SUPERFICIAL CUTTING ALGORITHM : OPTIMIZATION FOR THE USE OF MATERIAL AND MINIMIZATION OF WASTE WITHIN MANUFACTURING COMPANIES WITH REQUIREMENTS OF DETERMINED RECTANGULAR CUTTING PIECES

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ABSTRACT

This document of scientific means, goes all over the new methodology of the innovative algorithm of superficial cuts used to solve problems related to the optimization of materials and reduction of waste in a furniture factory whose business model involves the design of any furniture piece according to the client's wishes. This algorithm was designed and developed by researchers in ITESM Campus Leon, Mexico. This method contains every constraint necessary in order to provide effectiveness regarding the cutting planning process and diminishing its time. The model delivers solutions in real and polynomial time, and the cost of solving it is low since the demo version of the software can be used. This tool could be useful to any other organization from the same industry or with a similar cutting process.

It is intended to find a computational alternative that contributes in the manufacturing of a wooden piece of furniture, in a way that minimizes the waste of material to a maximum of 10% from the total of utilized product, with a low monetary investment. Also, it is expected that the analyzed company will make total use of this tool in no longer than three months.

KEYWORDS
Superficial cutting algorithm, GAMS, cutting-stock problem.

INTRODUCTION

During the process of manufacture of various products made out from glass, wood, paper, aluminum, and even in construction industries, there always have been cutting related problems. As it is known, the cutting process is essential for this kind of industries, but it's the less standardized one. This causes a lot of monetary losses due to material.This is the reason why it has been considered that the process has to be optimized. The reduction of waste is one of the most important objectives of any company. (Yang, 2006)

It was decided to conduct this study of standardization in the process of manual cutting, to improve the utilization of the material and waste reduction. The application of the optimization takes place in a factory dedicated to the design and manufacture of custom furniture, each project involves a unique design, inspired by the tastes and needs of the customer. However, the factory is dedicated to authentic projects where almost everything is majorly handmade, so every project is constantly exposed to human decision makingthat involves common mistakes and almost always leads to monetary losses,generated by the lack of use of resources in an optimal manner.

Initially, it was intended to use the cutting-stock algorithm, but this algorithm only covers linear cuts. A new algorithm was created, which presents a logical argumentation, where the parts that will be cut, are arranged in such way that the material is fully used.

In the next table it's shown some advantages of the superficial cutting algorithm, as some disadvantages of the cutting stock problem.

Advantages Superficial Algorithm	DisadvantagesCutting Stock Problem
The algorithm allows to make superficial cuts.	Cutting-stock uses less iterations.
It is way more efficient respecting of the exploitation of resources, since it is not produced any kind of arbitrary patterns like the cutting stock.	The Cutting-Stock offers a greater simplicity on the interpretation of the analysis of sensibility.
The algorithm results to be more comprehensible for the people who do not have sufficient technical knowledge of Operational Research.	In the of superficial cutting algorithm, the variables could increase considerably as bigger projects emerge demanding more complex and powerful softwares

Table 1.1

In the next table it is shown the advantages of the cutting-stock problem against the disadvantages of the superficial cutting algorithm.

AdvantagesCutting Stock Problem	Disadvantages Superficial Algorithm
The cutting-stock is done with the minimum number of iterations.	In the superficial cutting algorithm exist as many iterations as the number of pieces to cut.
The analysis of sensibility has a simple interpretation.	The interpretation of the variables is a considerably more complicated.

Table 1.2

Through this research it is expected to improve the efficiency and the time of the planning phase of the cutting process which is now implemented at the analyzed company. Nevertheless, it is important to take into account the scope of it. The created algorithm processes just known pieces with rectangular dimensions. Since the most important aspect of this algorithm is the interpretation of the results, the company most have employees capable to manage the software in use (in this case GAMS).

The reason why the optimization software GAMS was used, is because its various advantages, as offering the facility to change to more complex versions, being that the format that the PC program offers is similar to the software of a parallel supercomputer. GAMS offers the possibility to solve multiple versions of one same model of linear programming, non linear programming, mix programming and nonlinear mix programming. Unlike other softwares, GAMS can solve complex programs, than Lingo or Lindo, because the software allows to deposit parameters, which simplifies the solving of matricial problems. Moreover, the way to deposit data reduces the capturing time, also GAMS can be used as demo for unlimited time.

The objective of this research is to maximize the utility of the company, reduce the waste of raw material and facilitate the arrangement of the parts for the cutting process. Using the superficial cutting algorithm, the cutting process will be standardized, optimizing this procedure for the reduction of wastage. This will allow the company to have a better and more competitive price and to win more clients, aspiring to the diversification of their product, the professionalization of the company and better methods.

