MODELLING OF A BULK QUEUE SYSTEM IN TRIANGULAR FUZZY NUMBERS USING A- CUT

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ABSTRACT

In this paper we study to model the queue system with batch arrival. The arrival and service rates both are considered fuzzy in nature. The performance measures of the model have been analysed in triangular fuzzy numbers. α – cut and Zadeh extension principle has been applied to transfer the fuzzy queue measures into crisp measures. The numerical example has been given to deduce the efficiency of the model.

Key Words: *Membership function, Poisson and exponential probability distribution, triangular fuzzy numbers,*
α – cut interval analysis, threshold effect, queue characteristics etc.

1. INTRODUCTION

Uncertainty affects decision making and appears in a number of different forms. The concept of information is fully connected with the concept of uncertainty. The general frame work of fuzzy reasoning allows handling much of this uncertainty on employing fuzzy sets, which represent uncertainty in the range [0, 1]. The concept of fuzzy sets was first introduced by Zadeh (1965, 1975). Fuzzy queue model was first of all introduced by Li & Lee (1989). The model further developed by J.J. Buckley (1990), Negi and Lee (1992), S. P. Chenn (2006), Pourdarwish & Shokry (2009), T.P. Singh etal (2011, 2012, 2013), Kusum & T. P.Singh(2012) R. Shrinivasan (2014) etc. under different augmentations and parametric constraints.Recently Meenu etal (2015) analysed fuzzy queue model on batch arrival with threshold effect. The objective of the paper is to model a bulk queue system with FCFS discipline in triangular fuzzy numbers using α – cut representation through DSW algorithm. The evaluation of threshold effect has also been observed.

The fuzzy set decomposes into distinct level points through α – cut technique. The approximate method DSW (Dong Shah Wong) algorithm used to define a membership function of the performance measures in the model. The evaluation of threshold eliminates small membership values to avoid unnecessary computations. The fuzzified exponential distribution has been used in the study. We are of the opinion that the parameters fuzzy arrival rate and fuzzy service rate can be best describe by linguistic term as slow, average, high etc. make the study closer to real world situation.

2. TECHNIQUE HOW TO APPLY α-CUT:

Let us consider a classical single server queueing model infinite calling source and first come first served discipline.

The inter arrival time A and the service time S are described by the following fuzzy sets

$$A = \{(a, \mu_{\tilde{A}}(a)) : a \in X\}$$

$$S = \{(s, \mu_{\tilde{S}}(s)) : s \in Y\}$$

Here X is the classical set of the inter arrival time and Y is the classical set of the service time.

$\tilde{\mu}_A(a)$ is the membership function of the inter arrival time.

$\tilde{\mu}_S(s)$ is the membership function of the service time.

The α – cut of inter arrival time and service time are represented as

$$A(\alpha) = \{a \in X : \tilde{\mu}_A(a) \geq \alpha\}$$

$$S(\alpha) = \{s \in Y : \tilde{\mu}_S(s) \geq \alpha\}$$

Using α – cuts we have to define the membership function $\mu_{\tilde{P}(A,S)}$ as follows

$$\mu_{\tilde{P}(A,S)}(x) = \begin{cases} \dfrac{x - a_1}{a_2 - a_1}, & a_1 \leq x \leq a_2 \\ \dfrac{a_3 - x}{a_3 - a_2}, & a_2 \leq x \leq a_3 \\ 0, & \text{otherwise} \end{cases}$$

2.1 INTERVAL ANALYSIS ARITHMETIC

Let I_1 and I_2 be two interval numbers defined by ordered of real numbers with lower and upper bounds.

$I_1 = [a, b]$, $a \leq b$ & $I_2 = [c, d]$, $c \leq d$.

Define a general arithmetic property with the symbol

$* = [+, -, \times, \div]$ symbolically the operation $I_1 * I_2 = [a, b] * [c, d]$ represents another interval.

