac{d_3}{\theta_3}\left(e^{\theta_3(t_4-t_3)}-1\right)-d_3\left(t_4-t_3\right)\right]+$$

$$d_{c2}\left[\frac{d_4}{\theta_4}\left(e^{\theta_4(t_5-t_4)}-1\right)-d_4\left(t_5-t_4\right)\right]$$

Expected total cost E(TC) = Setup cost +E(Holding cost) +E(Deteriorating cost)

$$Min\,E\left(TC\right)=s_c+h_{c1}\left[\begin{array}{l}\frac{\left(a_1m_1/\left(m_1-1\right)\right)}{\theta_1}\left(e^{\theta_1(t_2-t_1)}-1\right)-\frac{\left(a_1m_1/\left(m_1-1\right)\right)}{\theta_1}\left(t_2-t_1\right)+\\ \frac{\left(a_2m_2/\left(m_2-1\right)\right)}{\theta_2}\left(e^{\theta_2(t_3-t_2)}-1\right)-\frac{\left(a_2m_2/\left(m_2-1\right)\right)}{\theta_2}\left(t_3-t_2\right)+\\ \frac{\left(a_3m_3/\left(m_3-1\right)\right)}{\theta_3}\left(e^{\theta_3(t_4-t_3)}-1\right)-\frac{\left(a_3m_3/\left(m_3-1\right)\right)}{\theta_3}\left(t_4-t_3\right)\end{array}\right]+$$

$$h_{c2}\left[\frac{\left(a/m\right)}{\theta_4}\left(e^{\theta_4(t_5-t_4)}-1\right)-\frac{\left(a/m\right)}{\theta_4}\left(t_5-t_4\right)\right]+$$

$$d_{c1}\left[\begin{array}{l}\frac{\left(a_1m_1/\left(m_1-1\right)\right)}{\theta_1}\left(e^{\theta_1(t_2-t_1)}-1\right)-\left(a_1m_1/\left(m_1-1\right)\right)\left(t_2-t_1\right)+\\ \frac{\left(a_2m_2/\left(m_2-1\right)\right)}{\theta_2}\left(e^{\theta_2(t_3-t_2)}-1\right)-\left(a_2m_2/\left(m_2-1\right)\right)\left(t_3-t_2\right)+\\ \frac{\left(a_3m_3/\left(m_3-1\right)\right)}{\theta_3}\left(e^{\theta_3(t_4-t_3)}-1\right)-\left(a_3m_3/\left(m_3-1\right)\right)\left(t_4-t_3\right)\end{array}\right]+$$

$$d_{c2}\left[\frac{\left(a/m\right)}{\theta_4}\left(e^{\theta_4(t_5-t_4)}-1\right)-\left(a/m\right)\left(t_5-t_4\right)\right]$$

Subject to

$$\theta_i\left(t_{i+1}-t_i\right)\le\frac{I_{mi}}{k_i},0<k_i<1,\ i=1,\ 2,3,4. \qquad -------(13)$$

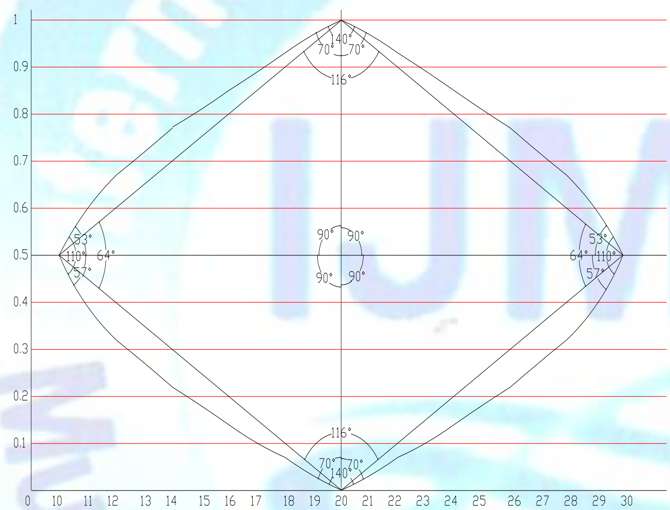
By usingQuadratic Programming problem the optimal values of t_1, t_2, t_3, $t_4$and t_5 have been obtained from the expected total cost E(TC(t_1,t_2,t_3,t_4,t_5)).