METHOD

Subjects

In this research, the measurements from the wooden pieces of a desk were used as a sample. However, the main intention of this model is for it to be replicated with any type of furniture, from any material, in any measurements (only if it applies according to the project scope) by only changing the inputs.
Therefore, this experimental implementation involves using the theory of the algorithm as such and put it into practice with a real piece of furniture, in this case the desk.
The data required was the dimensions from the cutting pieces, the number of pieces needed from every type of cutting piece, and also the dimensions of the sheet of wood from where the pieces will be cut. It is as well important to consider the type of material, since normally, one only piece of furniture may contain different types of wood and other products and every type of material must have a separate model.

Procedure

The algorithm is broken down and described next.

First of all, it is essential set the pieces in order according to their area, the biggest ones go first. This algorithm is somehow manual, since to obtain the most optimal result, it is recommended to make one iteration for every different piece. Once having selected the first piece to start with (it should be the largest one),the total quantity of wood sheets needed to cut that particular piece is calculated as simple as dividing the sheet's area by the cutting piece's area.

As a next step, one should locate the pieces in the cutting sheet. It is intended that the excess area remains in the most uniform way possible, which is why the ideal location of the pieces may allow the excess area to have an "L" shape. This could be achieved if one locates the longer edge of the cutting piece in line with the longer edge of the cutting sheet.

After assigning the first piece, it is important to analyze the remaining area of the cutting sheet. If the area of this surplus or any of its lateral dimensions are smaller than the area and/or the lateral dimensions of the smallest cutting piece, this surplus must be blocked as it is no longer eligible to keep cutting pieces out from it. On the opposite side, if this excess area is large enough it must be divided into at least two areas of rectangular shape. The dimensions of this new areas must be considered in the next iteration as other sheets available to cut.

This cycle must be repeated till all the pieces are done cutting.

This desk has nine pieces. The cutting sheets have a standard dimension of 122x144 cm. The dimensions of the cutting pieces are also in centimeters and can be observed in the following models.

MODEL 1:
SETS

i-number of types of pieces
k- number of available cutting sheets
p- number of surplus areas

PARAMETERS
a_i– area of every piece kind i
T_k–area of every sheet kind k
x_i– edge x of every piece kind i
y_i– edge y of every piece kind i
h_k – edge h of every sheet kind k
b_k– edge b of every sheet kind k
c_i– quantity of pieces kind i required to cut

OBJECTIVE FUNCTION

$$\text{Min } z= \sum_{k=1}^{n} e_k$$

VARIABLES

z

POSITIVE VARIABLES

e_k – excess area from every cutting sheet

W_k –occupied area from every cutting sheet

n_{ik}– quantity of pieces kind i cut out from sheet kind k

$l1_{pk}$ – edge 1 (regarding the base) of remaining area p in sheet k

$l2_{pk}$ –edge 2(regarding the height) of remaining area p in sheet k

f_{pk} – area of the remaining area p in sheet k

BINARY VARIABLES

xh_{ik}– edge x of cutting piece iasigned to edge h of cutting sheet k

yh_{ik}– edge y of cutting piece iasigned to edge h of cutting sheet k

xb_{ik}– edge x of cutting piece iasigned to edge b of cutting sheet k

yb_{ik}– edge y of cutting piece iasigned to edge b of cutting sheet k

CONSTRAINTS

[bin1/ equation binary balance 1]

$xb_{ik}+yb_{ik} = 1$

[bin2/ equation binary balance 2]

$xh_{ik}+yh_{ik} = 1$

[bin3/ equation binary balance 3]

$xh_{ik}-yb_{ik} = 0$

[bin4/ equation binary balance 4]

$xb_{ik}+yh_{ik} = 0$

[cuts/ quantity of cut pieces must be equal to the quantity of required pieces]

$$\sum_{k=1}^{n} n_{ik} = c_i$$

[la1/ edge 1 of remaining area]

$l1_{pk} =b_k– (xb_{ik}* x_i * 1) – (yb_{ik}* y_i* 1)$

[la2/ edge 2 of remaining area

$l2_{pk} =h_k– (xh_{ik}* x_i * 3) – (yh_{ik}* y_i* 3)$

[ar/ remaining area in every sheet k]

$f_{pk}= l1_{pk} * h_k+ (l2_{pk} * (b_k - l2_{pk}))$

[ate/total of remaining area]

$$e_k = \sum_{p=1}^{n} f_{pk}$$

[ato/ total of occupied area]

$$\sum_{i=1}^{n} (a_i * n_{ik}) = W_k$$

[hoj/the total area of the sheet must be equal to the remaining area+ occupied area.]