The interval calculation depends on the magnitudes and signs of the elements a, b, c, d.

$$[a, b] + [c, d] = [a + c, b + d]$$

$$[a, b] - [c, d] = [a - d, b - c]$$

$$[a, b] \times [c, d] = [\min(ac, ad, bc, bd), max(ac, ad, bc, bd)]$$

$$[a, b] \div [c, d] = [a, b] \times \left[\frac{1}{d}, \frac{1}{c}\right], provided\ 0 \notin [c, d]$$

$$\alpha[a, b] = \begin{cases} [\alpha a, \alpha b] for\ \alpha > 0 \\ [\alpha b, \alpha a] for\ \alpha < 0 \end{cases}$$

2.2 DSW ALGORITHM:

The DSW algorithm consists of the following steps:

- Select a $\alpha\ cut\ value\ where\ 0 \leq \alpha \leq 1$.
- Find the interval in the input membership functions that correspond to this c.

- Using standard binary interval operations compute the interval for the output membership function for the selected α cut level.
- Repeat above steps for different values of α to complete α cut representation of the solution.

3. MODEL DESCRIPTION:

Considering a single service channel with Poisson input of batch size x and fuzzified exponential inter-arrival service model of a queue system with infinite capacity and FCFS service discipline i.e. in notation form the model is of $M^{[x]}/M/1/\infty/FCFS$ type. Let λ_x be arrival rate of Poisson Process of batch size x and c_x be assigned probability. The number of customers in any arrival is a random variable x where $c_x = \frac{\lambda_x}{\lambda}$, where λ is composite arrival rate of all the batches of size x i. e. $\lambda = \sum_{i=1}^{\infty} \lambda_i$.

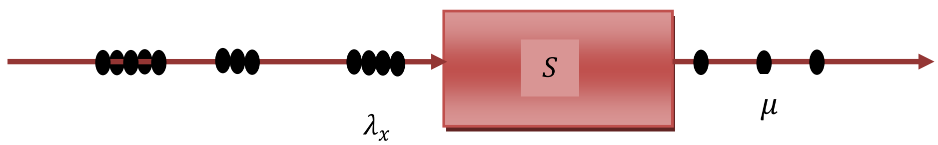

Figure 1: Bulk Queue Model with Single Service Channel

Define $P_n(t)$ probability that there are n units in the system at any instant t. The differential difference equations governing the model can be derived easily using general Birth Death arguments.

$$P_n^{'}(t) = -(\lambda+\mu)P_n(t) + \mu P_{n+1}(t) + \lambda \sum_{k=1}^{n} P_{n-k}(t)c_k \qquad , \qquad n\geq 1$$

$$P_0^{'}(t) = -\lambda P_0(t) + \mu P_1(t) \qquad n=0$$

IN STEADY STATE:

The steady state condition is reached when the behaviour of the system becomes independent of the time. When t→∞ the steady state equations are

$$0=-(\lambda+\mu) P_n + \mu P_{n+1} + \lambda \sum_{k=1}^{n} P_{n-k} c_k \qquad n\geq 1$$

$$0 = -\lambda P_0 + \mu P_1 \qquad n=0$$

The various performance measures of this model are derived by Meenu Mittal etal (2015).

1. The steady state Probability P_n is given by

$$P_n = (1-\rho)\{\alpha + (1-\alpha)\rho\}^{n-1}(1-\alpha)\rho \;\; ; \;\; n \geq 0 \;\; \text{where } \rho = \frac{\lambda}{\mu(1-\alpha)}$$

2. Expected batch size E(x) for two parameters λ_1 & λ_2 is given by

$$E(x) = \frac{(\lambda_1 + 2\lambda_2)}{\lambda}$$

3. Expected queue length L_q for two parameters λ_1 & λ_2 is given by

$$L_q = \left[\frac{\rho_1 + 3\rho_2}{1-\rho_1-2\rho_2}\right] \text{ where } \rho = \rho_1 + 2\rho_2 \; ; \; \rho_1 = \frac{\lambda_1}{\mu}, \rho_2 = \frac{\lambda_2}{\mu}$$