4. L - Ubud with rhombus Fuzzy Number and its agreement index method

Definition :(L-U bud with rhombus fuzzy number)

A L-U bud with rhombus fuzzy number $\tilde{A}$ described as a normalized convex fuzzy subset on the real line R whose membership function $\mu_{\tilde{A}}(x)$ is defined as follows

$$\mu_{\tilde{A}}(x)=\begin{cases} 0.5 & at \quad x=a,c \\ \frac{1}{2}\left\{1-\left(\frac{x-a}{b-a}\right)^{\frac{n}{n+1}}\right\} & at \quad a\le x\le b \\ \frac{1}{2}\left\{1-\left(\frac{c-x}{c-b}\right)^{\frac{n}{n+1}}\right\} & at \quad b\le x\le c \\ \frac{1}{2}\left\{1+\left(\frac{x-a}{b-a}\right)^{\frac{n}{n+1}}\right\} & at \quad a\le x\le b \\ \frac{1}{2}\left\{1+\left(\frac{c-x}{c-b}\right)^{\frac{n}{n+1}}\right\} & at \quad b\le x\le c \\ 0 \ and \ 1 & at \quad x=b \end{cases}$$ for all $n \geq 0$.



L-U bud with rhombus fuzzy number

This type of fuzzy number is denoted as $\tilde{A}=[a,b,c;w_A]$ where $w_A=0.5$, whose membership function $\mu_{\tilde{A}}(x)$ satisfies the following conditions:

1. $\mu_{\tilde{A}}(x)$ is a continuous mapping from R to the closed interval [0,1].
2. $\mu_{\tilde{A}}(x)$ is a convex function, $\mu_{\tilde{A}}(x)=0,1 \ at \ x=b, \ \mu_{\tilde{A}}(x)=0.5 \ at \ x=a,c.$
3. $\mu_{\tilde{A}}(x)$ is strictly decreasing as well as increasing and continuous function on [a, b] and [b, c].
4. When n=0, the membership function gives the four points (a,0.5), (b,0), (b,1), (c,0.5).
5. The two line joining of the points (a,0.5), (c,0.5) and (b,0), (b,1) are intersect orthogonally.
6. By joining the four points (a,0.5), (b,0), (c,0.5), (b,1), the rhombus is formed.
7. When n > 0, the L-U bud shape is formed by joining the membership values.
8. The opposite angle of the L-U bud curve is decreased in horizontal wise and increased in vertical wise, when the value of n > 0.

Properties:

1.Opposite angles are equal.

2.The horizontal and vertical diagonal bisect each other and meet at 90^0.

3. In the horizontal and vertical diagonal, the base of the adjacent angles are equal.

4.The length of the horizontal diagonal is twice of the length of the vertical diagonal.

5. Agreement index method

Let us define the concept of a fuzzy upper bound. A function $h(x)$; $x \in R$ such that

$$h(x) = \begin{cases} 1 & at \quad x \le x_1 \\ \dfrac{x_2 - x}{x_2 - x_1} & at \quad x_1 \le x \le x_2 \\ 0 & at \quad x \ge x_2 \end{cases}$$

$h(x)$ represent by a fuzzy subset $H \subset R.$ consider a fuzzy number $A \subset R,$ which we call the agreement index of A with regard to H, the ratio being defined as

$$i(A,H) = \frac{area\ of\ \ A \cap H}{area\ of\ \ A} \in [0,1].$$

When **A is nonfuzzy**, that is, $\mu(A) = \begin{cases} 1, & x = n \\ 0, & x \ne n \end{cases}$ then, $i(A,H) = h(n).$

The agreement index of a L-U bud fuzzy number is of the form

When **H isnonfuzzy**, that is,

$h(x) = \begin{cases} 1, & x \le x_1 \\ 0, & x > x_1 \end{cases}$ In this case compute the area of A to the left of x_1.

$$i_G(A,H) = \frac{1}{I_7}(I_1 + I_2 + I_3 + I_4 + I_5 + I_6)$$

where, $I_1 = \left(\dfrac{(c - x_2)(x_2 - x_a)}{x_2 - x_1}\right), I_2 = \left(\dfrac{(x_2 - a)(x_a - x_b)}{x_2 - x_1}\right), I_3 = \left(\dfrac{{x_a}^2 - {x_b}^2 + 2x_2(x_a - x_b)}{2(x_2 - x_1)}\right),$

$$I_4 = \left(\frac{n(c-b)}{2(2n+1)}\right)\left(1 - 2\left(\frac{x_2 - x_a}{x_2 - x_1}\right)\right)^{\frac{2n+1}{n}}, I_5 = \left(\frac{n(a-b)}{2(2n+1)}\right)\left(2\left(\frac{x_2 - x_b}{x_2 - x_1}\right) - 1\right)^{\frac{2n+1}{n}},$$

$$I_6 = \left(\frac{n(a-c)}{2(2n+1)}\right), I_7 = \left(\frac{(c-a)(n+1)}{2n+1}\right)$$