$T_k =W_k+ e_k$

The base model solved in the software GAMS defines the first iteration in the following way:

EXPERIMENTAL LINEAR PROGRAMMING CODE

```
sets
p/1/
i/1/
k/1/
;


parameters
a(i)  area of cutting piece type i
    /1 2204.24
     /

T(k) area of cutting sheets type k
    /1 29768

     /

x(i)    edge x of cutting piece type i
    /1 47.2
     /

y(i)  edge y of cutting piece type i
    /1 46.7
     /

h(k)  edge h of cutting sheets type k
    /1 244
     /
```

```
b(k)  edge b of cutting sheets type k
    /1 122
     /

C(i) quantity of cutting pieces type i required
    /1 3
     /;

variables
z

positive variables
e(k) remaining area in every sheet
W(k) occupied area in every sheet
n(i,k) quantity of cutting pieces type i cut out from sheet type k
l1(p,k) edge 1 (regarding the base) of remaining area p in sheet k
l2(p,k) edge 2(regarding the height) of remaining area p in sheet k
f(p,k) area of the remaining area p in sheet k


binary variables
xh(i,k)  edge x of cutting piece i asigned to edge h of cutting sheet k
yh(i,k)  edge y of cutting piece i asigned to edge h of cutting sheet k
xb(i,k)  edge x of cutting piece i asigned to edge b of cutting sheet k
yb(i,k)  edge y of cutting piece i asigned to edge b of cutting sheet k
;

equations
FO objective function minimizing the remaining area
```

```
bin1(i,k) equation binary balance 1
bin2(i,k) equation binary balance 2
bin3 (i,k) equation binary balance 3
bin4 (i,k) equation binary balance 4
cortes(i)  quantity of cut pieces must be equal to the quantity of required pieces
la1(i,p,k) edge 1 of remaining area
la2(i,p,k) edge 2 of remaining area
ar(p,k)   remaining area in every sheet k
ate(k)    total of remaining area
ato(k)    total of occupied area
hoj(k)    the total area of the sheet must be equal to the remaining area+ occupied area.

;

FO.. z =e= sum(k,e(k));
bin1(i,k).. xb(i,k)+yb(i,k)  =e= 1;
bin2(i,k).. xh(i,k)+yh(i,k)  =e= 1;
bin3(i,k).. xh(i,k)-yb(i,k)  =e= 0;
bin4(i,k).. xb(i,k)-yh(i,k)  =e= 0;
cortes(i).. sum(k, n(i,k))=e= C(i);
la1(i,p,k)..  l1(p,k)=e= b(k)-(xb(i,k)*x(i)*1)-(yb(i,k)*y(i)*1)  ;
la2(i,p,k)..  l2(p,k)=e= h(k)-(xh(i,k)*x(i)*3)-(yh(i,k)*y(i)*3)  ;
ar(p,k)..   f(p,k)  =e=  l1(p,k)*h(k)+(l2(p,k)*(b(k)-l2(p,k)));
ate(k).. e(k)=e=sum(p,f(p,k));
ato(k).. sum(i,  a(i)*n(i,k))=e=W(k);
hoj(k).. T(k)=e= W(k)+e(k)

model superficie /all/;
```

```
model superficie /all/;

solve superficie using RMINLP minimizing z;

display  z.l, n.l , W.l , e.l ,l1.l,l2.l,xh.l,xb.l,yb.l,yh.l ;
```

Prototype 1.1

.

Afterwards, in the following iterations, the variables l1(p,k) and l2(p,k) will be the dimensions of cutting sheets. The following iterations will proceed in the same way just adding data to the parameters kind k, belonging to the specifications of the cutting sheets.

After analyzing all the following iterations, the final result could be compared with the one from the following model that represents the maximum utilization of the cutting sheets. However, this model does not ensure the good positioning and arrangement of the cutting pieces.

MODEL 2:

SETS

- i-number of types of pieces
- k- number of available cutting sheets
- p- number of surplus areas

PARAMETERS

a_i– area of every piece kind i
T_k–area of every sheet kind k
x_i– edge x of every piece kind i
y_i– edge y of every piece kind i
h_k – edge h of every sheet kind k
b_k– edge b of every sheet kind k
c_i– quantity of pieces kind i required to cut

OBJECTIVE FUNCTION

$$\text{Min } z= \sum_{k=1}^{n} e_k$$

VARIABLES

z

POSITIVE VARIABLES

e_k – excess area from every cutting sheet
W_k –occupied area from every cutting sheet
n_{ik}– quantity of pieces kind i cut out from sheet kind k

CONSTRAINTS

[cuts/ quantity of cut pieces must be equal to the quantity of required pieces]

$$\sum_{k=1}^{n} n_{ik} = c_i$$

[ato/ total of occupied area]

$$\sum_{i=1}^{n}(a_i * n_{ik}) = W_k$$

[hoj/the total area of the sheet must be equal to the remaining area+ occupied area.]