4. Expected batch size E(x) for three parameters λ_1 , λ_2 & λ_3 is given by

$$E(x) = \frac{(\lambda_1 + 2\lambda_2 + 3\lambda_3)}{\lambda}$$

5. Expected queue length for three parameters λ_1 , λ_2 & λ_3 is given by

$$L_q = \left[\frac{\rho_1 + 3\rho_2 + 6\rho_3}{1 - \rho_1 - 2\rho_2 - 3\rho_3}\right] \text{where } \rho = \rho_1 + 2\rho_2 + 3\rho_3 \; ; \; \rho_1 = \frac{\lambda_1}{\mu}, \rho_2 = \frac{\lambda_2}{\mu} \; ; \; \rho_3 = \frac{\lambda_3}{\mu}$$

6. Mean queue length in fuzzy parameters is given by

$$\tilde{L}_q(\alpha) = \left[\frac{\tilde{\rho}_{(1,L)} + 3\tilde{\rho}_{(2,L)}}{1 - \tilde{\rho}_{(1,L)} - 2\tilde{\rho}_{(2,L)}}; \frac{\tilde{\rho}_{(1,U)} + 3\tilde{\rho}_{(2,U)}}{1 - \tilde{\rho}_{(1,U)} - 2\tilde{\rho}_{(2,U)}}\right]$$

7. By using Little's formulae, we may find other effective measures in fuzzy parameters

(a) waiting time in queue:

$$\widetilde{W}_q(\alpha) = \left[\frac{1}{\tilde{\lambda}_U}\left(\frac{\tilde{\rho}_{(1,L)} + 3\tilde{\rho}_{(2,L)}}{1 - \tilde{\rho}_{(1,L)} - 2\tilde{\rho}_{(2,L)}}\right); \frac{1}{\tilde{\lambda}_L}\left(\frac{\tilde{\rho}_{(1,U)} + 3\tilde{\rho}_{(2,U)}}{1 - \tilde{\rho}_{(1,U)} - 2\tilde{\rho}_{(2,U)}}\right)\right] \text{ Where } \tilde{\lambda} = \sum_{i=1}^{\infty} \tilde{\lambda}_i$$

(b)waiting time in system

$$\widetilde{W}_s(\alpha) = \left[\frac{1}{\tilde{\lambda}_U}\left(\frac{\tilde{\rho}_{(1,L)} + 3\tilde{\rho}_{(2,L)}}{1 - \tilde{\rho}_{(1,L)} - 2\tilde{\rho}_{(2,L)}}\right) + \frac{1}{\tilde{\mu}_U}; \frac{1}{\tilde{\lambda}_L}\left(\frac{\tilde{\rho}_{(1,U)} + 3\tilde{\rho}_{(2,U)}}{1 - \tilde{\rho}_{(1,U)} - 2\tilde{\rho}_{(2,U)}}\right) + \frac{1}{\tilde{\mu}_L}\right]$$

4. NUMERICAL ILLUSTRATIONS:

4.1 For Two Parameters in triangular Fuzzy Numbers:

Consider both arrival and service rates as triangular fuzzy numbers as

$\tilde{\lambda}_1 = (1,2,3,), \tilde{\lambda}_2 = (2,3,4,), \tilde{\mu} = (33,34,35)$ such that

$$\tilde{\rho} = \frac{(\tilde{\lambda}_1 + 2\tilde{\lambda}_2)}{\tilde{\mu}} < 1$$

Applying α-cut, the interval of confidence at possibility level α as

$\tilde{\lambda}_1 = (\alpha + 1, 3 - \alpha), \tilde{\lambda}_2 = (\alpha + 2, 4 - \alpha), \tilde{\mu} = (\alpha + 33, 35 - \alpha), \tilde{\lambda} = (2\alpha + 3, 7 - 2\alpha)$

$$\tilde{\rho}_1 = \left(\frac{\alpha + 1}{35 - \alpha}, \frac{3 - \alpha}{33 + \alpha}\right) ; \tilde{\rho}_2 = \left(\frac{\alpha + 2}{35 - \alpha}, \frac{4 - \alpha}{33 + \alpha}\right)$$

$$\tilde{L}_q(\alpha) = \left(\frac{7 + 4\alpha}{30 - 4\alpha}, \frac{15 - 4\alpha}{22 + 4\alpha}\right)$$