6. Inventory Model in Fuzzy Environment

The proposed inventory model in fuzzy environment is

$$Min\,E\left(\tilde{T}C\left(t_1,t_2,t_3,t_4,t_5\right)\right)=\tilde{s}_c+\tilde{h}_{c1}\left[\begin{array}{l}\frac{\left(a_1m_1/\left(m_1-1\right)\right)}{\theta_1}\left(e^{\theta_1\left(t_2-t_1\right)}-1\right)-\frac{\left(a_1m_1/\left(m_1-1\right)\right)}{\theta_1}\left(t_2-t_1\right)+\\ \frac{\left(a_2m_2/\left(m_2-1\right)\right)}{\theta_2}\left(e^{\theta_2\left(t_3-t_2\right)}-1\right)-\frac{\left(a_2m_2/\left(m_2-1\right)\right)}{\theta_2}\left(t_3-t_2\right)+\\ \frac{\left(a_3m_3/\left(m_3-1\right)\right)}{\theta_3}\left(e^{\theta_3\left(t_4-t_3\right)}-1\right)-\frac{\left(a_3m_3/\left(m_3-1\right)\right)}{\theta_3}\left(t_4-t_3\right)\end{array}\right]+$$

$$\tilde{h}_{c2}\left[\frac{\left(a/m\right)}{\theta_4}\left(e^{\theta_4\left(t_5-t_4\right)}-1\right)-\frac{\left(a/m\right)}{\theta_4}\left(t_5-t_4\right)\right]+$$

$$\tilde{d}_{c1}\left[\begin{array}{l}\frac{\left(a_1m_1/\left(m_1-1\right)\right)}{\theta_1}\left(e^{\theta_1\left(t_2-t_1\right)}-1\right)-\left(a_1m_1/\left(m_1-1\right)\right)\left(t_2-t_1\right)+\\ \frac{\left(a_2m_2/\left(m_2-1\right)\right)}{\theta_2}\left(e^{\theta_2\left(t_3-t_2\right)}-1\right)-\left(a_2m_2/\left(m_2-1\right)\right)\left(t_3-t_2\right)+\\ \frac{\left(a_3m_3/\left(m_3-1\right)\right)}{\theta_3}\left(e^{\theta_3\left(t_4-t_3\right)}-1\right)-\left(a_3m_3/\left(m_3-1\right)\right)\left(t_4-t_3\right)\end{array}\right]+$$

$$\tilde{d}_{c2}\left[\frac{\left(a/m\right)}{\theta_4}\left(e^{\theta_4\left(t_5-t_4\right)}-1\right)-\left(a/m\right)\left(t_5-t_4\right)\right]$$

Subject to

$\theta_i\left(t_{i+1}-t_i\right)\le\frac{I_{mi}}{k_i},0<k_i<1,i=1,\ 2,3,4.$

where ~ represents the fuzzification of the parameters and

$\tilde{s}_c=(s_{c1},s_{c2},s_{c3});\ \tilde{h}_{c1}=(h_{11},h_{12},h_{13});\tilde{h}_{c2}=(h_{21},h_{22},h_{23});\ \tilde{d}_{c1}=(d_{11},d_{12},d_{13});\tilde{d}_{c2}=(d_{21},d_{22},d_{23});$

By using the agreement index of L - Ubudwith rhombus fuzzy number the total cost is defuzzified.

$$Min\,i_G\left(E\left(TC\left(t_1,t_2,t_3,t_4,t_5\right)\right),H\right)=$$

$$i_G\left(s_c,H\right)+i_G\left(h_{c1},H\right)\left[\begin{array}{l}\frac{\left(a_1m_1/\left(m_1-1\right)\right)}{\theta_1}\left(e^{\theta_1\left(t_2-t_1\right)}-1\right)-\frac{\left(a_1m_1/\left(m_1-1\right)\right)}{\theta_1}\left(t_2-t_1\right)+\\ \frac{\left(a_2m_2/\left(m_2-1\right)\right)}{\theta_2}\left(e^{\theta_2\left(t_3-t_2\right)}-1\right)-\frac{\left(a_2m_2/\left(m_2-1\right)\right)}{\theta_2}\left(t_3-t_2\right)+\\ \frac{\left(a_3m_3/\left(m_3-1\right)\right)}{\theta_3}\left(e^{\theta_3\left(t_4-t_3\right)}-1\right)-\frac{\left(a_3m_3/\left(m_3-1\right)\right)}{\theta_3}\left(t_4-t_3\right)\end{array}\right]+$$