$T_k = W_k + e_k$

[eq/ balance between the cutting and the required pieces]

$n_{ik} < c_i$

[efi/ total efficiency , utilised area vs remaining area]

$$Ef = \frac{\sum_{k=1}^{n} W_k}{\sum_{k=1}^{n} T_k}$$

EXPERIMENTAL LINEAR PROGRAMMING CODE II

```
sets
i/1*9/
k/1*2/
;


parameters
a(i)    area of cutting piece type i
    /1 2204.24
     2 2121
     3 1961.4
     4 1868
     5 1482.68
     6 707
     7 700
     8 303
     9 300
     /

T(k)   area of cutting sheets type k
    /1 29768
     2 29768

     /
```

```
x(i)    edge x of cutting piece type i
  /  1 47.2
     2 42
     3 42
     4 40
     5 36.7
     6 17.5
     7 17.5
     8 7.5
     9 7.5
     /

y(i)    edge y of cutting piece type i
  /  1 46.7
     2 50.5
     3 46.7
     4 46.7
     5 40.4
     6 40.4
     7 40
     8 40.4
     9 40
     /


h(k) edge h of cutting sheets type k
    /1 244
     2 244

     /
```

```
b(k)  edge b of cutting sheets type k
    /1 122
     2 122

     /

C(i)  quantity of cutting pieces type i required
  /  1 3
     2 6
     3 6
     4 1
     5 6
     6 8
     7 8
     8 4
     9 4
     /

;

variables
z

positive variables
e(k)  remaining area in every sheet
W(k) occupied area in every sheet
ef efficiency of utilization of cutting sheets

integer variables
n(i,k) quantity of cutting pieces type i cut out from sheet type k
```

```
equations
FO  objective function minimizing the remaining area
ato(k)    total of occupied area
hoj(k)  the total area of the sheet must be equal to the remaining area+ occupied area
eq(i,k)  balance entre piezas cortadas y requeridas
cortes(i) quantity of cut pieces must be equal to the quantity of required pieces
efi total efficiency of occupied area vs remaining area
;

FO.. z =e= sum(k,e(k));
cortes(i).. sum(k, n(i,k))=e= C(i);
ato(k).. sum(i, a(i)*n(i,k))=e=W(k);
hoj(k).. T(k)=e= W(k)+e(k);
eq(i,k).. n(i,k)=l= C(i);
efi.. ef=e= sum(k, W(k))/ sum(k, T(k)) ;

model superficie /all/;

solve superficie using MIP minimizing z;

display  z.l, n.l , W.l , e.l, ef.l  ;
```

Prototype 2.1

EXPERIMENTAL RESULTS

- Firstiterationmodel 1
COMPILATION TIME = 0.000 SECONDS 3 MB 24.4.6 r52609 WEX-WEI
GAMS 24.4.6 r52609 Released Jun 26, 2015 WEX-WEI x86 64bit/MS Windows 05/04/16 19:41:00 Page 4
General Algebraic Modeling System
Model Statistics SOLVE superficie Using RMINLP From line 90
MODEL STATISTICS
BLOCKS OF EQUATIONS 12 SINGLE EQUATIONS 12
BLOCKS OF VARIABLES 11 SINGLE VARIABLES 11
NON ZERO ELEMENTS 26 NON LINEAR N-Z 1
DERIVATIVE POOL 20 CONSTANT POOL 17
CODE LENGTH 7 DISCRETE VARIABLES 4
GENERATION TIME = 0.016 SECONDS 4 MB 24.4.6 r52609 WEX-WEI
EXECUTION TIME = 0.016 SECONDS 4 MB 24.4.6 r52609 WEX-WEI
General Algebraic Modeling System
Execution
---- 92 VARIABLE z.L = 23155.280
---- 92 VARIABLE n.L quantity of cutting pieces type i cut out from sheet t

ype k
1
1 3.000
---- 92 VARIABLE W.L occupied area in every sheet
1 6612.720

---- 92 VARIABLE e.L remaining area in every sheet
1 23155.280
---- 92 VARIABLE l1.L edge 1 (regarding the base) of remaining area p insh
eet k
1
1 75.300
---- 92 VARIABLE l2.L edge 2(regarding the height) of remaining area p in s
heet k
1
1 102.400
---- 92 VARIABLE xh.L edge x of cutting piece iasigned to edge h of cuttin
g sheet k
1
1 1.000
---- 92 VARIABLE xb.L edge x of cutting piece iasigned to edge b of cuttin
g sheet k
(ALL 0.000)
---- 92 VARIABLE yb.L edge y of cutting piece iasigned to edge b of cuttin
g sheet k
1
1 1.000
---- 92 VARIABLE yh.L edge y of cutting piece iasigned to edge h of cuttin

g sheet k
(ALL 0.000)
EXECUTION TIME = 0.000 SECONDS 3 MB 24.4.6 r52609 WEX-WEI

n(1,1)	W(1)	e(1)	l1(1,1)	l2(1,1)
3	6612.72 cm2	23155.28 cm2	75.3 cm	102.4 cm

Table 1.3

After all the iterations were finished, the final result specified that all the pieces were cut out only from two sheets. Pieces number 1,2, 4, 8 , 9 and six from number 6 will be cut out from sheet number one, and pieces number 3, 5, 7 and the remaining two from number 6 will be cut out from sheet number 2.