$$\widetilde{W}_q(\alpha) = \left[\left(\frac{1}{7 - 2\alpha}\left(\frac{7 + 4\alpha}{30 - 4\alpha}\right)\right), \left(\frac{1}{2\alpha + 3}\left(\frac{15 - 4\alpha}{22 + 4\alpha}\right)\right)\right]$$

$$\widetilde{W}_s(\alpha) = \left[\left(\frac{1}{7 - 2\alpha}\left(\frac{7 + 4\alpha}{30 - 4\alpha}\right) + \frac{1}{35 - \alpha}\right), \left(\frac{1}{2\alpha + 3}\left(\frac{15 - 4\alpha}{22 + 4\alpha}\right) + \frac{1}{33 + \alpha}\right)\right]$$

With the help of MATLAB we get the following data table by putting various values of α

Table 1: The α –cuts of $\tilde{L}_q$, $\widetilde{W}_q(\alpha)$& $\widetilde{W}_s(\alpha)$ at α values for two parameters

S. No.	α	$\tilde{L}_q(\alpha)$ $[\tilde{L}_{qlower}, \tilde{L}_{qupper}]$	$\widetilde{W}_q(\alpha)$ $[\widetilde{W}_{qlower}, \widetilde{W}_{qupper}]$	$\widetilde{W}_s(\alpha)$ $[\widetilde{W}_{s\ lower}, \widetilde{W}_{s\ upper}]$
1	0	[0.2333, 0.6818]	[0.0333,0.2273]	[0.0619 ,0.2576]
2	0.1	[0.2500, 0.6518]	[0.0368, 0.2037]	[0.0654, 0.2339]
3	0.2	[0.2671, 0.6228]	[0.0405,0.1832]	[0.0692, 0.2133]
4	0.3	[0.2847, 0.5948]	[0.0445, 0.1652]	[0.0733, 0.1953]
5	0.4	[0.3028, 0.5678]	[0.0488, 0.1494]	[0.0777, 0.1794]
6	0.5	[0.3214, 0.5417]	[0.0536, 0.1354]	[0.0826, 0.1653]
7	0.6	[0.3406, 0.5164]	[0.0587, 0.1230]	[0.0878, 0.1527]
8	0.7	[0.3603, 0.4919]	[0.0643, 0.1118]	[0.0935, 0.1415]
9	0.8	[0.3806, 0.4683]	[0.0705, 0.1018]	[0.0997, 0.1314]
10	0.9	[0.4015, 0.4453]	[0.0772, 0.0928]	[0.1065, 0.1223]
11	1	[0.4231, 0.4231]	[0.0846, 0.0846]	[0.1140, 0.1140]