$$i_G\left(h_{c2},H\right)\left[\frac{\left(a/m\right)}{\theta_4}\left(e^{\theta_4\left(t_5-t_4\right)}-1\right)-\frac{\left(a/m\right)}{\theta_4}\left(t_5-t_4\right)\right]+$$

$$i_G\left(d_{c1},H\right)\left[\begin{array}{l}\dfrac{\left(a_1m_1/\left(m_1-1\right)\right)}{\theta_1}\left(e^{\theta_1\left(t_2-t_1\right)}-1\right)-\left(a_1m_1/\left(m_1-1\right)\right)\left(t_2-t_1\right)+\\ \dfrac{\left(a_2m_2/\left(m_2-1\right)\right)}{\theta_2}\left(e^{\theta_2\left(t_3-t_2\right)}-1\right)-\left(a_2m_2/\left(m_2-1\right)\right)\left(t_3-t_2\right)+\\ \dfrac{\left(a_3m_3/\left(m_3-1\right)\right)}{\theta_3}\left(e^{\theta_3\left(t_4-t_3\right)}-1\right)-\left(a_3m_3/\left(m_3-1\right)\right)\left(t_4-t_3\right)\end{array}\right]+$$

$$i_G\left(d_{c2},H\right)\left[\frac{\left(a/m\right)}{\theta_4}\left(e^{\theta_4\left(t_5-t_4\right)}-1\right)-\left(a/m\right)\left(t_5-t_4\right)\right]$$

Subject to

$$\theta_i\left(t_{i+1}-t_i\right)\le\frac{I_{mi}}{k_i},0<k_i<1,i=1,2,3,4. \qquad --------(14)$$

By using Quadratic programming problem the optimal values of t_1, t_2, t_3, t_4 and t_5 have been obtained from the expected total cost $i_G(E(TC(t_1,t_2,t_3,t_4,t_5)),H)$.

7. Numerical Example

The Corn is cultivated, it takes time to grownup in three months. In the three months gap some baby carrots are cultivated for every month. The following values of the parameter in proper unit were considered as input for the numerical analysis of the above problem, $\alpha=5,\beta=10,\ a_1=0.4,a_2=0.2,a_3=0.1\ \theta_1=3,\theta_2=1,\theta_3=2,\theta_4=1,k_1=0.2,k_2=0.9,k_3=0.4,k_4=0.6,s_c=1000.$

The datum are taken in fuzzy

$\tilde{h}_{c1}=(1.6,2.0,2.4)$; $\tilde{h}_{c2}=(1.8,2.2,2.6)$; $\tilde{d}_{c1}=(0.13,0.15,0.17)$; $\tilde{d}_{c2}=(0.16,0.19,0.22)$;

Using the matlab software the optimal values of t_1, t_2, t_3, t_4, t_5 are obtained. Substituting the optimal values of t_1, t_2, t_3, t_4, t_5 in equations (6),(8),(10),(12),(13) and (14) the optimal values of I_{m1}, I_{m2}, I_{m3}, I_{m4} and the expected total cost are obtained.The results are compared with crisp value.

Comparison table for the inventory model:

Model	Maize		Baby carrot		t_1* /weeks	t_2* /weeks	t_3* /weeks	t_4* /weeks	t_5* /weeks	I^*_{m1} /kg	I^*_{m2} /kg	I^*_{m3} /kg	I^*_{m4} /kg	E(TC)
	h_{c1}	d_{c1}	h_{c2}	d_{c2}										
Crisp	1.6	0.13	1.8	0.16	8.9993	10.9001	12.7805	14.5206	17.1191	25.89	25.14	20.37	384	5567.2
Crisp	2.0	0.15	2.2	0.19	8.9992	10.9001	12.7806	14.5343	17.1094	25.89	25.14	20.99	372	5696.5
Crisp	2.4	0.17	2.6	0.22	8.9991	10.9001	12.7806	14.5601	17.1097	25.90	25.14	21.64	364	5615.2
Fuzzy	1.85	0.16	2.15	0.17	8.9799	10.8801	12.7706	14.5402	17.1195	25.98	25.15	20.999	376	5538.3

Observation:

From the above table, it should be noted that compare to crisp model, the fuzzy model is very effective method in the sense that the expected total cost is obtained in fuzzy model is less than the crisp model.

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