RECOMMENDED FINAL CUTTINGUSING SUPERFICIAL ALGORITHM

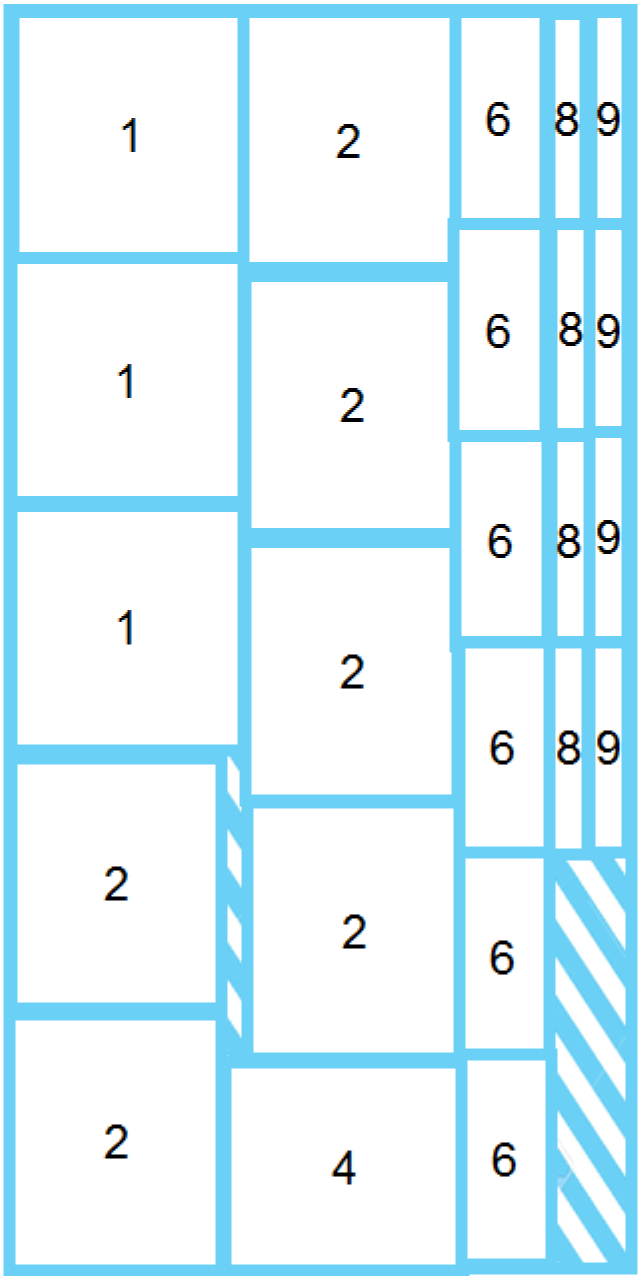


Figure 1.1

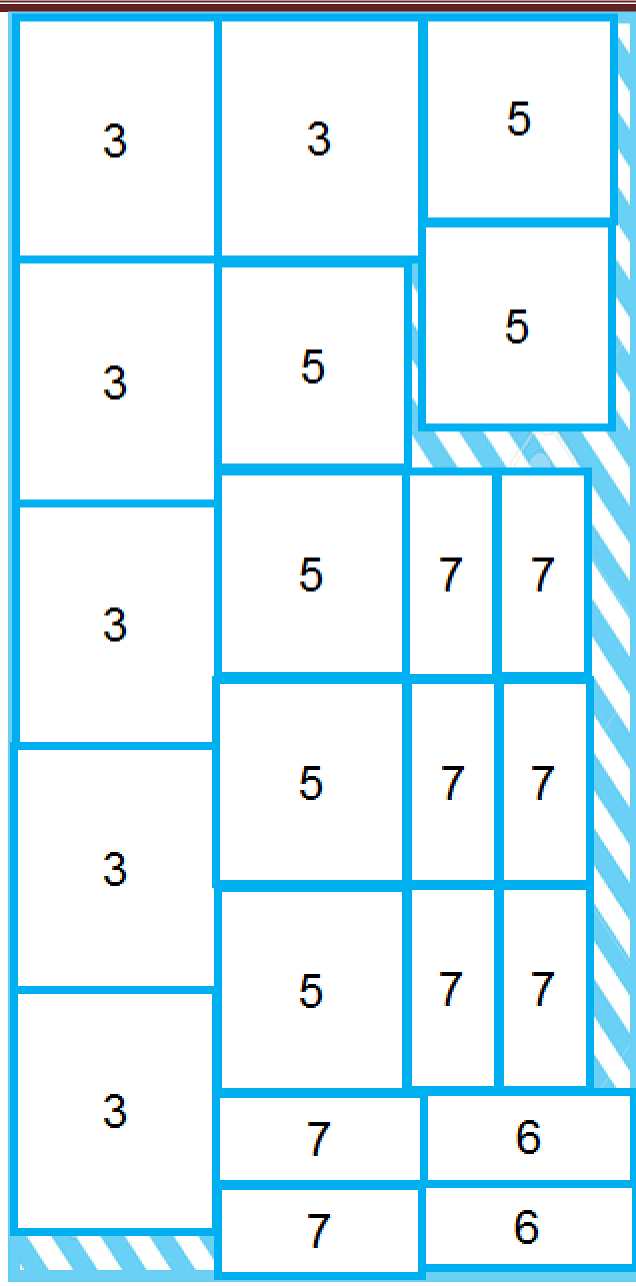


Figure 1.2

- The second model showed the following results:

COMPILATION TIME = 0.000 SECONDS 3 MB 24.4.6 r52609 WEX-WEI

GAMS 24.4.6 r52609 Released Jun 26, 2015 WEX-WEI x86 64bit/MS Windows 05/03/16 11:19:30 Page 4

General Algebraic Modeling System

Model Statistics SOLVE superficie Using MIP From line 109

MODEL STATISTICS

BLOCKS OF EQUATIONS 6 SINGLE EQUATIONS 33
BLOCKS OF VARIABLES 5 SINGLE VARIABLES 24
NON ZERO ELEMENTS 66 DISCRETE VARIABLES 18

GENERATION TIME = 0.016 SECONDS 4 MB 24.4.6 r52609 WEX-WEI

EXECUTION TIME = 0.016 SECONDS 4 MB 24.4.6 r52609 WEX-WEI
G e n e r a l A l g e b r a i c M o d e l i n g S y s t e m
E x e c u t i o n

---- 111 VARIABLE z.L = 3996.800

---- 111 VARIABLE n.L quantity of cutting pieces type i cut out from sheet t
ype k

	1	2
1		3.000
2		6.000
3	6.000	
4		1.000
5	6.000	
6		8.000
7	8.000	
8		4.000
9		4.000

---- 111 VARIABLE W.L occupied area in every sheet

1 26264.480, 2 29274.720

---- 111 VARIABLE e.L remaining area in every sheet

1 3503.520, 2 493.280

--111 VARIABLE ef. =0.933efficiencyof utilization of cutting sheets

Number of pieces kind "i"cut out from sheet kind "k"	
Variable	Results
n(1,1)	0
n(2,1)	0
n(3,1)	6
n(4,1)	0
n(5,1)	6
n(6,1)	0
n(7,1)	8
n(8,1)	0
n(9,1)	0
n(1,2)	3
n(2,2)	6
n(3,2)	0
n(4,2)	1
n(5,2)	0
n(6,2)	8
n(7,2)	0
n(8,2)	4
n(9,2)	4

Table 1.4

Occupied area in sheet k	
Variable	Result
W(1)	26264.48 cm2
w(2)	29274.72 cm2

Table 1.5

Remaining area in sheet k	
Variable	**Result**
e(1)	3503.520 cm2
e(2)	493.280 cm2

Table 1.6
z=3996.8 cm2