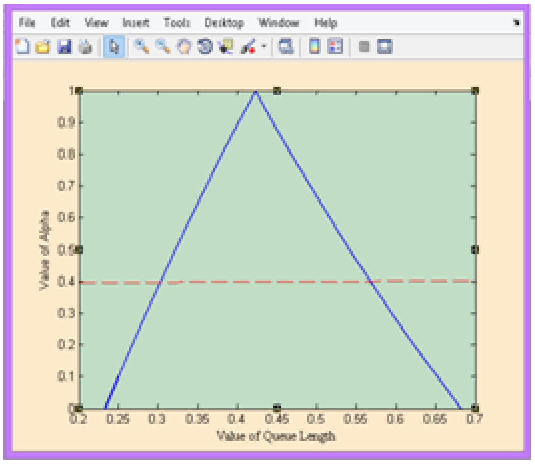

Figure 2: α –cuts of $\tilde{L}_q$

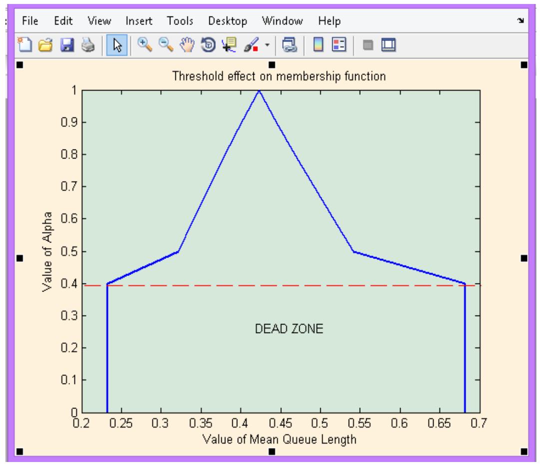

Figure 3: Threshold effect for $\tilde{L}_q$

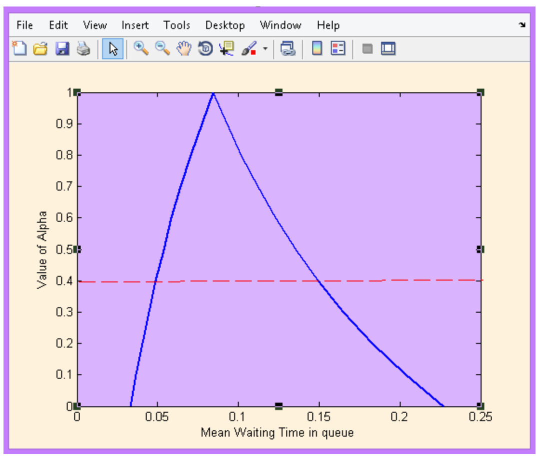

Figure 4: α –cuts of $\tilde{W}_q$

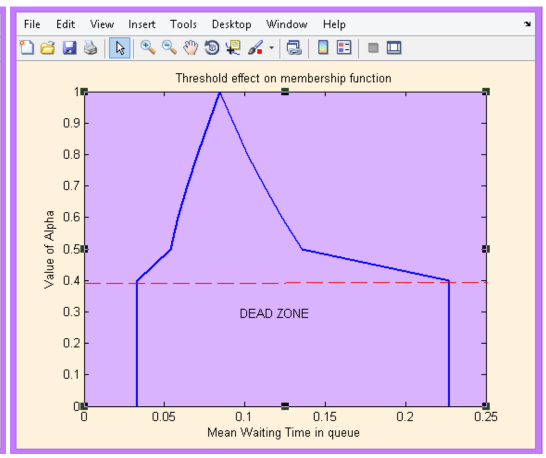

Figure 5: Threshold effect for $\tilde{W}_q$

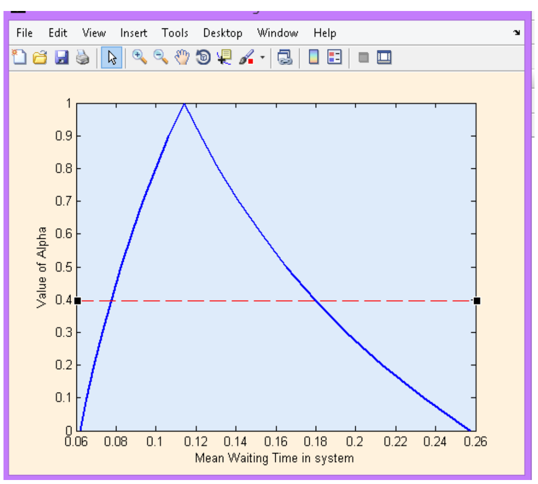

Figure 6: α –cuts of $\tilde{W}_s$

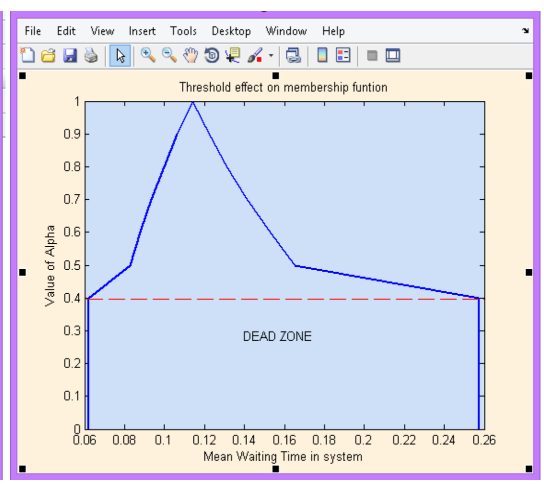

Figure 7: Threshold effect for $\tilde{W}_s$

4.2 For Three Parameters in Triangular Fuzzy Numbers:

Consider $\tilde{\lambda}_1 = (1,2,3,), \tilde{\lambda}_2 = (2,3,4,),\ \tilde{\lambda}_3 = (3,4,5)\ , \tilde{\mu} = (35,36,37)$

Such that $\tilde{\rho} = \frac{(\tilde{\lambda}_1+2\tilde{\lambda}_2+3\tilde{\lambda}_3)}{\tilde{\mu}} < 1$

Applying α-cut, we get,

$\tilde{\lambda}_1 = (\alpha+1, 3-\alpha), \tilde{\lambda}_2 = (\alpha+2, 4-\alpha), \tilde{\lambda}_3 = (\alpha+3, 5-\alpha), \tilde{\lambda} = (3\alpha+6, 12-3\alpha),$

$\tilde{\mu} = (\alpha+35, 37-\alpha)$

$$\tilde{\rho}_1 = \left(\frac{\alpha+1}{37-\alpha}, \frac{3-\alpha}{35+\alpha}\right);\tilde{\rho}_2 = \left(\frac{\alpha+2}{37-\alpha}, \frac{4-\alpha}{35+\alpha}\right);\tilde{\rho}_3 = \left(\frac{\alpha+3}{37-\alpha}, \frac{5-\alpha}{35+\alpha}\right)$$

$$\tilde{L}_q(\alpha) = \left(\frac{25+10\alpha}{23-7\alpha}, \frac{45-10\alpha}{7\alpha+9}\right)$$

$$\tilde{W}_s(\alpha) = \left[\left(\frac{1}{12-3\alpha}\left(\frac{25+10\alpha}{23-7\alpha}\right) + \frac{1}{37-\alpha}\right), \left(\frac{1}{3\alpha+6}\left(\frac{45-10\alpha}{7\alpha+9}\right) + \frac{1}{35+\alpha}\right)\right]$$

With the help of MATLAB we get the following data table by putting various values of α

Table- 2: The α –cuts of $\tilde{L}_q$ & $\tilde{W}_s(\alpha)$ at α values for three parameters

S. No.	α	$\tilde{L}_q(\alpha)$	$\tilde{W}_q(\alpha)$	$\tilde{W}_s(\alpha)$
1	0	[1.0870, 5.0000]	[0.0906,0.8333]	[0.1176, 0.8619]
2	0.1	[1.1659, 4.5361]	[0.0997, 0.7200]	[0.1268, 0.7485]
3	0.2	[1.2500, 4.1346]	[0.1096, 0.6265]	[0.1368, 0.6549]
4	0.3	[1.3397, 3.7838]	[0.1207, 0.5484]	[0.1479, 0.5767]
5	0.4	[1.4356, 3.4746]	[0.1329, 0.4826]	[0.1603,0.5108]
6	0.5	[1.5385, 3.2000]	[0.1465, 0.4267]	[0.1739, 0.4548]
7	0.6	[1.6489, 2.9545]	[0.1617, 0.3788]	[0.1891,0.4069]
8	0.7	[1.7680, 2.7338]	[0.1786,0.3375]	[0.2061, 0.3655]
9	0.8	[1.8966, 2.5342]	[0.1976, 0.3017]	[0.2252, 0.3296]
10	0.9	[2.0359, 2.3529]	[0.2189, 0.2705]	[0.2466, 0.2983]
11	1	[2.1875, 2.1875]	[0.2431, 0.2431]	[0.2708, 0.2708]