This model as well suggests only two cutting sheets, but with a different arrange. Only piece number 6 is distributed in a different way.

RESULTS ANALYSIS

In the first model, the variables that indicate the arrangement and location of the pieces in the cutting sheet or remaining area are the binary variables xh, yh, yb, and xb. This variables will take the number 1 value in the case where that specific edge from the piece must correspond to that specific edge of the sheet. For example, in the previous shown iteration, the variable xh=1 meaning that the edge x of that piece must correspond to the edge h of the cutting sheet.

This particular analysis cannot be observed in the second model, but the ideal result would be that regardless of the pieces arrangement, the total remaining area should be the same in both models. And in this experiment, that was the case since the remaining area was in both of 3996.8 cm2 and both had an efficiency of 93.3%.
Then, the second model could work as a verification of the first one.

For bigger scale problems, meaning problems with a larger number of cutting pieces and different types of furniture, it is important to take into account the following considerations.
First group up all the pieces from the same material. After that, in order to know the ideal number of total sheets to cut, the second model should be used as a first approach. Then, the first model comes to action to provide the best arrangement of the cutting pieces. If there are too many pieces to cut of each type of piece it would be better to look for homogeneity in the cut patterns, saying it is better to have many pieces from one type in one sheet than have them all mixed up. This will help the cutting process to be faster and to look for a uniform remaining area as well.