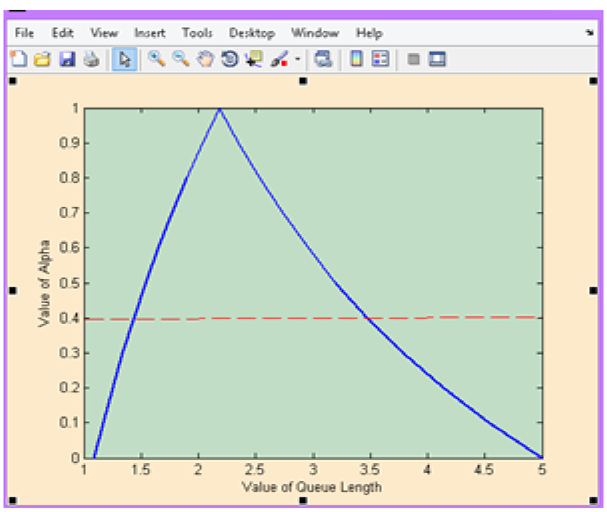

Figure 8: α –cuts of $\tilde{L}_q$

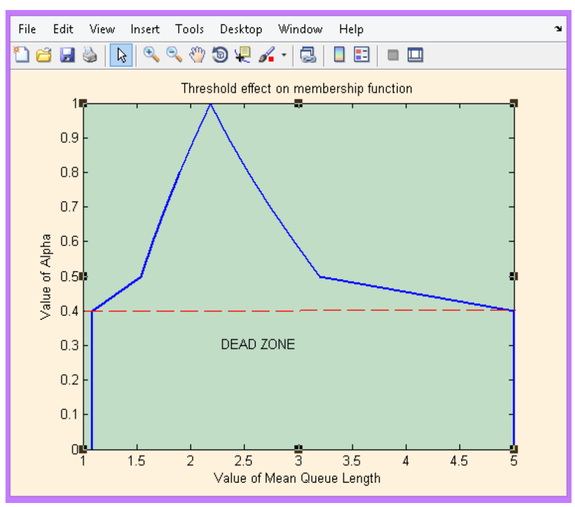

Figure 9: Threshold effect for $\tilde{L}_q$

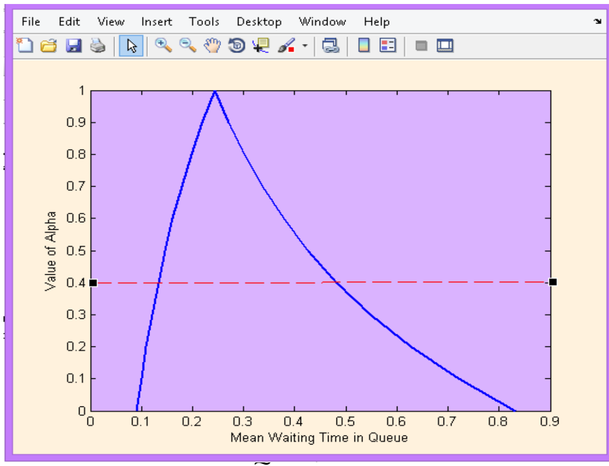

Figure 10: α –cuts of $\tilde{W}_q$

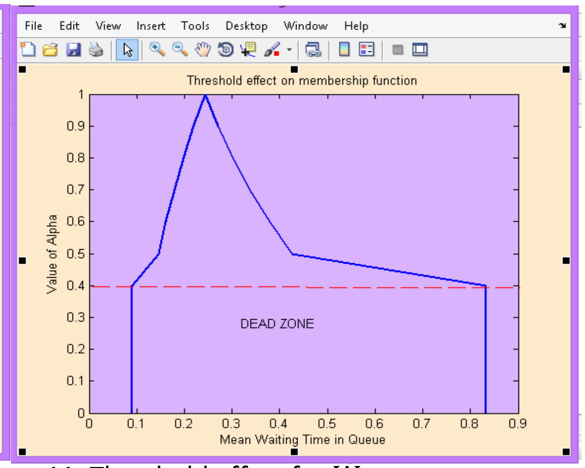

Figure 11: Threshold effect for W_q

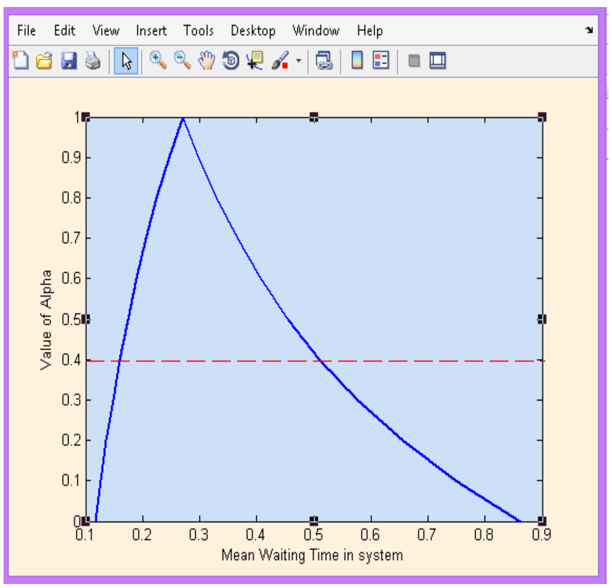

Figure 12: α –cuts of $\tilde{W}_s$

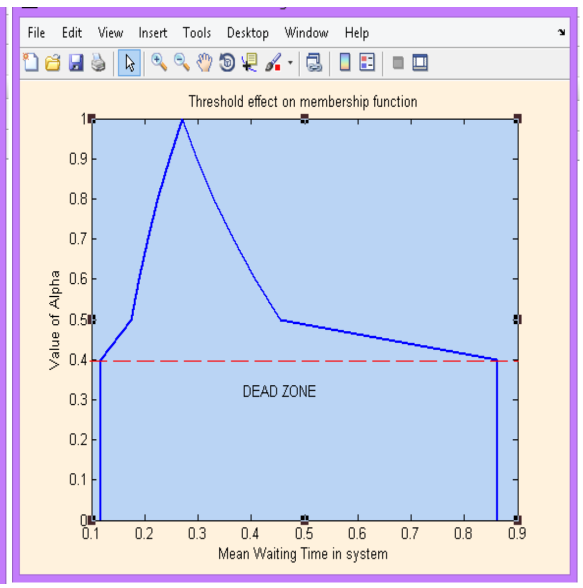

Figure 13: Threshold effect for $\tilde{W}_s$

5. ANALYTICAL STUDY OF THE MODEL:

In this model, we performed α-cuts with respect to arrival rates, service rates, expected number of customers in queue, waiting time in queue as well as waiting time in system at eleven distinct α – levels: 0, 0.1,0.2,0.3,0.4................................0.9 & 1. The crisp interval for fuzzy expected number of customer in queue, mean waiting time in queue and mean waiting time in system at different possibilities of α – level for two parameters as well as three parameters have been shown in table 1 & table 2 respectively. The data have been presented through Graphical diagrams in which threshold effect on membership functions have also been evaluated. Threshold effect creates a dead zone as shown in graphs. From table 1 mean number of customers in queue is 0.4231 and the impossibility value falls outside interval [0.2333, 0.6818] for two parameters while from table 2 for three parameters, mean number of customers in queue is 2.1875 and the impossibility value falls outside interval [1.0870, 5.0000] for three parameters. These results are beneficial for designing a queueing system.

6. CONCLUSION:

We find that when arrival rates $\tilde{\lambda}_1$, $\tilde{\lambda}_2$, $\tilde{\lambda}_3$ and service rate $\tilde{\mu}$ are in triangular fuzzy numbers, the performance measures in the system are expressed by fuzzy number that completely conserve the fuzziness of input information. From the tables & diagrams, it is clear that as the values of α increase, the lower bound of queue length as well as waiting time in queue as well as in system increase and corresponding upper bound decreases and therefore, uncertainty decreases and the value becomes crisp value. The evaluation of threshold through α-cut avoid unnecessarily computations in order to eliminate small membership function (value).The effect of threshold creates dead zone and the α-cut thresh hold creates a sudden drop in evaluation of membership functions that is not desired. To avoid a sudden drop, we can suggest to a non linear threshold where the membership value goes down to zero in a slower pace. With these data the value of α=.4 can be considered as the most appropriate value where cut can be applied.

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