CONCLUSIONS

- An optimality computational tool was established for the solution of cutting problems within the conditions previously mentioned.
- This tools give results for: the number of pieces that fit in one sheet, starting from the dimensions given, manufactured with wood for the elaboration of office furniture.
- This results are obtained in real polynomial time
- The objective of creating a tool that expedites the cutting problem in an efficiently.
- The tool can be used in small or large scale establishing patterns and rules of operation in the program.
- The tool can be used in any type of industry that requires it.
- Implementation is very useful and so easy to improve to large scale.
- Training to workers and managers to use it would be desirable.
- The cost of implementation artificial algorithm is very competitive.
- The line programming code can spread to another needs or requirements.
- A second step could be to handle materials in different geometric forms like triangle, circle or another forms.

APPENDIX

The following images belong to the analyzed company, so as the design and break down of the shown pieces.

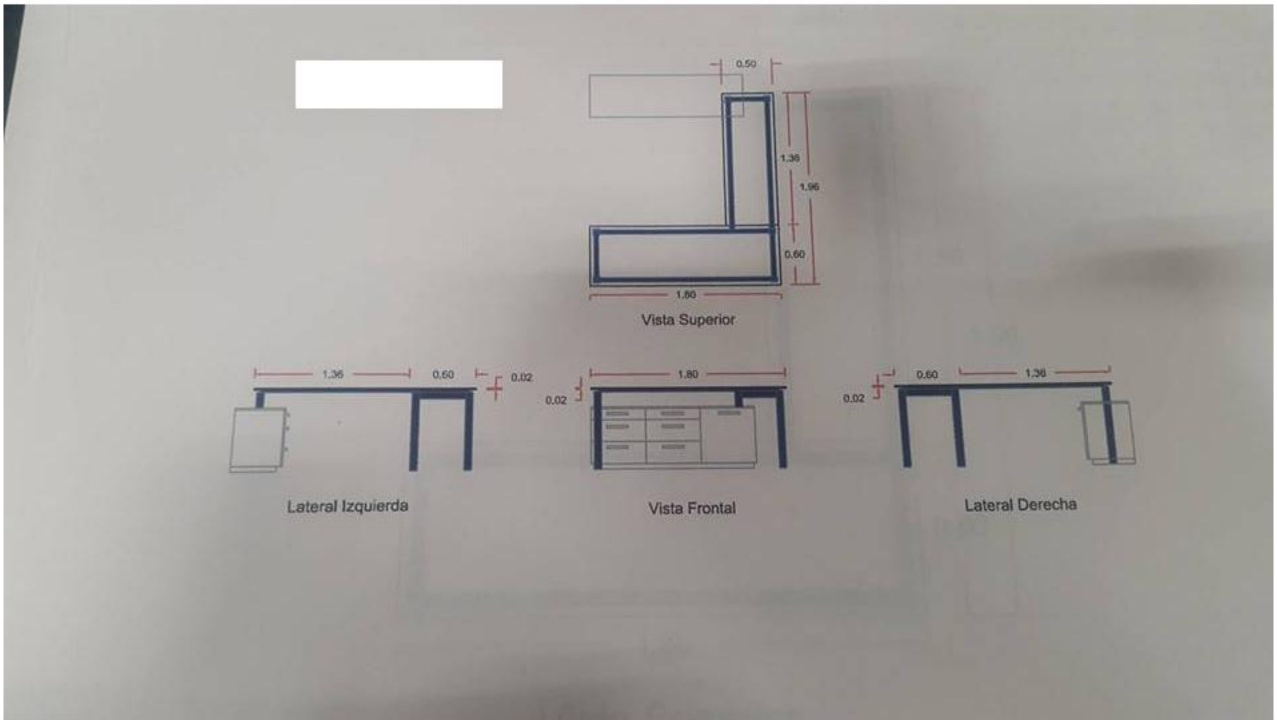
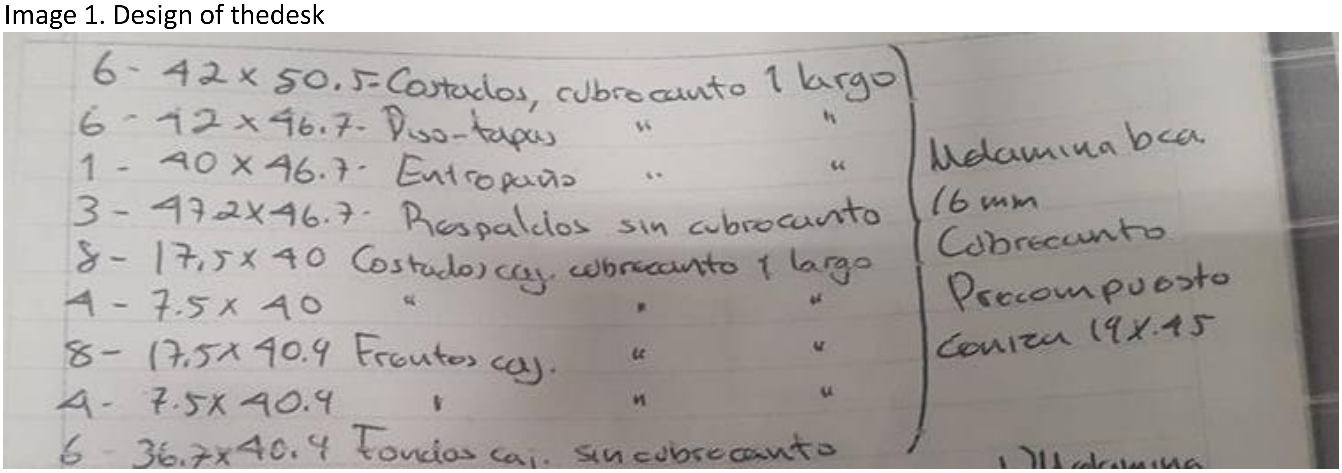


Image 1. Design of thedesk


6 - 42 x 50.5 = Costados, cubrecanto 1 largo
6 - 42 x 46.7 - Piso-tapas " "
1 - 40 x 46.7 - Entrepaños " "
3 - 47.2 x 46.7 - Respaldos sin cubrecanto
8 - 17.5 x 40 Costados caj. cubrecanto 1 largo
4 - 7.5 x 40 " " "
8 - 17.5 x 40.4 Frentes caj. " "
4 - 7.5 x 40.4 " " "
6 - 36.7 x 40.4 Fondos caj. sin cubrecanto

Melamina bca.
16 mm
Cubrecanto
Precompuesto
Ceniza 19 X .45

Image 2. Description of the cutting pieces

Image 3. Cuttingarea

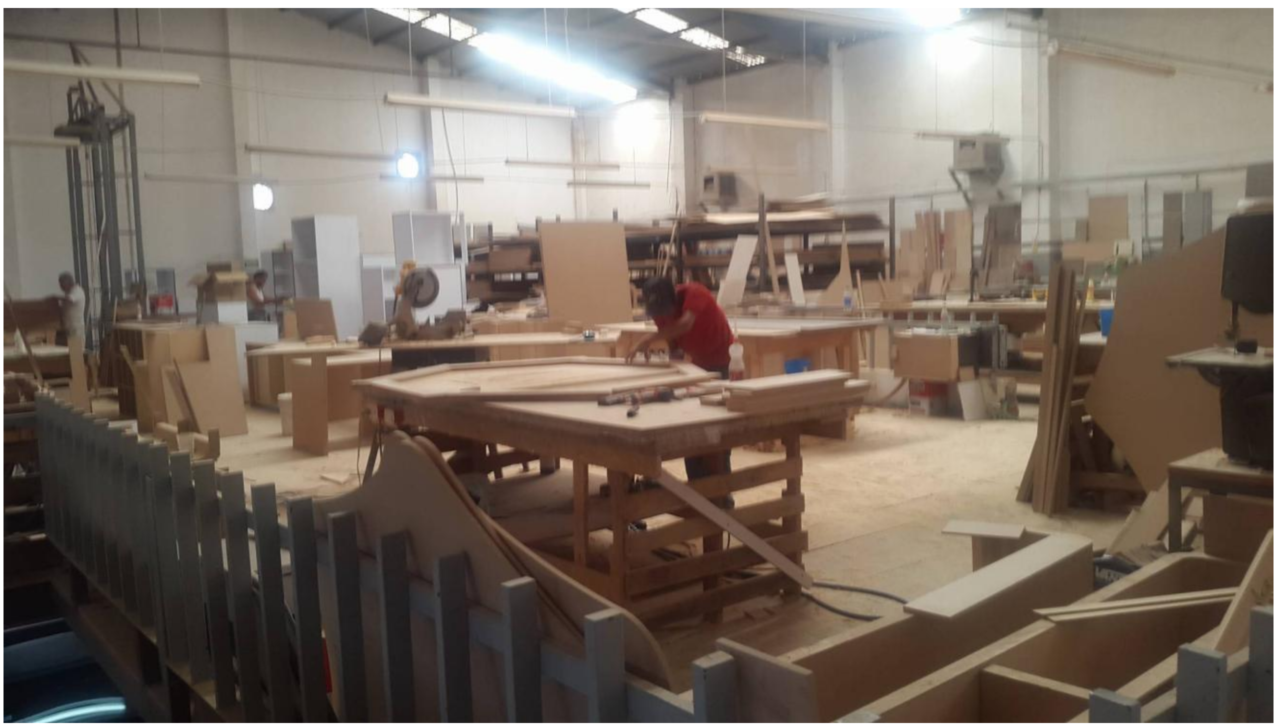


Image 4. Cuttingarea

Imagen 5. Packing area

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